Chapter 4: Counting Methods

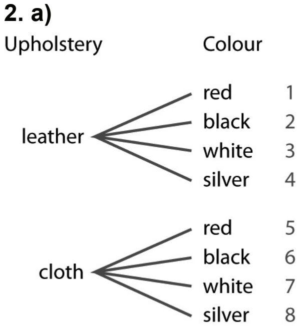
Lesson 4.1: Counting Principles, page 235

1. a)

	khaki	black
red	red/khaki	red/black
blue	blue/khaki	blue/black
green	green/khaki	green/black

Each x represents a different combination. There are 6 x's; therefore, there are six different variations of the outfit to choose from.
b) The number of outfit variations, O, is related to the number of shirts and the number of shorts:
O = (number of shirts) · (number of shorts)
$O = 3 \cdot 2$
$O = 6$
There are six different variations of the outfit to choose from. This matches the part a) result.

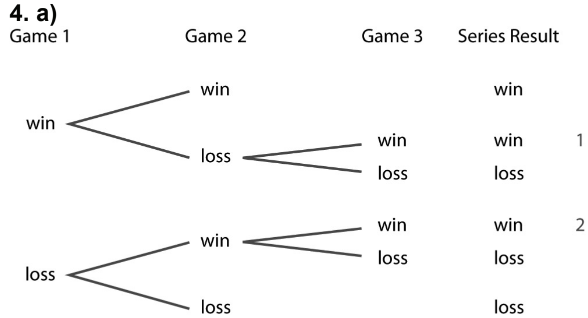
2. a)



Therefore, there are 8 upholstery-colour choices that are available.
b) The number of upholstery-colour choices, U, is related to the number of colours and the number of kinds of upholstery:
U = (number of colours) · (number of upholstery)
$U = 4 \cdot 2$
$U = 8$
There are 8 upholstery-colour choices that are available. This matches the part a) result.

3. a) The Fundamental Counting Principle does not apply because tasks in this situation are related by the word OR.
b) The Fundamental Counting Principle does apply because tasks in this situation are related by the word AND.
c) The Fundamental Counting Principle does not apply because tasks in this situation are related by the word OR.
d) The Fundamental Counting Principle does apply because tasks in this situation are related by the word AND.

4. a)



b) By looking at the tree diagram, I can see there are 2 ways in which Kim's team can win the series despite losing one game.

5. The number of colour-size variations, C, is related to the number of colours and the number of sizes:
C = (number of colours) · (number of sizes)
$C = 5 \cdot 4$
$C = 20$
There are 20 colour-size variations that are available.

6. The number of computer systems, S, the employees can build for their customers is related to the number of desktop computers (dc), the number of monitors (m), the number of printers (p), and the number of software packages (sp):
S = (# of dc) · (# of m) · (# of p) · (# of sp)
$S = 5 \cdot 4 \cdot 6 \cdot 3$
$S = 360$
Therefore, the employees can build 360 different computer systems for their customers.

7. The number of possible meals, M, is related to the number of soups (s), the number of sandwiches (sw), the number of drinks (dr), and the number of desserts (d):
M = (# of s) · (# of sw) · (# of dr) · (# of d)
$M = 3 \cdot 5 \cdot 4 \cdot 2$
$M = 120$
Therefore, there are 120 different meal possibilities.

8. Event A: Selecting a rap CD
OR
Event B: Selecting a classic rock CD
$n(A \cup B) = n(A) + n(B)$
$n(A \cup B) = 8 + 10$
$n(A \cup B) = 18$
Therefore, Charlene can select from 18 CDs to play in her car stereo that will match Tom's musical tastes.

9. a) The number of different PIN combinations, C, is related to the number of digits from which to select for each digit of the PIN, P:
$C = P_1 \cdot P_2 \cdot P_3 \cdot P_4 \cdot P_5$
$C = 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$
$C = 59\,049$
There are 59 049 different five-digit PIN combinations.

b) The number of different PIN combinations, N, is related to the number of digits from which to select for each digit of the PIN, D:
$N = D_1 \cdot D_2 \cdot D_3 \cdot D_4 \cdot D_5$
$N = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5$
$N = 15\ 120$
There are only 15 120 different five-digit PIN combinations in which the digits cannot repeat.

10. The number of different bytes that can be created, N, is related to the number of digits from which to choose for each digit of the byte, B:
$N = B_1 \cdot B_2 \cdot B_3 \cdot B_4 \cdot B_5 \cdot B_6 \cdot B_7 \cdot B_8$
$N = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
$N = 256$
Therefore, 256 different bytes can be created.

11. a) The number of different upper-case letter possibilities, N, is related to the number of upper-case letters from which to choose for each odd position of the country's postal code, P:
$N = P_1 \cdot P_3 \cdot P_5$
$N = 26 \cdot 26 \cdot 26$
$N = 17\ 576$
The number of different digit possibilities, D, is related to the number of digits from which to choose for each even position of the country's postal code, P:
$D = P_2 \cdot P_4 \cdot P_6$
$D = 10 \cdot 10 \cdot 10$
$D = 1000$
The number of different postal codes that are possible in this country, C, is related to the number of upper-case letter possibilities, N, and the number of digit possibilities, D:
$C = N \cdot D$
$C = 17\ 576 \cdot 1000$
$C = 17\ 576\ 000$
Therefore, 17 576 000 postal codes are possible.
b) The number of different upper-case letter possibilities, N, is related to the number of upper-case letters from which to choose for each odd position of the country's postal code, P:
$N = P_1 \cdot P_3 \cdot P_5$
$N = 21 \cdot 21 \cdot 21$
$N = 9261$
The number of different digit possibilities, D, remains the same since all digits can be used. The number of different postal codes that are possible in Canada, C, is related to the number of uppercase letter possibilities, N, and the number of digit possibilities, D:
$C = N \cdot D$
$C = 9261 \cdot 1000$
$C = 9\ 261\ 000$
Therefore, 9 261 000 postal codes are possible in Canada.

12. To answer this question, I need to determine how many digit combinations there are for the last four digits of one of these two phone numbers, and then multiply it by 2.
The number of digit combinations, C, is related to the number of possible digits for each of the last four digits of one of the phone numbers, P:
$C = P_1 \cdot P_2 \cdot P_3 \cdot P_4$
$C = 10 \cdot 10 \cdot 10 \cdot 10$
$C = 10\ 000$
The number of phone numbers is $2C$ since there are two given templates for the phone numbers in the question.
$2C = 2(10\ 000)$
$2C = 20\ 000$
Therefore, 20 000 different phone numbers are possible for this town.

13. The number of different codes, C, is related to number of positions from which to select for each switch of the garage door opener, G:
$C = G_1 \cdot G_2 \cdot G_3 \cdot G_4 \cdot G_5 \cdot G_6 \cdot G_7 \cdot G_8 \cdot G_9$
$C = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
$C = 19\ 683$
Therefore, 19 683 different codes are possible.

14. Event A: Selecting a pickup truck OR
Event B: Selecting a passenger van OR
Event C: Selecting a car OR
Event D: Selecting a sports utility vehicle
$n(A \cup B \cup C \cup D) = n(A) + n(B) + n(C) + n(D)$
$n(A \cup B \cup C \cup D) = 8 + 10 + 35 + 12$
$n(A \cup B \cup C \cup D) = 65$
Therefore, a customer has 65 choices when renting just one vehicle.

15. a) Multiply the number of sizes of the crust, by the number of types of the crust, by the number of types of cheese, by the number of types of tomato sauce.
$2 \cdot 2 \cdot 2 \cdot 2 = 16$
Multiply this number by the number of different toppings.
$16 \cdot 20 = 320$
Therefore, there are 320 different pizzas that can be made with any crust, cheese, tomato sauce, and 1 topping.
b) Multiply the number of types of cheese by the number of types of tomato sauce.
$2 \cdot 2 = 4$
Therefore, there are 4 different pizzas that can be made with a thin whole-wheat crust, tomato sauce, cheese, and no toppings.

16. a) The number of different upper-case letter possibilities, N, is related to the number of upper-case letters from which to choose for each of the first three positions of the Alberta licence plate, P:
$N = P_1 \cdot P_2 \cdot P_3$
$N = 24 \cdot 24 \cdot 24$
$N = 13\ 824$
The number of different digit possibilities, D, is related to the number of digits from which to choose for each of the last three positions of the Alberta licence plate, P:
$D = P_4 \cdot P_5 \cdot P_6$
$D = 10 \cdot 10 \cdot 10$
$D = 1000$
The number of different possible Alberta licence plates, C, is related to the number of upper-case letter possibilities, N, and the number of digit possibilities, D:
$C = N \cdot D$
$C = 13\ 824 \cdot 1000$
$C = 13\ 824\ 000$
So, 13 824 000 Alberta licence plates are possible.
b) The number of different upper-case letter possibilities, N, remains the same since the number of letters in the plates and the number of letters that can be used is the same as in a).
The number of different digit possibilities, D, is related to the number of digits from which to choose for each of the last four positions of the Alberta licence plate, P:
$D = P_4 \cdot P_5 \cdot P_6 \cdot P_7$
$D = 10 \cdot 10 \cdot 10 \cdot 10$
$D = 10\ 000$
The number of different possible Alberta licence plates, C, is related to the number of upper-case letter possibilities, N, and the number of digit possibilities, D:
$C = N \cdot D$
$C = 13\ 824 \cdot 10\ 000$
$C = 138\ 240\ 000$
$138\ 240\ 000 - 13\ 824\ 000 = 124\ 416\ 000$
So, 124 416 000 more licence plates are possible.

17. e.g., If multiple tasks are related by AND, it means the Fundamental Counting Principle can be used and the total number of solutions is the product of the solutions to each task. For example: A 4-digit PIN involves choosing the 1st digit AND the 2nd digit AND the 3rd digit AND the 4th digit. So the number of solutions is $10 \cdot 10 \cdot 10 \cdot 10 = 10\ 000$. OR means the solution must meet at least one condition so you must add the number of solutions to each condition, and then subtract the number of solutions that meet all conditions. For example: Calculating the number of 4-digit PINs that start with 3 OR end with 3. The solution is the number of PINs that start with 3, plus the number of PINs that end with 3, minus the number of PINs that both start and end with 3:
$1000 + 1000 - 100 = 1900$.

18. a) i) Event A: Drawing a king OR
Event B: Drawing a queen
$n(A \cup B) = n(A) + n(B)$
$n(A \cup B) = 4 + 4$
$n(A \cup B) = 8$
$\text{Likelihood} = \frac{8}{52}$
$\text{Likelihood} = \frac{2}{13}$
Therefore, there is a 2 in 13 chance that a king or a queen will be drawn.
ii) Event A: Drawing a diamond
OR
Event B: Drawing a club
$n(A \cup B) = n(A) + n(B)$
$n(A \cup B) = 13 + 13$
$n(A \cup B) = 26$
$\text{Likelihood} = \frac{26}{52}$
$\text{Likelihood} = \frac{1}{2}$
Therefore, there is a 1 in 2 chance that a diamond or a club will be drawn.
iii) Event A: Drawing an Ace OR
Event B: Drawing a spade
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$n(A \cup B) = 4 + 13 - 1$
$n(A \cup B) = 16$
$\text{Likelihood} = \frac{16}{52}$
$\text{Likelihood} = \frac{4}{13}$
Therefore, there is a 4 in 13 chance that an ace or a spade will be drawn.
b) No, e.g., because the Fundamental Counting Principle only applies when tasks are related by the word AND.

19. e.g., To begin, there are 90 two-digit numbers. There are 10 with a 1 in the tens column, 10 with a 2 in the tens column, and this pattern continues until I reach the 10 with a 9 in the tens column. Next, I must determine the numbers that are divisible by either 2 or 5. I know that every other number is even and thus divisible by 2. This means that $\frac{90}{2}$ or 45 of the two-digit numbers are divisible by 2. The two-digit numbers that are divisible by 5 can be found by starting at the first two-digit number, 10, and then counting by 5 until I get to a three-digit number.
By doing this, I can determine that the two-digit numbers that are divisible by 5 are: 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95. There are 18 of them. I see that half of them or 9 are even and thus divisible by 2.

Therefore, there are 9 numbers that are divisible by 5 and not by 2. If I add this together with the number of two-digit numbers that are divisible by two (45), I see that there are 54 two-digit numbers divisible by 2 or 5. Whatever is leftover from the two digit numbers are the ones that are not divisible by either 2 or 5. This amount is:
90 – 54 = 36. Thus, there are 36 two-digit numbers that are not divisible by either 2 or 5.

20. The number of different outcomes for a student's test, N, is related to the number of possible answers for each question on the test, T:
$N = T_1 \cdot T_2 \cdot T_3 \cdot T_4 \cdot T_5 \cdot T_6 \cdot T_7 \cdot T_8 \cdot T_9 \cdot T_{10}$
$N = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
$N = 1024$
A perfect score is only 1 out of these 1024 outcomes; therefore, there is a 1 in 1024 chance that the student will get a perfect score.

21. This question is solved by constant application of the Fundamental Counting Principle.
If an item from each category is selected:
$O = 3 \cdot 5 \cdot 4 \cdot 2$
$O = 120$
If no soup is selected:
$O = 5 \cdot 4 \cdot 2$
$O = 40$
If no sandwich is selected:
$O = 3 \cdot 4 \cdot 2$
$O = 24$
If no drink is selected:
$O = 3 \cdot 5 \cdot 2$
$O = 30$
If no dessert is selected:
$O = 3 \cdot 5 \cdot 4$
$O = 60$
If no soup or sandwich is selected:
$O = 4 \cdot 2$
$O = 8$
If no soup or drink selected:
$O = 5 \cdot 2$
$O = 10$
If no soup or dessert is selected:
$O = 5 \cdot 4$
$O = 20$
If no sandwich or drink is selected:
$O = 3 \cdot 2$
$O = 6$
If no sandwich or dessert is selected:
$O = 3 \cdot 4$
$O = 12$
If no drink or dessert is selected:
$O = 3 \cdot 5$
$O = 15$
If only a soup, sandwich, drink or dessert is selected:
O = 3, 5, 4, 2
$T_{\text{otal}} = 120 + 40 + 24 + 30 + 60 + 8 + 10 + 20 + 6 + 12 + 15 + 3 + 5 + 4 + 2$
$T_{\text{otal}} = 359$
Therefore, 359 meals are possible if you do not have to choose an item from a category.

Lesson 4.2: Introducing Permutations and Factorial Notation, page 243

1. a) $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$6! = 720$

b) $9 \cdot 8! = 9 \cdot (8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)$
$9 \cdot 8! = 9 \cdot 40320$
$9 \cdot 8! = 362880$

c) $\frac{5!}{3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1}$
$\frac{5!}{3!} = 5 \cdot 4 \cdot \frac{3}{3} \cdot \frac{2}{2} \cdot \frac{1}{1}$
$\frac{5!}{3!} = 5 \cdot 4 \cdot \frac{3!}{3!}$
$\frac{5!}{3!} = 5 \cdot 4 \cdot 1$
$\frac{5!}{3!} = 20$

d) $\frac{8!}{7!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$\frac{8!}{7!} = 8 \cdot \frac{7}{7} \cdot \frac{6}{6} \cdot \frac{5}{5} \cdot \frac{4}{4} \cdot \frac{3}{3} \cdot \frac{2}{2} \cdot \frac{1}{1}$
$\frac{8!}{7!} = 8 \cdot \frac{7!}{7!}$
$\frac{8!}{7!} = 8 \cdot 1$
$\frac{8!}{7!} = 8$

e) $3! \cdot 2! = (3 \cdot 2 \cdot 1) \cdot (2 \cdot 1)$
$3! \cdot 2! = 6 \cdot 2$
$3! \cdot 2! = 12$

f) $\frac{9!}{4!3!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(4 \cdot 3 \cdot 2 \cdot 1) \cdot (3 \cdot 2 \cdot 1)}$
$\frac{9!}{4!3!} = \frac{9}{3} \cdot \frac{8}{2} \cdot \frac{7}{1} \cdot 6 \cdot 5 \cdot \frac{4}{4} \cdot \frac{3}{3} \cdot \frac{2}{2} \cdot \frac{1}{1}$
$\frac{9!}{4!3!} = \frac{9}{3} \cdot \frac{8}{2} \cdot 7 \cdot 6 \cdot 5 \cdot \frac{4!}{4!}$
$\frac{9!}{4!3!} = \frac{9}{3} \cdot \frac{8}{2} \cdot 7 \cdot 6 \cdot 5 \cdot 1$
$\frac{9!}{4!3!} = 3 \cdot 4 \cdot 7 \cdot 6 \cdot 5$
$\frac{9!}{4!3!} = 2520$