$$P(F)=\frac{n(F)}{n(O)}$$
$$P(F)=\frac{1584}{42504}$$
$$P(F)=\frac{6}{161}$$
The probability that a dealt hand contains five cards of the same colour is $\frac{6}{161}$, or about 0.037 267 or 3.727%.

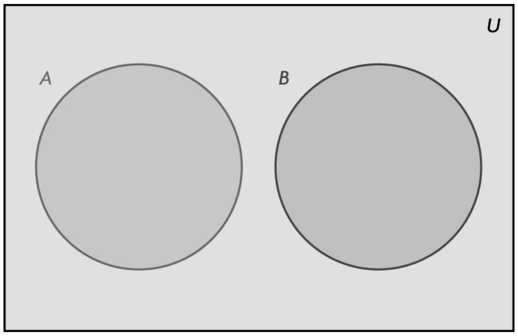
c) Let F represent a hand containing a four of a kind, and let O represent all euchre hands.
There are 6 different ways to have four of a kind in euchre (one for each rank of card). There are 20 ways to place the fifth card. Therefore, there are 120 ways to have a four of a kind in a dealt hand.
The total number of hands is ${}_{24}C_5$, or 42 504.
$$P(F)=\frac{n(F)}{n(O)}$$
$$P(F)=\frac{120}{42504}$$
$$P(F)=\frac{5}{1771}$$
The probability that a dealt hand will have four of a kind is $\frac{5}{1771}$, or about 0.002 823 or 0.282%.

8. Let A represent a playlist in which Emanuella's six favourite songs are played together. Let O represent all playlists.
The number of ways to arrange Emanuella's favourite songs so that they are together is ${}_6P_6 \cdot 25$, or $6! \cdot 25$. The number of ways to arrange the other 24 songs is ${}_{24}P_{24}$, or 24!. The number of ways to arrange 30 songs is ${}_{30}P_{30}$, or 30!. Therefore, the total number of playlists that contain the six favourite songs together is $25 \cdot 6! \cdot 24!$, and the total number of playlists is 30!.
$$P(A)=\frac{n(A)}{n(O)}$$
$$P(A)=\frac{6!\cdot 25\cdot 24!}{30!}$$
$$P(A)=\frac{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 25\cdot 24!}{30\cdot 29\cdot 28\cdot 27\cdot 26\cdot 25\cdot 24!}$$
$$P(A)=\frac{6\cdot 5\cdot 4\cdot 3\cdot 2}{30\cdot 29\cdot 28\cdot 27\cdot 26}$$
$$P(A)=\frac{6\cdot 5}{30}\cdot\frac{4}{28}\cdot\frac{3}{27}\cdot\frac{2}{26}\cdot\frac{1}{29}$$
$$P(A)=\frac{1}{7\cdot 9\cdot 13\cdot 29}$$
$$P(A)=\frac{1}{23751}$$
The probability that all 6 of Emanuella's favourite songs will be played together is $\frac{1}{23\,751}$, or about 0.000 042 1 or 0.004 21%.

9. Let C represent Stella dropping 2 loonies and one other coin. Let O represent all of the combinations of 3 coins possible.
Stella has 6 loonies and 6 other coins. The number of ways that 2 loonies and 1 other coin could be dropped is ${}_6C_2 \cdot {}_6C_1$.
$$n(C)={}_6C_2\cdot{}_6C_1$$
$$n(C)=\frac{6!}{(6-2)!\cdot 2!}\cdot\frac{6!}{(6-1)!\cdot 1!}$$
$$n(C)=\frac{6!}{4!\cdot 2!}\cdot\frac{6!}{5!\cdot 1!}$$
$$n(C)=\frac{6\cdot 5\cdot 4!}{4!\cdot 2\cdot 1}\cdot\frac{6\cdot 5!}{5!\cdot 1}$$
$$n(C)=\frac{6\cdot 5}{2}\cdot 6$$
$$n(C)=3\cdot 5\cdot 6$$
$$n(C)=90$$
The total number of ways that 3 coins can be dropped is ${}_{12}C_3$.
$$n(O)={}_{12}C_3$$
$$n(O)=\frac{12!}{(12-3)!\cdot 3!}$$
$$n(O)=\frac{12!}{9!\cdot 3!}$$
$$n(O)=\frac{12\cdot 11\cdot 10\cdot 9!}{9!\cdot 3\cdot 2\cdot 1}$$
$$n(O)=\frac{12\cdot 11\cdot 10}{3\cdot 2}$$
$$n(O)=\frac{1320}{6}$$
$$n(O)=220$$
Now determine the probability.
$$P(C)=\frac{n(C)}{n(O)}$$
$$P(C)=\frac{90}{220}$$
$$P(C)=\frac{9}{22}$$
The probability that exactly two of the dropped coins are loonies is $\frac{9}{22}$, or about 0.409 or 40.9%.

Lesson 5.4: Mutually Exclusive Events, page 338

1. a) Let A represent rolling a sum of 2. Let B represent rolling a sum of 8.
$A = \{1, 1\}$, $B = \{4, 4\}$.



b) A and B are mutually exclusive, because there is no way for the same two numbers to add up to both 2 and 8.

c) Outcome Table:

		Die 1			
	SUM	**1**	**2**	**3**	**4**
Die 2	**1**	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

Total number of outcomes = 16

$P(\text{sum of 2 or 8}) = \frac{2}{16}$

$P(\text{sum of 2 or 8}) = \frac{1}{8}$

The probability that Zach will roll a sum of 2 or 8 is $\frac{1}{8}$, 0.125 or 12.5%.

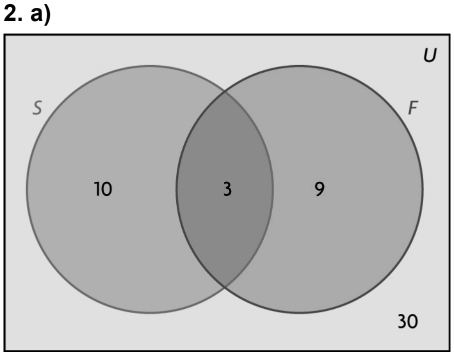
d) Doubles: {(1,1), (2,2), (3,3), (4,4)}
Sum of 6: {(2,4), (3,3), (4,2)}

$P(\text{doubles or sum of 6}) = \frac{6}{16}$

$P(\text{doubles or sum of 6}) = \frac{3}{8}$

The probability Zach will roll doubles or a sum of 6 is $\frac{3}{8}$, 0.375 or 37.5%.

2. a)



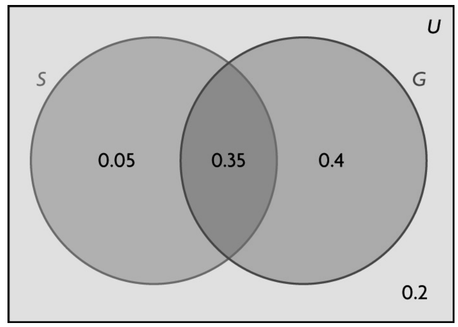
b) The events are not mutually exclusive. For example, the king of spades is both a face card and a spade.

c) $P(\text{face card or spade}) = \frac{22}{52}$

$P(\text{face card or spade}) = \frac{11}{26}$

The probability of drawing a face card or spade is $\frac{11}{26}$, or about 0.423 or 42.3%.

3. a) Let G represent going to the gym, and let S represent going shopping



b) No, these events are not mutually exclusive.
$P(G) + P(S) + P(\text{neither}) = 0.75 + 0.4 + 0.2 = 1.35$.
This exceeds 1, so there are favourable outcomes for events G and S that are common.

c) $P(G \cup S) = 1 - P(\text{neither})$
$P(G \cup S) = 1 - 0.2$
$P(G \cup S) = 0.8$

The probability that Maria will do at least one of these activities on Saturday is 0.8 or 80%.

4. a) No. e.g., 2 is both an even number and a prime number.

b) Yes. e.g., You cannot roll a sum of 10 and a roll of 7 at the same time.

c) Yes. e.g., You cannot walk and ride to school at the same time.

5. a) $P(A) = \frac{n(A)}{n(C)}$

$P(A) = \frac{144\ 945}{389\ 045}$

$P(A) = \frac{28\ 989}{77\ 809}$

The probability a person who is Métis lives in Alberta or British Columbia is $\frac{28\,989}{77\,809}$, or about 0.373 or 37.3%.

b) $P(M) = \frac{n(M)}{n(C)}$

$P(M) = \frac{119\ 920}{389\ 045}$

$P(M) = \frac{23\ 984}{77\ 809}$

The probability that a person who is Métis lives in Manitoba or Saskatchewan is $\frac{23\,984}{77\,809}$, or about 0.308 or 30.8%.

c) Yes, because these two events are mutually exclusive, so $P(A \cap M)$ is equal to 0.
d) The odds in favour of a person who is Métis living in one of the four western provinces are 264 865 : 124 180, or 52 973 : 24 836.
6. There are 12 stuffed dogs and bears, and 28 other prizes. Therefore, the odds in favour of winning either a stuffed dog or a stuffed bear are 12 : 28, or 3 : 7.
7. a) Let F represent the dice rolls with a sum of 5, let N represent the dice rolls with a sum of 9. Let O represent the set of all dice rolls.
F: {(1,4), (2,3), (3,2), (4,1)}
N: {(3,6), (4,5), (5,4), (6,3)}
$n(F \cup N) = 8$
$n(O) = 36$
The odds against the sum equaling 5 or 9 are 28 : 8, or 7 : 2.
b) Let E represent the dice rolls in which both dice are even numbers, and let S represent the dice rolls with a sum of 8. Let O represent the set of all dice rolls.
E: {(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)}
S: {(2,6), (3,5), (4,4), (5,3), (6,2)}
(2,6), (4,4) and (6, 2) are in both sets.
$n(E \cup S) = n(E) + n(S) - n(E \cap S)$
$n(E \cup S) = 9 + 5 - 3$
$n(E \cup S) = 11$
$n(O) = 36$
The odds against both dice being even numbers or the sum being 8 are 25 : 11.
8. a) Let S represent studying and V represent playing video games.
$P(S \cup V) = P(S) + P(V) - P(S \cap V)$
$0.8 = 0.4 + 0.6 - P(S \cap V)$
$P(S \cap V) = 0.2$
The probability that John will do both activities is 0.2 or 20%.
b) No. Since $P(S \cap V) \neq 0$, then $n(S \cap V) \neq 0$, so the sets of favourable outcomes for S and V are not disjoint.
9. a) No. e.g., One athlete won two ore more medals at the Summer and Winter Olympics.
b) Total number of medal winners = 307
The odds in favour of a Canadian medal winner winning two or more medals at the Summer Olympics are 21 : (307 – 21) or 21 : 286.
c) $n(S \cup W) = 20 + 47 + 1$
$n(S \cup W) = 68$
Total number of medal winners = 307.
The odds in favour of the athlete having won two or more medals is 68 : (307 – 68) or 68 : 239.
10. e.g., Tricia has a probability of 0.3 of cycling to school on any given day, and a probability of 0.2 of getting a ride from her older brother, Steve. Otherwise, she walks to school. What is the probability that she does not walk to school on any given day? (0.5)
11. e.g., There are 67 Grade 10 students that take art and 37 that take photography. If there are 87 students, how many take both? (17)
12. a) Let G represent wearing glasses and H represent having a hearing loss.
If 68% of seniors have a hearing loss, and 10% of these people do not wear glasses, then 10% · 68%, or 6.8% of seniors have a hearing loss but do not wear glasses. This means that 61.2% of seniors wear glasses and have a hearing loss.
$P(H \setminus G) = 6.8\%$
$P(G \cap H) = 61.2\%$
$P(G \setminus H) = P(G) - P(G \cap H)$
$P(G \setminus H) = 76\% - 61.2\%$
$P(G \setminus H) = 14.8\%$
The probability that this person will wear glasses and not have hearing aids is 14.8%.
b) Let G represent wears glasses and H represent having a hearing loss.
$P((G \cup H)') = 100\% - (76\% + 6.8\%)$
$P((G \cup H)') = 17.2\%$
The probability that this person will not wear glasses and not have hearing loss is 17.2%.
13. a) Let E represent the eights, and let K represent the kings. Let O represent all cards.

$n(E) = 4$
$n(K) = 4$
$n(E \cap K) = 0$
$n(E \cup K) = n(E) + n(K)$
$n(E \cup K) = 8$
$n(O) = 52$

$P(E \cup K) = \frac{n(E \cup K)}{n(O)}$
$P(E \cup K) = \frac{8}{52}$
$P(E \cup K) = \frac{2}{13}$

The probability of drawing an eight or a king is $\frac{2}{13}$, or about 0.154 or 15.4%.
b) Let R represent the red cards, and let F represent the face cards. Let O represent all cards.
$n(R) = 26$
$n(F) = 12$
$n(R \cap F) = 6$
$n(R \cup F) = n(R) + n(F) - n(R \cap F)$
$n(R \cup F) = 26 + 12 - 6$
$n(R \cup F) = 32$
$n(O) = 52$
$P(R \cup F) = \frac{n(R \cup F)}{n(O)}$
$P(R \cup F) = \frac{32}{52}$
$P(R \cup F) = \frac{8}{13}$
The probability of drawing a red card or a face card is $\frac{8}{13}$, or about 0.615 or 61.5%.
14. Let D represent the households that have one or more dogs, and let C represent the households that have one of more cats. Let O represent all Prairie households. $P(D) = 37\%$ $P(C) = 31\%$
a) $P(D \cup C) = 100\% - P((D \cup C)')$
$P(D \cup C) = 100\% - 47\%$
$P(D \cup C) = 53\%$
The probability that a Prairie household has a cat or dog is 53%.

b) $P(D \cup C) = P(D) + P(C) - P(D \cap C)$
$53\% = 37\% + 31\ \% - P(D \cap C)$
$53\% = 68\% - P(D \cap C)$
$P(D \cap C) = 15\%$
$P(C \setminus D) = P(C) - P(D \cap C)$
$P(C \setminus D) = 31\% - 15\%$
$P(C \setminus D) = 16\%$
The probability that a Prairie household has one or more cats, but no dogs, is 16%.
c) $P(D \setminus C) = P(D) - P(D \cap C)$
$P(D \setminus C) = 37\% - 15\%$
$P(D \setminus C) = 22\%$
The probability that a Prairie household has one or more dogs, but no cats, is 22%.
15. Let M represent snow on Monday and let T represent snow on Tuesday.
$P(M) = 60\%$
$P(T) = 40\%$
$P(M \cap T) = 20\%$
$P(M \cup T) = P(M) + P(T) - P(M \cap T)$
$P(M \cup T) = 60\% + 40\% - 20\%$
$P(M \cup T) = 80\%$
The probability that it will snow on Monday or on Tuesday is 80%.
16. Let S represent damage to the computer's power supply and let C represent damage to other components.
$P(S) = 0.15\%$
$P(C) = 0.30\%$
$P(S \cap C) = 0.10\%$
$P(S \cup C) = P(S) + P(C) - P(S \cap C)$
$P(S \cup C) = 0.15\% + 0.30\% - 0.10\%$
$P(S \cup C) = 0.35\%$
No. e.g., Since the probability of any form of damage is 0.35%, the computer does not need a surge protector.
17. Let the following variables represent the following blood types:
OP: type O+
ON: type O–
AP: type A+
AN: type A–
BP: type B+
BN: type B–
ABP: type AB+
ABN: type AB–
a) i) $P(\text{type O}) = P(OP) + P(ON)$
$P(\text{type O}) = 38\% + 7\%$
$P(\text{type O}) = 45\%$
The probability that a randomly selected Canadian has type O blood is 45%.
ii) $P(\text{negative}) = P(ON) + P(AN) + P(BN) + P(ABN)$
$P(\text{negative}) = 7\% + 6\% + 2\% + 1\%$
$P(\text{negative}) = 16\%$
The probability that a randomly selected Canadian has a negative blood type is 16%.
iii) $P(\text{A or B}) = P(AP) + P(AN) + P(BP) + P(BN)$
$P(\text{A or B}) = 34\% + 6\% + 9\% + 2\%$
$P(\text{A or B}) = 51\%$
The probability that a randomly selected Canadian has type A or B blood is 51%.

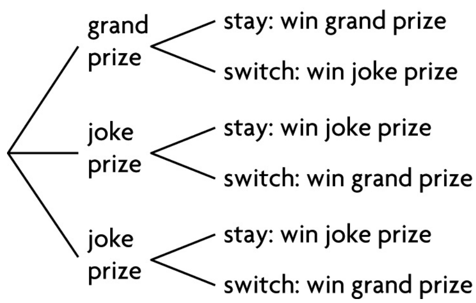
b) $P(\text{Dani}) = P(AP) + P(AN) + P(\text{A}BP) + P(\text{A}BN)$
$P(\text{Dani}) = 34\% + 6\% + 3\% + 1\%$
$P(\text{Dani}) = 44\%$
The probability that Dani can donate blood to the next person who needs a transfusion is 44%.
c) $P(\text{Richard}) = P(ABN) + P(AN) + P(BN) + P(\text{O}N)$
$P(\text{Richard}) = 1\% + 6\% + 2\% + 7\%$
$P(\text{Richard}) = 16\%$
The probability that Richard will be able to receive blood is 16%.
19. e.g., To determine the probability of two events that are not mutually exclusive, you must subtract the probability of both events occurring after adding the probabilities of each event. Example: Female students at a high school may play hockey or soccer. If the probability of a female student playing soccer is 62%, the probability of her playing in goal is 4%, and the probability of her either playing soccer or in goal is 64%, then the probability of her playing in goal at soccer is 62% + 4% – 64% = 2%.
19. A: student who likes rap music
B: student who likes blues music
C: student who likes rock music
O: all students
$P(A \setminus (B \cup C)) = 30\%$
$P(B \setminus (A \cup C)) = 13\%$
$P(C \setminus (A \cup B)) = 20\%$
$P((A \cap C) \setminus B) = 14\%$
$P((B \cap C) \setminus A) = 10\%$
The assumption is made that 100% of students like one of the three genres of music given.
$P(O) = P(A \setminus (B \cup C)) + P(B \setminus (A \cup C))$
$+ P(C \setminus (A \cup B)) + P((A \cap B) \setminus C) + P((A \cap C) \setminus B)$
$+ P((B \cap C) \setminus A) + P(A \cap B \cap C)$
$100\% = 30\% + 13\% + 20\% + P((A \cap B) \setminus C) + 14\%$
$+ 10\% + P(A \cap B \cap C)$
$100\% = 87\% + P((A \cap B) \setminus C) + P(A \cap B \cap C)$
$13\% = P((A \cap B) \setminus C) + P(A \cap B \cap C)$
The probability that a randomly selected student will like either all three types of music, or will like rap and blues, but not rock is 13%.
20. a) $P(A \cup B \cup C) = P(A) + P(B) + P(C)$
b) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$.

Applying Problem-Solving Strategies, page 343

A. e.g., I don't believe that it will change the probability, because nothing really changes.

B. There's a $\frac{2}{3}$ chance of choosing a joke prize, and a $\frac{1}{3}$ chance of choosing the grand prize.

Treating the joke prize and the small prize as equivalent:



After you select, Monty opens a door with a joke prize. If you originally selected the grand prize, and you switch, you will lose. But if you originally selected one of the two joke prizes, and you switch, you will win. So, if you switch, you will win $\frac{2}{3}$ of the time. The better strategy is to switch.

Lesson 5.5: Conditional Probability, page 350

1. a) These two events are dependent.
b) Let *R* represent the red die showing 4, let *S* represent rolling a sum that is greater than 7.

$$P(R) = \frac{1}{6}$$
$$P(S|R) = \frac{1}{2}$$
$$P(S \cap R) = P(R) \cdot P(S|R)$$
$$P(S \cap R) = \frac{1}{6} \cdot \frac{1}{2}$$
$$P(S \cap R) = \frac{1}{12}$$

The probability that Austin will win a point is $\frac{1}{12}$, or about 0.0833 or 8.33%.

2. a) These two events are dependent.
b) Let *A* represent the first card being a diamond, and let *B* represent the second card being a diamond.

$$P(A) = \frac{13}{52} \qquad P(A \cap B) = P(A) \cdot P(B|A)$$
$$P(A) = \frac{1}{4} \qquad P(A \cap B) = \frac{1}{4} \cdot \frac{4}{17}$$
$$P(B|A) = \frac{12}{51} \qquad P(A \cap B) = \frac{1}{17}$$
$$P(B|A) = \frac{4}{17}$$

The probability that both cards are diamonds is $\frac{1}{17}$, or about 0.0588 or 5.88%.

3. a) These two events are independent.
b) Let *D* represent drawing a diamond.

$$P(D) = \frac{1}{4}$$
$$P(D \cap D) = \frac{1}{4} \cdot \frac{1}{4}$$
$$P(D \cap D) = \frac{1}{16}$$

The probability that both cards are diamonds is $\frac{1}{16}$, 0.0625 or 6.25%.

4. a) i) Let *B* represent Lexie pulling a black sock from her drawer.

$$P(B) = \frac{6}{14} \qquad P(B \cap B) = P(B) \cdot P(B \mid B)$$
$$P(B) = \frac{3}{7} \qquad P(B \cap B) = \frac{3}{7} \cdot \frac{5}{13}$$
$$P(B \mid B) = \frac{5}{13} \qquad P(B \cap B) = \frac{15}{91}$$

The probability of drawing two black socks is $\frac{15}{91}$, or about 0.165 or 16.5%.

ii) Let *W* represent Lexie pulling a white sock from her drawer.

$$P(W) = \frac{8}{14} \qquad P(W \cap W) = P(W) \cdot P(W|W)$$
$$P(W) = \frac{4}{7} \qquad P(W \cap W) = \frac{4}{7} \cdot \frac{7}{13}$$
$$P(W|W) = \frac{7}{13} \qquad P(W \cap W) = \frac{4}{13}$$

The probability of drawing two white socks is $\frac{4}{13}$, or about 0.308 or 30.8%.

iii) Let *B* represent pulling a black sock, and let *W* represent pulling a white sock. Let *A* represent drawing a pair of socks.

$$P(A) = P(B \cap B) + P(W \cap W)$$
$$P(A) = \frac{15}{91} + \frac{4}{13}$$
$$P(A) = \frac{43}{91}$$

The probability of drawing a pair of socks is $\frac{43}{91}$, or about 0.473 or 47.3%.

b) No, the answers would not change, because there is still no replacement.