There are 220 different ways that a 5-person committee can be selected if David and Susan must be on the committee.

d) Case 1: 2 boys and 3 girls

$${}_6C_2 \cdot {}_8C_3 = \frac{6!}{2!4!} \cdot \frac{8!}{3!5!}$$

$${}_6C_2 \cdot {}_8C_3 = 840$$

Case 2: 1 boy and 4 girls

$${}_6C_1 \cdot {}_8C_4 = \frac{6!}{1!5!} \cdot \frac{8!}{4!4!}$$

$${}_6C_1 \cdot {}_8C_4 = 420$$

Case 3: 0 boys and 5 girls

$${}_6C_0 \cdot {}_8C_5 = \frac{6!}{0!6!} \cdot \frac{8!}{5!3!}$$

$${}_6C_0 \cdot {}_8C_5 = 56$$

Let C represent the number of 5-person committees with more girls than boys:
C = 840 + 420 + 56
C = 1316
There are 1316 different ways that a 5-person committee can be selected if there must be more girls than boys.

8. $\frac{5!}{2!2!} = 30$

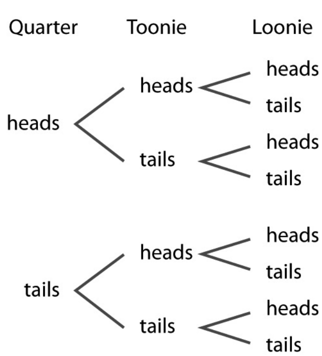
There are 30 different arrangements of the letters in the word TEETH.

9. 5! · 4! = 2880
There are 2880 different arrangements possible.

Chapter Review, page 293

1. e.g., The Fundamental Counting Principle is used when a counting problem has different tasks related by the word AND. For example, you can use it to figure out how many ways you can roll a 3 with a die and draw a red card from a deck of cards.

2.



The tree diagram shows there are 8 possible ways that the three coins can land.

3. Let A represent the number of sets of answers:
$A = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$
$A = 4^{10}$
A = 1 048 576
The student can give 1 048 576 different sets of answers.

4. a) $n + 2 \geq 0$ AND $n \geq 0$

$n \geq -2$

$\frac{(n+2)!}{n!} = 20$ is defined for $n \geq 0$, where $n \in$ I.

$$\frac{(n+2)!}{n!} = 20$$

$$\frac{(n+2)(n+1)(n)(n-1)...(3)(2)(1)}{(n)(n-1)...(3)(2)(1)} = 20$$

$$(n+2)(n+1) = 20$$

$$n^2 + n + 2n + 2 - 20 = 0$$

$$n^2 + 3n - 18 = 0$$

$$(n+6)(n-3) = 0$$

$n + 6 = 0$ or $n - 3 = 0$

$n = -6$ $\quad n = 3$

The root n = –6 is outside the restrictions on the variable in the equation, so it cannot be a solution. There is one solution, n = 3.

b) The simplified version of the equation is

$\frac{(n+1)!}{(n-1)!} = 132$.

$n + 1 \geq 0$ AND $n - 1 \geq 0$

$n \geq -1$ $\quad n \geq 1$

$\frac{(n+1)!}{(n-1)!} = 132$ is defined for $n \geq 1$, where $n \in$ I.

$$\frac{(n+1)!}{(n-1)!} = 132$$

$$\frac{(n+1)(n)(n-1)(n-2)...(3)(2)(1)}{(n-1)(n-2)...(3)(2)(1)} = 132$$

$$(n+1)(n) = 132$$

$$n^2 + n = 132$$

$$n^2 + n - 132 = 0$$

$$(n+12)(n-11) = 0$$

$n + 12 = 0$ or $n - 11 = 0$

$n = -12$ $\quad n = 11$

The root n = –12 is outside the restrictions on the variable in the equation, so it cannot be a solution. There is one solution, n = 11.

5. e.g., ${}_6P_6$ has a larger value. e.g., I know because $\frac{8!}{6!}$ is the factorial expression for the permutation expression ${}_8P_2$. Here, I have more objects than for ${}_6P_6$, but I am not using all of them. This leads to fewer possible arrangements, or in other words, a lower value for $\frac{8!}{6!}$.

6. Let O represent the number of orders:
$O = 12!$
$O = 479\ 001\ 600$
There are 479 001 600 different orders in which the singers could perform the 12 songs.

7. ${}_{25}P_3 = \frac{25!}{22!}$
${}_{25}P_3 = 13\,800$
There are 13 800 different ways a director of education, a superintendent of curriculum, and a superintendent of finance can be selected.

8. a) ${}_{25}P_{10} = \frac{25!}{15!}$
${}_{25}P_{10} = 11\ 861\ 676\ 288\ 000$
${}_{25}P_{10} = 1.186... \times 10^{13}$
There are 11 861 676 288 000 or about 1.2×10^{13} different ways the test can be created if there are no conditions.

b) ${}_{23}P_8 = \frac{23!}{15!}$
${}_{23}P_8 = 19\ 769\ 460\ 480$
${}_{23}P_8 = 1.976... \times 10^{10}$
There are 19 769 460 480 or about 2.0×10^{10} different ways the test can be created if the easiest question of the 25 is always first and the most difficult question is always last.

9. ${}_{52}P_5 = \frac{52!}{47!}$
${}_{52}P_5 = 311\,875\,200$
There are 311 875 200 different five-card arrangements possible.

10. a) Let A represent the number of arrangements:
$\frac{11!}{2!2!2!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$
$\frac{11!}{2!2!2!} = 11 \cdot 10 \cdot 9 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$\frac{11!}{2!2!2!} = 4\,989\,600$
There are 4 989 600 different arrangements that are possible if all the letters are used.

b) Let A represent the number of arrangements:
$\frac{10!}{2!2!2!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$
$\frac{10!}{2!2!2!} = 10 \cdot 9 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$\frac{10!}{2!2!2!} = 453\,600$
There are 453 600 different arrangements that are possible if all the letters are used, but each arrangement must begin with the C.

11. a) $\frac{14!}{2!3!4!5!} = 2\,522\,520$
There are 2 522 520 different ways Tina can stack the blocks in a single tower if there are no conditions.

b) $\frac{12!}{3!4!5!} = 27\,720$
There are 27 720 different ways Tina can stack the blocks in a single tower if there must be a yellow block at the bottom of the tower and a yellow block at the top.

12. ${}_{10}C_5 = \frac{10!}{5!5!}$ ${}_{11}C_7 = \frac{11!}{7!4!}$ ${}_{15}C_2 = \frac{15!}{2!13!}$
${}_{10}C_5 = 252$ ${}_{11}C_7 = 330$ ${}_{15}C_2 = 105$
Therefore, ${}_{11}C_7$ results in the greatest value.

13. ${}_{20}C_4 = \frac{20!}{4!16!}$
${}_{20}C_4 = 4845$
There are 4845 different selections of 4 books that Ruth can choose.

14. No. e.g., Each combination can be arranged in many different ways to make a permutation, so there are more permutations than combinations

15. a) ${}_{19}C_4 = \frac{19!}{4!15!}$
${}_{19}C_4 = 3876$
There are 3876 different ways that a committee of 4 people can be chosen if there are no conditions.

b) ${}_9C_2 \cdot {}_{10}C_2 = \frac{9!}{2!7!} \cdot \frac{10!}{2!8!}$
${}_9C_2 \cdot {}_{10}C_2 = 36 \cdot 45$
${}_9C_2 \cdot {}_{10}C_2 = 1620$
There are 1620 different ways that a committee of 4 people can be chosen if there must be an equal number of men and women on the committee.

c) ${}_{10}C_4 = \frac{10!}{4!6!}$
${}_{10}C_4 = 210$
There are 210 different ways that a committee of 4 people can be chosen if no men can be on the committee.

16. e.g., Let A represent the number of ways to assign teachers to the first group of 5:

$A = {}_{15}C_5$

$A = \frac{15!}{5!10!}$

$A = 3003$

Now there are 15 – 5 or 10 teachers left to assign. Let B represent the number of ways to assign the remaining teachers to the second group of 5:

$B = {}_{10}C_5$

$B = \frac{10!}{5!5!}$

$B = 252$

Now there are 10 – 5 or 5 teachers left to assign to the last group of 5. There is only 1 way that this can be done. Let T represent the total number of ways to assign the teachers:

$T = A \cdot B$

$T = 3003 \cdot 252$

$T = 756\ 756$

There are 756 756 different ways 15 teachers can be divided into 3 groups of 5.

17. e.g., The first point can be joined with 11 more points to form straight lines. The second point can then be joined with 10 more points to form straight lines (since it was already joined with the first point). The third point can be joined with 9 more points to form straight lines (since it was already joined with the first two points). This pattern continues on until I get to the second-last point that can only be joined with the last point (since it was already joined with the other 10 points). The last point cannot be joined any further since it is already joined to every other point in the circle. Using this observed pattern, I can calculate the number of straight lines (L):

$L = 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$

$L = 66$

There are 66 different ways the points can be joined to form straight lines.

18. a) Since there is one more boy than there are girls, the line must always follow this pattern: BGBGBGBGBGBGB. Thus the boys are arranged in 7 positions, and the girls in 6 positions.

$7! \cdot 6! = 3\ 628\ 800$

There are 3 628 800 ways in which the children can be arranged in one row if the boys and girls must alternate positions.

b) Group the triplets as one. There are 3! ways in which the triplets can arrange themselves. Let B represent the number of different arrangements:

$B = 11! \cdot 3!$

$B = 39\ 916\ 800 \cdot 6$

$B = 239\ 500\ 800$

There are 239 500 800 ways in which the children can be arranged in one row if the triplets must stand next to each other.

19. Case 1: 2 face cards and 3 non-face cards: ${}_{12}C_2 \cdot {}_{40}C_3$

Case 2: 3 face cards and 2 non-face cards: ${}_{12}C_3 \cdot {}_{40}C_2$

Case 3: 4 face cards and 1 non-face card: ${}_{12}C_4 \cdot {}_{40}C_1$

Case 4: 5 face cards and 0 non-face cards: ${}_{12}C_5 \cdot {}_{40}C_0$

Let H represent the number of hands with at least 2 face cards:

$H = {}_{12}C_2 \cdot {}_{40}C_3 + {}_{12}C_3 \cdot {}_{40}C_2 + {}_{12}C_4 \cdot {}_{40}C_1 + {}_{12}C_5 \cdot {}_{40}C_0$

$H = 66 \cdot 9880 + 220 \cdot 780 + 495 \cdot 40 + 792 \cdot 1$

$H = 844\ 272$

There are 844 272 different five-card hands with at least two face cards.

Chapter Task, page 295

A. Combinations. The order in which the dice are tossed does not matter (note that players toss all 8 dice simultaneously) nor does the way the dice are arranged when they land matter. What is important is the outcome of each toss—a combination of number of dice that land with the same side up AND number of dice that land with a different side up, for example, 7 dice land with the same face up AND 1 die with the opposite face up.

B. Each outcome can happen two ways. For example, 8 with the same side up could occur as 8 of the unmarked sides face up or 8 of the marked sides face up. That is why each calculation is the sum of two combination values:

8 dice land with the same side up

$= \binom{8}{8} + \binom{8}{8}$ or 1 + 1 or 2

7 dice land with the same side up

$= \binom{8}{7}\binom{1}{1} + \binom{8}{7}\binom{1}{1}$ or 8 + 8 or 16

6 dice land with the same side up

$= \binom{8}{6}\binom{2}{2} + \binom{8}{6}\binom{2}{2}$ or 28 + 28 or 56

5 dice land with the same side up

$= \binom{8}{5}\binom{3}{3} + \binom{8}{5}\binom{3}{3}$ or 56 + 56 or 112

4 dice land with the same side up

$= \binom{8}{4}\binom{4}{4} + \binom{8}{4}\binom{4}{4}$ or 70 + 70 or 140

3 dice land with the same side up

$= \binom{8}{3}\binom{5}{5} + \binom{8}{3}\binom{5}{5}$ or 56 + 56 or 112