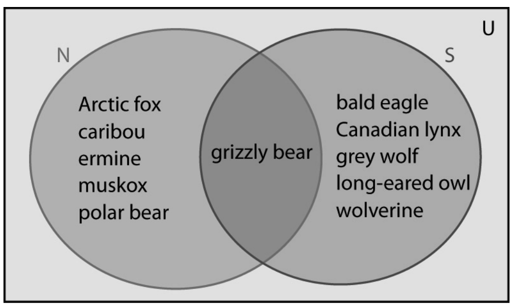
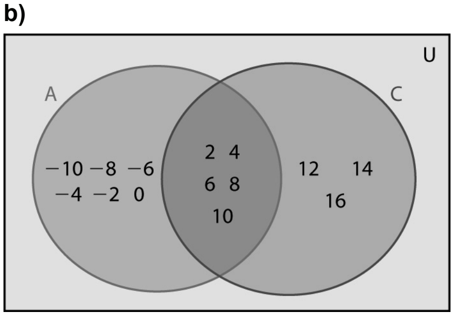
Lesson 3.3: Intersection and Union of Two Sets, page 172

1. a) $A = \{-10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10\}$
$B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A \cup B = \{-10, -8, -6, -4, -2, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
b) $n(A \cup B) = 16$
c) $A \cap B = \{0, 2, 4, 6, 8, 10\}$
d) $n(A \cap B) = 6$

2. a) Let A represent the universal set. Let N represent the set of tundra animals. Let S represent the set of southern animals.
N = {arctic fox, caribou, ermine, grizzly bear, muskox, polar bear}
S = {bald eagle, Canadian lynx, grey wolf, grizzly bear, long-eared owl, wolverine}
$N \cup S$ = {Arctic fox, caribou, ermine, muskox, polar bear, grizzly bear, bald eagle, Canadian lynx, grey wolf, long-eared owl, wolverine}
$T \cap S$ = {grizzly bear}
b)

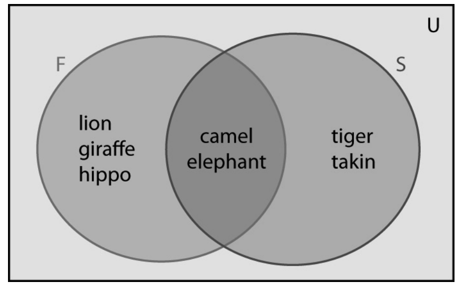


3. a) $A \cup C = \{-10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10, 12, 14, 16\}$
$n(A \cup C) = 14$
$A \cap C = \{2, 4, 6, 8, 10\}$
$n(A \cap C) = 5$
b)



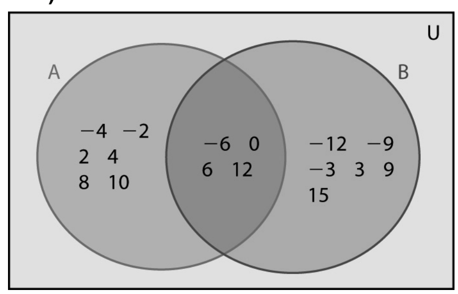
4. a) $T \cup C$ = {half-ton trucks, quarter-ton trucks, vans, SUVs, crossovers, 4-door sedans, 2-door coupes, sports cars, hybrids}
b) $n(T \cup C) = 9$
c) $T \cap C$ = {crossovers}

5. a) Let U represent the universal set. Let F represent the set of African animals. Let S represent the set of Asian animals



b) F = {lion, camel, giraffe, hippo, elephant}
S = {elephant, tiger, takin, camel}
$F \cup S$ = {lion, giraffe, hippo, camel, elephant, tiger, takin}
$F \cap S$ = {camel, elephant}

6. a)



b) $A \cup B = \{-12, -9, -6, -4, -3, -2, 0, 2, 3, 4, 6, 8, 9, 10, 12, 15\}$
$n(A \cup B) = 16$
$A \cap B = \{-6, 0, 6, 12\}$
$n(A \cap B) = 4$

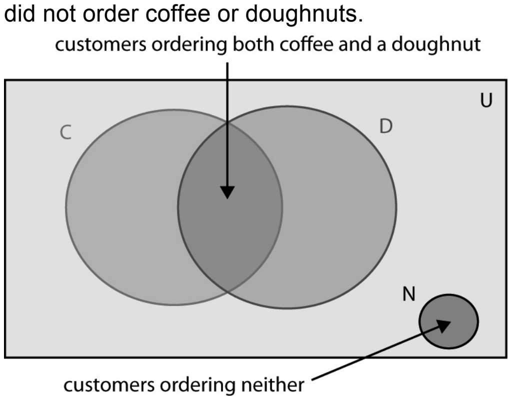
7. Let U represent the universal set. Let H represent the set of people who liked Sherlock Holmes. Let P represent the set of people who liked Hercule Poirot.
$n(H \cup P) = n(U) - n((H \cup P)')$
$n(H \cup P) = 25 - 4$
$n(H \cup P) = 21$
$n(H \cap P) = n(H) + n(P) - n(H \cup P)$
$n(H \cap P) = 16 + 11 - 21$
$n(H \cap P) = 6$
6 people like both detectives.
$n(H \text{ only}) = n(H) - n(H \cup P)$
$n(H \text{ only}) = 16 - 6$
$n(H \text{ only}) = 10$
10 people liked Sherlock Holmes only.
$n(P \text{ only}) = n(P) - n(H \cup P)$
$n(P \text{ only}) = 11 - 6$
$n(P \text{ only}) = 5$
5 people liked Hercule Poirot only.

8. Let U represent the universal set. Let V represent the set of people who liked vanilla ice cream. Let C represent the set of people who liked chocolate ice cream.
$n(C \cup V) = n(U) - n((C \cup V)')$
$n(C \cup V) = 80 - 9$
$n(C \cup V) = 71$
$n(C \text{ only}) = n(C \cup V) - n(V \text{ only}) - n(C \cap V)$
$n(C \text{ only}) = 71 - 11 - 20$
$n(C \text{ only}) = 40$
40 people like chocolate ice cream only.

9. Let U represent the universal set. Let K represent the set of people who like to ski. Let W represent the set of people who like to swim.
$n(K \cup W) = n(U) - n((K \cup W)')$
$n(K \cup W) = 26 - 5$
$n(K \cup W) = 21$
$n(K \cap W) = n(K) + n(W) - n(K \cup W)$
$n(K \cap W) = 19 + 14 - 21$
$n(K \cap W) = 12$
12 people like to ski and swim.

10. e.g., She could draw a Venn diagram showing the set of multiples of 2 and the set of multiples of 3. The intersection of the sets would be the multiples of 6.

11. a) U = {all customers surveyed}
C = {customers ordering coffee}
D = {customers ordering donuts}
N = {customers ordering neither coffee nor doughnuts}
b) For the following Venn diagram:
The rectangular area labelled U represents the universal set.
The shaded area labelled D represents the set of people who ordered doughnuts.
The shaded area labelled C represents the set of people who ordered coffee.
The shaded area labelled $D \cap C$ represents the set of people who ordered coffee and doughnuts.
The unshaded area labelled N represents those people did not order coffee or doughnuts.



c) Determine $n(D \cap C)$ using the information available.
$n(U) = 100$, $n(D) = 45$, $n(C) = 65$, $n((D \cup C)') = 10$
$n(D \cup C) = n(U) - n((D \cup C)')$
$n(D \cup C) = 100 - 10$
$n(D \cup C) = 90$
Therefore,
$n(D \cap C) = n(D) + n(C) - n(D \cup C)$
$n(D \cap C) = 45 + 65 - 90$
$n(D \cap C) = 20$
There were 20 people who ordered coffee and a doughnut.

12. Let U represent the universal set. Let T represent the set of seniors who watch television. Let R represent the set of seniors who listen to the radio.
$n(R \text{ only}) = n(U) - n(T)$
$n(R \text{ only}) = 100 - 67$
$n(R \text{ only}) = 33$
33 seniors prefer to listen to the radio only.

13. Let U represent the universal set. Let C represent the set of people who attended the Calgary Stampede. Let P represent the set of people who attended the PNE.
$n(C \cup P) = n(U) - n((C \cup P)')$
$n(C \cup P) = 56 - 14$
$n(C \cup P) = 42$
$n(C \cap P) = n(C) + n(P) - n(C \cup P)$
$n(C \cap P) = 30 + 22 - 42$
$n(C \cap P) = 10$
10 people had been to both the Calgary Stampede and the PNE.

14. Of the 54 people, 31 own their home, so $54 - 31 = 23$ people rent their home. Of that 23, 9 rent their house, so $23 - 9 = 14$ rent their condominium. Of the 30 people who live in a condominium, 14 rent, so $30 - 14 = 16$ must own the condominium in which they live.

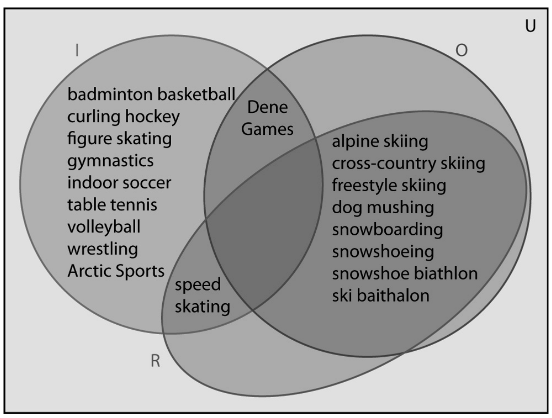
15. Let U represent the universal set. Let R represent the set of people who like reality shows. Let C represent the set of people who like contest shows.
$n(C \cup R) = n(U) - n((C \cup R)')$
$n(C \cup R) = 32 - 4$
$n(C \cup R) = 28$
$n(C \cap R) = n(C \cup R) - n(C \text{ only}) - n(R \text{ only})$
$n(C \cap R) = 28 - 13 - 9$
$n(C \cap R) = 6$
6 people like both type of shows.

16. No. e.g., The three numbers do not add up to 48. There is an overlap between sets B and C, but $B \not\subset C$.
The sum of the three values in the problem is 59.
$59 - 48 = 11$
11 students must drive a car and take a bus.
$31 - 11 = 20$
20 students drive a car but do not take a bus.
$16 - 11 = 5$
5 students take a bus but do not drive a car.
There are a total of $15 + 12 = 27$ students who do not take a bus.

17. a) Sets *A* and *B* are disjoint sets.
b) Sets *A* and *C* intersect.
c) Yes; *B* and *C*; e.g., *C* intersecting *A* and *A* and *B* being disjoint says nothing about the intersection, if any, of *B* and *C*.

18. e.g., The union of two sets is more like the addition of two numbers because all the elements of each set are counted together, instead of those present in both sets.

19. a) e.g., indoor, outdoor, races
b) e.g., *U* = {all sports}
I = {indoor sports} = {badminton, basketball, curling, figure skating, gymnastics, hockey, indoor soccer, speed skating, table tennis, volleyball, wrestling, Arctic Sports, Dene Games}
O = {outdoor sports} = {alpine skiing, cross-country skiing, freestyle skiing, snowshoe biathlon, ski biathlon, dog mushing, snowboarding, snowshoeing, Dene Games}
R = {races} = {speed skating, alpine skiing, cross-country skiing, biathlon, dog mushing, snowboarding, snowshoeing}
c)



d) Yes. e.g., My classmate sorted the games as individual, partner and team games.

History Connection, page 175

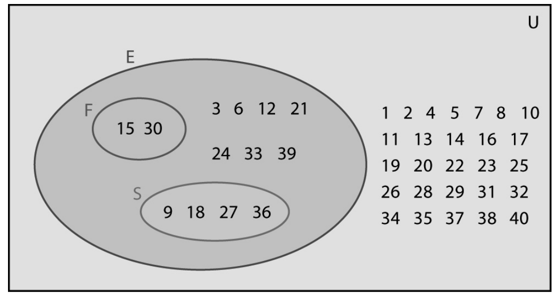
A. e.g., The “barber paradox” can be stated as follows: Suppose there is one male barber in a small town, and that every man in the town keeps himself clean-shaven. Some do so by shaving themselves and the others go to the barber. So, the barber shaves all the men who do not shave themselves. Does the barber shave himself? The question leads to a paradox: If he does not shave himself, then he must abide by the rule and shave himself. If he does shave himself, then according to the rule he will not shave himself.

B. e.g., One remarkable paradox that arises from Cantor’s work on set theory is the Banach-Tarski theorem, which states that a solid, three-dimensional ball can be split into a finite number of non-overlapping pieces, which can then be put back together in a different way to yield *two* identical copies of the original ball of the same size.

Mid-Chapter Review, page 178

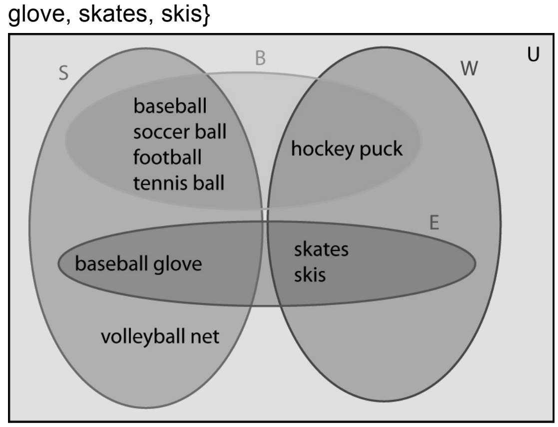
1. a) $V \subset N$, $M \subset N$, $F \subset N$, $F \subset M$
b) e.g., *N* = {all foods}, *V* = {fruits and vegetables}, *M* = {meats}, *F* = {fish}
c) No. e.g., Pasta is not part of *M* or *V*.
d) Sets *V* and *M* are disjoint, Sets *V* and *F* are disjoint.

2. a)



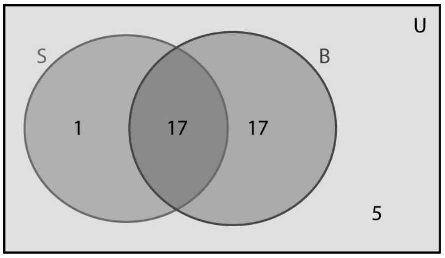
b) Sets *F* and *S* are disjoint sets.
c) i) False. e.g., 6 is in *E* but not *F*.
ii) True. e.g., All elements of *S* are in *E*.
iii) False. e.g., 9 is not a multiple of 15.
iv) True. e.g., *F* = {15, 30}.
v) True. e.g., A set is a subset of itself.

3. e.g., *S* = {summer sport equipment} = {baseball, soccer ball, football, tennis ball, baseball glove, volleyball net}
W = {winter sport equipment} = {hockey puck, skates, skis}
B = {sports balls} = {baseball, soccer ball, football, tennis ball, hockey puck}
E = {sports equipment worn on body} = {baseball glove, skates, skis}



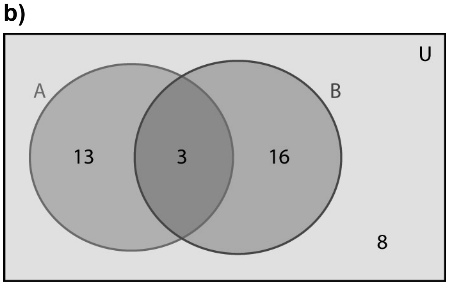
4. a) beverage or soup: 40 – 5 = 35
beverage and soup: 34 + 18 = 52
overlap: 52 – 35 = 17
17 students bought a beverage and soup.
b) only beverage: 34 – 17 = 17
only soup: 18 – 17 = 1
18 students bought only a beverage or only soup.

c)



5. a) sunglasses or hat: 20 – 5 = 15
sunglasses and hat: 13 + 6 = 19
overlap: 19 – 15 = 4
4 students are wearing sunglasses and a hat.
b) only sunglasses: 13 – 4 = 9
9 students are wearing sunglasses but not a hat.
c) only hat: 6 – 4 = 2
2 students are wearing a hat but not sunglasses.

6. a) e.g., Tanya did not put any elements in the intersection of A and B.
$n(A \cup B) = n(U) - n((A \cup B)')$
$n(A \cup B) = 40 - 8$
$n(A \cup B) = 32$
$n(A \cap B) = n(A) + n(B) - n(A \cup B)$
$n(A \cap B) = 16 + 19 - 32$
$n(A \cap B) = 3$
$n(A \setminus B) = n(A) - n(A \cap B)$
$n(A \setminus B) = 16 - 3$
$n(A \setminus B) = 13$
$n(B \setminus A) = n(B) - n(A \cap B)$
$n(B \setminus A) = 19 - 3$
$n(B \setminus A) = 16$
b)



7. Let U represent the universal set. Let D represent the set of students who have a dog. Let C represent the set of students who have a cat.
$n(C \cup D) = n(U) - n((C \cup D)')$
$n(C \cup D) = 20 - 4$
$n(C \cup D) = 16$
$n(C \cap D) = n(C) + n(D) - n(C \cup D)$
$n(C \cap D) = 8 + 8 - 16$
$n(C \cap D) = 0$
No students have a cat and a dog.

Lesson 3.4: Applications of Set Theory, page 191

1. $n(P) = p + 16$, $n(Q) = q + 21$, $n(R) = r + 18$
e.g., p Can be any number. Suppose $p = 14$. Then $n(P) = 30$.
$n(Q) = 30$, so $q = 30 - 21$ or 9
$n(R) = 30$, so r 301 – 18 or 12

2. a) $n((F \cup M) \setminus A) = 9 + 15 + 8$
$n((F \cup M) \setminus A) = 32$
b) $n((A \cup F) \setminus M) = 9 + 11 + 7$
$n((A \cup F) \setminus M) = 27$
c) $n((F \cup A) \cup (F \cup M))$
$= (9 + 11 + 7 + 9) + (15 + 8 + 4)$
$= 36 + 27$
$= 63$
d) $n(A \setminus F \setminus M) = 7$

3. e.g., Staff could look at how many David Smiths were on that bus route or they could look at the books in the bag and see how many David Smiths are taking courses that use those books.

4. P = {population surveyed}
$n(P) = 641$
L = {people wearing corrective lenses}
L' = {people not wearing corrective lenses}
$n(L') = 167$
G = {people wearing glasses}
C = {people wearing contact lenses}
$n(L) = n(P) - n(L')$
$n(L) = 641 - 167$
$n(L) = 474$
$n(G \cup C) = n(L)$
$n(G \cup C) = n(G) + n(C) - n(G \cap C)$
$474 = 442 + 83 - n(G \cap C)$
$51 = n(G \cap C)$
51 people might make use of a package deal. This is $\frac{51}{574}$ = 10.759…% or about 10.8% of all potential customers.

5. e.g., “Canadian Rockies,” “ski accommodations,” “weather forecast,” “Whistler.” By combining two or more of these terms, Jacques can search for the intersection of web pages related to these terms. For example, “ski accommodations” and “Canadian Rockies” is more likely to give him useful information for his trip than either of those terms on its own.