To reach my goal at a more reasonable age, the rules could be changed to allow a larger investment each year. Or, the rules could be changed to allow an investment of $5000 in one 5-year GIC each year, instead of every 5 years.

Lesson 1.4: Compound Interest: Present Value, page 40

1. Investment B will require a greater present value to be invested because the compounding frequency in less than for investment A.

$$P=\frac{A}{(1+i)^n}$$

Investment A: A = 10 000, $i=\frac{0.05}{12}$, n = 120

$$P=\frac{10\ 000}{\left(1+\frac{0.05}{12}\right)^{120}}$$

$P=6071.61$

The present value of investment A is $6071.61.

Investment B: A = 10 000, i = 0.0125, n = 40

$$P=\frac{10\ 000}{(1+0.0125)^{40}}$$

$P=6084.13$

The present value of investment B is $6084.13.

Investment B requires a higher present value.

2. a) Investment A:

$$\frac{A}{P}=\frac{10\ 000}{6071.61}$$

$$\frac{A}{P}=1.647...$$

Investment B:

$$\frac{A}{P}=\frac{10\ 000}{6084.13}$$

$$\frac{A}{P}=1.643...$$

The future value to present value ratio for Investment A is 1.647… and for investment B is 1.643…

b) The investment with annually compounded interest would have a higher ratio because the interest rate is higher and the principal is lower. With a 6% interest rate compounded annually and a future value of $10 000, the present value must be $5583.95. Since the principal is lower than both investment A and B, the ratio will be higher.

3. Row 1: Determine the present value.

$$P=\frac{A}{(1+i)^n}$$

A = 2500, i = 0.078, n = 8

The present value is $1370.85.

Row 2: Determine the annual interest rate.

The present value is $2000.

The annual interest rate is unknown.

The compounding period is semi-annual, or 2 times per year.

The term (in years) is 5.

The future value is $3500.

Using my calculator, the annual interest rate is 11.5%.

Row 3: Determine the present value.

$$P=\frac{A}{(1+i)^n}$$

A = 11 000, i = 0.006, n = 48

The present value is $8254.48.

Row 4: Determine the investment term.

The present value is 609.35.

The annual interest rate is 13.6%.

The compounding period is annual, or once per year.

The term (in years) is unknown.

The future value is $100 000.00.

Using my calculator, the term of the investment is 39.999… or 40 years.

Row 5: Determine the annual interest rate.

The present value is $16 150.00.

The annual interest rate is unknown.

The compounding period is monthly, or 12 times per year.

The term (in years) is 2.

The future value is $23 500.00.

Using my calculator, the annual interest rate is 18.9%.

4. a) $P=\frac{A}{(1+i)^n}$

A = 250 000, i = 0.085, n = 20

$$P=\frac{250\ 000}{(1+0.085)^{20}}$$

$P=48\ 904.097...$

Mac should invest $48 904.10 now to have $250 000 in 20 years.

b) 250 000 – 48 904.10 = 201 095.90

The investment will earn $201 095.90 in interest in 20 years.

5. a) The present value is $9000.

The annual interest rate is unknown.

The compounding period is quarterly, or 4 times per year.

The term (in years) is 2.

The future value is $17 000.

Joseppie would need an annual interest rate of 33.1% to meet his goal. This is not reasonable. Current interest rates for savings accounts are 0.5% to 1.25%.

b) The present value is $9000.
The annual interest rate is 12%.
The compounding period is quarterly, or 4 times per year.
The term (in years) is unknown.
The future value is $17 000.
Using my calculator, it will take Joseppie 5.4 years to have $17 000.

6. $P = \dfrac{A}{(1+i)^n}$

A = 17 500, i = 0.028, n = 20
Claire has to invest $10 073.39 now to have $17 500 in ten years.

7. a) Option A: *The present value is unknown.*
The annual interest rate is 4.80%.
The compounding period is annual, or once per year.
The term (in years) is 6 years.
The future value is $24 000.
$n = 6 \cdot 1 = 6$

$$P = \frac{A}{(1+i)^n}$$

$$P = \frac{24\ 000}{(1+0.0480)^6}$$

$$P = 18\ 115.217...$$

The present value of option A is $18 115.22.
Interest earned: 24 000 – 18 115.22 = 5884.78
Option B: *The present value is unknown.*
The annual interest rate is 4.75%.
The compounding period is semi-annual, or 2 times per year.
The term (in years) is 6 years.
The future value is $24 000.
$n = 6 \cdot 2 = 12$

$$P = \frac{A}{(1+i)^n}$$

$$P = \frac{24\ 000}{(1+0.0475)^{12}}$$

$$P = 18\ 108.574...$$

The present value of option B is $18 108.57.
Interest earned: 24 000 – 18 108.57 = 5891.43
Option C: *The present value is unknown.*
The annual interest rate is 4.70%.
The compounding period is quarterly, or 4 times per year.
The term (in years) is 6 years.
The future value is $24 000.
$n = 6 \cdot 4 = 24$

$$P = \frac{A}{(1+i)^n}$$

$$P = \frac{24\ 000}{(1+0.0470)^{24}}$$

$$P = 18\ 132.351...$$

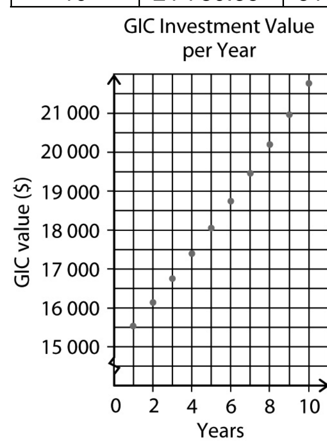
Interest earned: 24 000 – 18 132.35 = 5867.65

Option	Rate of Return
A	$\dfrac{5884.78}{18\ 115.22} = 0.324\ 853...$
B	$\dfrac{5891.43}{18\ 108.57} = 0.325\ 339...$
C	$\dfrac{5867.65}{18\ 132.35} = 0.323\ 601...$

Option B has the greatest rate of return at 32.53%. Sasha should choose option B so that she earns the most interest on her investment.
b) Sasha would earn $5891.43 on her investment by choosing option B.

8. Option A: e.g., Increase the interest rate to 7.6% in part b) and decrease the interest rate to 1.9% in part c).

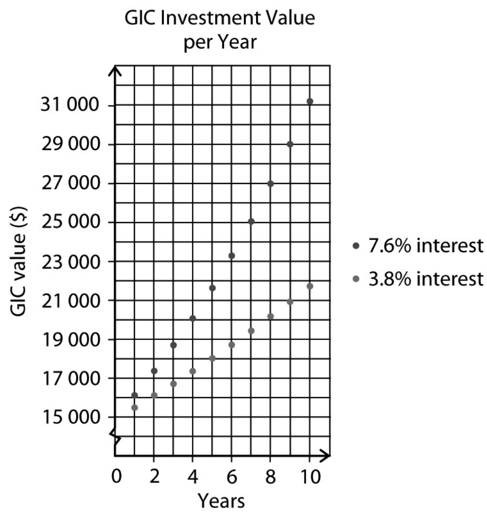
	a)	b)	c)
Principal ($)	15 000	15 000	15 000
Interest Rate per Annum	0.038	0.076	0.019
Periods per Year	1	1	1
Value at End of Year			
2	16 161.66	17 366.64	15 575.42
4	17 413.28	20 106.68	16 172.90
6	18 761.84	23 279.03	16 793.31
8	20 214.83	26 951.90	17 437.52
10	21 780.35	31 204.27	18 106.44



Time (years)	Value of Investment ($)		
	a)	b)	c)
0	15 000	15 000	15 000
2	16 161.66	17 366.64	15 575.42
4	17 413.28	20 106.68	16 172.90
6	18 761.84	23 279.03	16 793.31
8	20 214.83	26 951.90	17 437.52
10	21 780.35	31 204.27	18 106.44

d) The graphs start at the same point. Increasing the interest rate made the graph steeper and it increased faster. Decreasing the interest rate made the graph less steep and it increased more slowly.
Option B: e.g., Decrease the principal to $20 000 in part b) and increase the principal to $30 000 in part c).

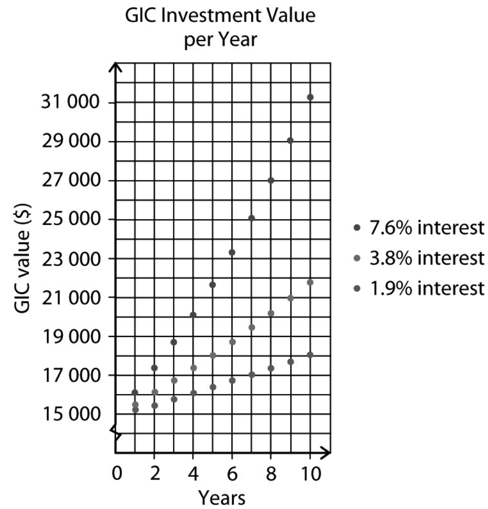
	a)	b)	c)
Principal ($)	26 000	20 000	30 000
Interest Rate per Annum	0.062	0.062	0.062
Periods per Year	2	2	2
Value at End of Year			
1	27 636.99	21 259.22	31 888.83
2	29 377.04	22 597.72	33 896.58
3	31 226.65	24 020.50	36 030.75
4	33 192.71	25 532.85	38 299.28
5	35 282.55	27 140.43	40 710.64



Time (years)	Value of Investment ($)		
	part a)	part b)	part c)
0	26 000	20 000	30 000
1	27 636.99	21 259.22	31 888.83
2	29 377.04	22 597.72	33 896.58
3	31 226.65	24 020.50	36 030.75
4	33 192.71	25 532.85	38 299.28
5	35 282.55	27 140.43	40 710.64

d) e.g., Changing the principal does not change the slope of the graph. Increasing the principal moved the graph up and decreasing the principal moved the graph down.
Option C: e.g., Decrease the interest rate to 3% in part b) and decrease the interest rate to 2% in part c).

	a)	b)	c)
Principal ($)	8000	8000	8000
Interest Rate per Annum	0.041	0.03	0.002
Periods per Year	4	4	4
Value at End of Year			
2	8680.02	8492.79	8325.66
4	9417.85	9015.94	8664.57
6	10 218.39	9571.31	9017.28
8	11 086.99	10 160.89	9384.34



Time (years)	Value of Investment ($)		
	part a)	part b)	part c)
0	8000.00	8000.00	8000.00
2	8680.02	8492.79	8325.66
4	9417.85	9015.94	8664.57
6	10 218.39	9571.31	9017.28
8	11 086.99	10 160.89	9384.34

d) The graphs start at the same point. Decreasing the interest rate made the graphs less steep and they increased more slowly. The distance between points in the same year became larger as time increased.

9. Option C will allow Blake to invest the least and still meet his goal. It had the highest annual interest rate and the second most frequent compounding period.
Option A: *The present value is unknown.*
The annual interest rate is 12.6%.
The compounding period is annual, or once per year.
The term (in years) is 40 years.
The future value is 1 000 000.
$n = 40 \cdot 1 = 40$

$$P = \frac{A}{(1+i)^n}$$

$$P = \frac{1\ 000\ 000}{(1+0.126)^{40}}$$

$$P = 8678.893...$$

The present value of option A is $8678.89.
Option B: *The present value is unknown.*
The annual interest rate is 11.9%.
The compounding period is semi-annual, or 2 times per year.
The term (in years) is 40 years.
The future value is 1 000 000.
$n = 40 \cdot 2 = 80$

$P=\frac{A}{(1+i)^n}$

$P=\frac{1\ 000\ 000}{(1+0.119)^{40}}$

$P=9815.741...$

The present value of option B is $9815.74.

Option C: *The present value is unknown.*

The annual interest rate is 13.2%.

The compounding period is quarterly, or 4 times per year.

The term (in years) is 40 years.

The future value is 1 000 000.

$n = 40 \cdot 4 = 160$

$P=\frac{A}{(1+i)^n}$

$P=\frac{1\ 000\ 000}{(1+0.132)^{160}}$

$P=5545.600...$

The present value of option C is $5545.60.

Option D: *The present value is unknown.*

The annual interest rate is 11.53%.

The compounding period is weekly, or 52 times per year.

The term (in years) is 40 years.

The future value is 1 000 000.

$n = 40 \cdot 52 = 2080$

$P=\frac{A}{(1+i)^n}$

$P=\frac{1\ 000\ 000}{(1+0.11153)^{2080}}$

$P=9982.772...$

The present value of option D is $9982.77.

Option C has the lowest present value, so it is the best option for Blake.

10. Franco made the greater original investment because investments with annual compound interest earn less than investments with monthly compounded interest (and the same annual interest rate).

	Franco	David
Future Value ($)	25 000	25 000
Interest Rate per Annum	0.069	0.069
Periods per Year	1	12
Number of Years	30	30
Present Value ($)	3377.60	3173.40

3377.60 – 3173.40 = 204.20

Franco invested $204.20 more than David.

11. a) The present value is $3000.

The annual interest rate is unknown.

The compounding period is quarterly, or 4 times per year.

The term (in years) is 10.

The future value is $7000.

I used the financial application on my calculator: Lucy needs an annual interest rate of 8.56% to meet her goal.

b) $\frac{A}{P}=\frac{7000}{3000}$

$\frac{A}{P}=2.333...$

e.g., The ratio would decrease if the interest were compounded annually. A lower compounding frequency would reduce the future value but not change the present value, making the ratio smaller.

The present value is $3000.

The annual interest rate is 8.56%.

The compounding period is annual, or once per year.

The term (in years) is 10 years.

$n = 10 \cdot 1 = 10$

The future value is unknown.

$A = P(1 + i)^n$

$A = 3000(1 + 0.0856)^{10}$

$A = 6820.553\ldots$

The future value of the investment with interest compounded annually is $6820.55.

$\frac{A}{P}=\frac{6820.55}{3000}$

$\frac{A}{P}=2.273...$

To two decimal places, the ratio would decrease to 2.27.

12. *The present value is unknown.*

The annual interest rate is 5.3%.

The compounding period is monthly, or 12 times per year.

The term (in years) is 0.75 years.

The future value is $4765.30.

$n = 0.75 \cdot 12 = 9$

$P=\frac{A}{(1+i)^n}$

$P=\frac{4765.30}{(1+0.053)^{9}}$

$P=4579.995...$

The present value of Daniel's investment is $4579.995… or $4580.

4765.30 – 4579.995… = 185.304…

The account has earned $185.30 in interest.

13. *The present value is unknown.*

The annual interest rate is 5.5%.

The compounding period is semi-annual, or 2 times per year.

The term (in years) is 10 years.

The future value is $15 000.

$n = 10 \cdot 2 = 20$

$P=\frac{A}{(1+i)^n}$

$P=\frac{15\ 000}{(1+0.055)^{20}}$

$P=8718.76...$

The present value of the investment is $8718.76.

$$\frac{8718.76}{3} = 2906.25$$

Each sibling will need to contribute $2906.25 to the GIC.

14. e.g., In an investment, you agree to loan a sum of money to another entity (like a company); the amount you loan is called the *present value* of the principal. The *interest rate* dictates the amount of money they pay you for the loan, for a given time period, called the *term*. *Simple interest* pays you a percentage of the loaned amount at the end of the term. With *compound interest,* the interest is paid out more often, defined by the *compounding frequency*. You don't get the *compound interest* immediately, but effectively loan the entity the interest as well, until the end of the term. The *present value* plus the interest you earn is called the *future value*. A higher *interest rate* and a higher *compounding frequency* will earn you more interest.

15. I will use a present value of $100 so my future value will be $300.
The present value is $100.
The annual interest rate is unknown.
The compounding period is quarterly, or 4 times per year.
The term (in years) is 12.
The future value is $300.
I used the financial application on my calculator: An interest rate of 9.26% will allow the investment to triple every 12 years.

16. a) The present value is $1000.
The annual interest rate is 5%.
The compounding period is annual, or once per year.
The term (in years) is 1 year.
The future value is unknown.
The future value of the investment is $1050.
b) i) The present value is $1000.
The annual interest rate is unknown.
The compounding period is semi-annual, or 2 times per year.
The term (in years) is 1 year.
The future value is $1050.
The annual interest rate is 4.94%.
ii) The present value is $1000.
The annual interest rate is unknown.
The compounding period is quarterly, or 4 times per year.
The term (in years) is 1 year.
The future value is $1050.
The annual interest rate is 4.91%.
iii) The present value is $1000.
The annual interest rate is unknown.
The compounding period is monthly, or 12 times per year.
The term (in years) is 1 year.
The future value is $1050.
The annual interest rate is 4.89%.

c) e.g., By choosing a lower interest rate with more frequent compounding, you can take advantage of the power of compound interest and earn the same interest as you could at a higher interest rate with less frequent compounding. This is useful when interest rates are low.

Mid-Chapter Review, page 45

1. $A = P + Prt$
A is $477.56; P is $450; r *is* 2.04% or 0.0204
$477.56 = 450 + (450)(0.0204)(t)$
$27.56 = 9.18t$
$t = 3.002\ldots$
Paula held the investment for 3 years.

2. a) $A = P + Prt$
A is $7200; P is $6000; r *is* 6.4% or 0.064
$7200 = 6000 + (6000)(0.064)(t)$
$1200 = 384t$
$t = 3.125$
It will take 3.125 years for the investment to earn $1200 in interest.
b) If paid annually, the interest will be paid out at the end of the next full year, or in 4 years.
c) If paid quarterly, the interest will be paid out at the end of the next full quarter, or in 3.25 years (3 years and 4 months).

3. a) Katherine: P is $5000; r is 4.867 75% or 0.048 677 5; t is 1, 20 times
$A = P(1 + (0.048\ 677\ 5)(1))$
Calculate the interest, using the value of A as the new value of P for each new year. Use a table to organise the answers. (Some values have been omitted.)

Year	Principal ($)	Year-end Value ($)
1	5000.00	5243.39
5	6046.97	6341.32
10	7669.16	8042.47
15	9726.52	10199.98
20	12 335.79	12 936.27

Katherine's account will be worth $12 936.27 after 20 years.
Brad: P is $5000; r is 5.5% or 0.055; t is 1, 5, 10, 15, 20
$A = P(1 + rt)$

Year	Principal ($)	Year-end Value ($)
1	5000	5275
5	5000	6375
10	5000	7750
15	5000	9125
20	5000	10500

Brad's account will be worth $10 500 after 20 years.