Total if A is the first letter:
$21 + 12 = 33$
Therefore, if the first letter is A, there are 33 possible arrangements.
If the first letter is L, S, or K, there are three possibilities for the second letter: A, and 2 of L, S, and K (the ones that are not the first letter).
If A is the second letter:
3 possibilities for the third letter: A, and 2 of L, S, and K
The A has 3 possibilities for the fourth letter and the other two letters have 2. $3 + 2(2) = 7$
If A is not the second letter:
2 possibilities for the third letter
The A has 2 possibilities for the fourth letter and the other letter has 1. $2 + 1 = 3$
Total for both second letters that are not A:
$3(2) = 6$
Total for one of three times where first letter is L, S, or K:
$7 + 6 = 13$
Total when first letter is L, S, or K:
$3(13) = 39$
Total arrangements:
$39 + 33 = 72$
Therefore, 72 four-letter arrangements can be made using all of the letters in the word ALASKA.

20. If I have an O as the first letter, there are 4 possibilities for the second letter, each of which has 3 possibilities for the third letter. $4(3) = 12$
Therefore, there are 12 possible arrangements when O is the first letter.
If the first letter is B, K, or S:
There are 3 possibilities for the second letter: O, and two of B, K, and S (the ones that are not the first letter). O has 3 possibilities for the third letter while the other 2 have 2. $3 + 2(2) = 7$
Total if the first letter is B, K, or S:
$3(7) = 21$
Total arrangements:
$21 + 12 = 33$
Therefore, 33 three-letter arrangements can be made using all of the letters in the word BOOKS.

History Connection, page 290

A. Yes. Each number from 0 to 127 is assigned a different character or symbol on the keyboard. Since the numbers already have an established order, the characters and symbols assigned to these numbers do, as well.
B. Yes. Each number in ASCII (pronounced “askey”) must be converted into a string of 0s and 1s to create the binary code, so order matters. Each 0 or 1 is associated with a position in the string. A different permutation of 0s and 1s represents a different number in the ASCII code system.
C. There are 128 numbers in ASCII that must be represented by a string of 0s and 1s. You need to determine the length of the string needed to create 128 different arrangements of 0s and 1s. You can begin by thinking about a string of length of 5.
A box diagram, □□□□□, can help you determine the number of ASCII numbers you can represent.
Within each box you can place a 0 or a 1. There are two choices for each box, since repetition of 0s and 1s is allowed. So for a string length of 5, there are $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$ or 32 ASCII numbers that can be represented. Obviously, the string must be longer for 128 numbers. If n represents the string length, and 128 numbers must be represented, then $2^n = 128$. By trial and error, $n = 7$.
A binary string of length 7 is needed to represent each ASCII code.

Chapter Self-Test, page 291

1. a) Let N represent the number of different serial numbers:
$N = 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 3$
$N = 2\ 028\ 000$
There are 2 028 000 different serial numbers possible, if repetition of characters is allowed.
b) Let N represent the number of different serial numbers:
$N = 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \cdot 3$
$N = 1\ 296\ 000$
There are 1 296 000 different serial numbers possible, if no repetition is allowed.

2. Event A: Drawing a spade
Event B: Drawing a diamond
$n(A \cup B) = n(A) + n(B)$
$n(A \cup B) = 13 + 13$
$n(A \cup B) = 26$
Therefore, there are 26 ways to draw 1 card that is a spade or a diamond.

3. a) $n + 9 \geq 0$
$n \geq -9$
$(n + 10)(n + 9)!$ is defined for $n \geq -9$, where $n \in$ I.
$(n + 10)(n + 9)! = (n + 10)[(n + 9)(n + 8)\ldots(3)(2)(1)]$
$(n + 10)(n + 9)! = (n + 10)(n + 9)(n + 8)\ldots(3)(2)(1)$
$(n + 10)(n + 9)! = (n + 10)!$
b) $n - 2 \geq 0$ AND $n \geq 0$
$n \geq 2$
$\frac{(n-2)!}{n!}$ is defined for $n \geq 2$, where $n \in$ I.

$$\frac{(n-2)!}{n!} = \frac{(n-2)(n-3)\ldots(3)(2)(1)}{n(n-1)(n-2)(n-3)\ldots(3)(2)(1)}$$

$$\frac{(n-2)!}{n!} = \frac{1}{n(n-1)}$$

$$\frac{(n-2)!}{n!} = \frac{1}{n^2 - n}$$