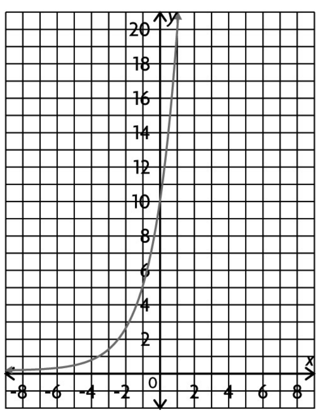
Chapter 7: Exponential and Logarithmic Functions

Lesson 7.1: Exploring the Characteristics of Exponential Functions, page 439

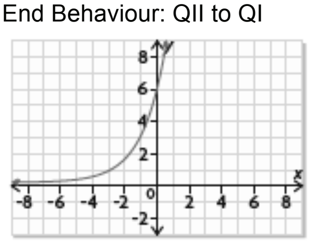
1. a) No, linear
b) Yes
c) No, quadratic
d) No, cubic
e) Yes
f) No, quadratic

2. b) No *x*-intercepts; *y*-intercept: $y = 1$
End behaviour: QII to QI
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
e) No *x*-intercepts; *y*-intercept: $y = 1$
End behaviour: QII to QI
Domain: $\{ x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$

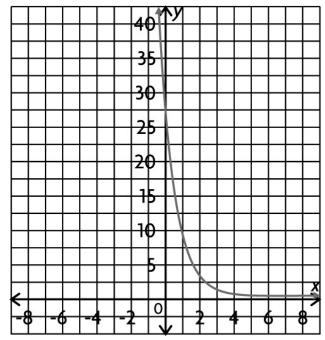
3. a) Number of *x*-intercepts: 0;
y-intercept: $y = 10$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI



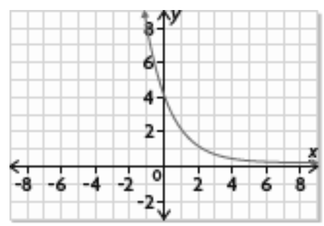
b) Number of *x*-intercepts: 0
y-intercept: $y = 6$
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI



c) Number of *x*-intercepts: 0
y-intercept: $y = 27$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI



d) Number of *x*-intercepts: 0
y-intercept: $y = 4$
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI



Lesson 7.2: Relating the Characteristics of an Exponential Function to Its Equation, page 448

1. a)

Term	First Difference	Second Difference
14 – 7	7	
28 – 14	14	7
56 – 28	28	14
112 – 56	56	28
224 – 112	112	56

I noticed that the difference between consecutive *y*-values doubles from one pair of values to the next. I know that there must be a base value of 2 being raised to an exponent and then multiplied by some constant term in the equation. The equation looks like $y = a(2^x)$. Thus, the answer is yes, because for each unit increase in *x*, the value of *y* doubles.
b)

Term	First Difference	Second Difference
3072 – 768	2304	
768 – 192	576	1728
192 – 48	144	432
48 – 12	36	108
12 – 3	9	27

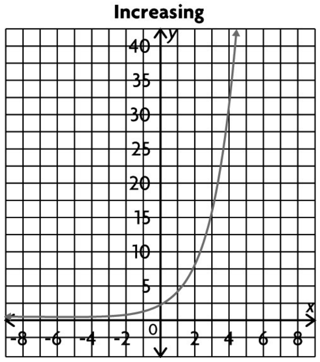
I noticed that the difference between consecutive *y*-values is not constant. The function is not linear.

I noticed each y-value is $\frac{1}{4}$ the previous y-value as x increases by 1. The function must be an exponential function with a base value of $\frac{1}{4}$ being raised to an exponent and then multiplied by some constant term in the equation. Thus, the answer is yes because for each unit increase in x, the value of y is divided by 4.

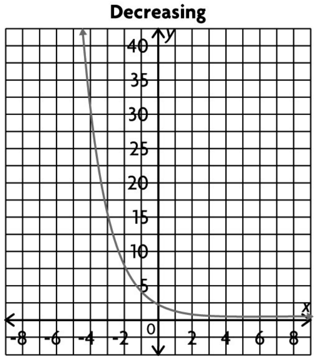
2.a) Number of x-intercepts: 0
y-intercept: $y = 4$; Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI
b) Number of x-intercepts: 0
y-intercept: $y = 2$; Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y > 0, y \in R\}$;
End Behaviour: QII to QI
c) Number of x-intercepts: 0; y-intercept: $y = 7$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI
d) Number of x-intercepts: 0; y-intercept: $y = 3$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI

3. e.g. Increasing exponential functions increase as x increases, whereas decreasing exponential functions decrease as x increases.

Increasing



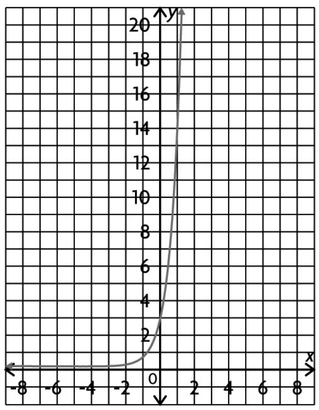
Decreasing



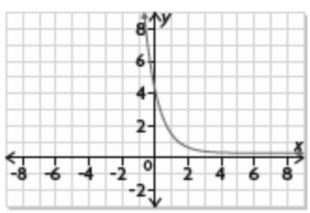
4. a) The y-intercept is 5 and the function is increasing.
$y = 5(2)^x$
$y = 5(2)^{(0)}$
$y = 5(1)$
$y = 5$
The function is increasing because the base is greater than 1.
b) The y-intercept is 2 and the function is decreasing.
$y = 2(0.5)^x$
$y = 2(0.5)^{(0)}$
$y = 2(1)$
$y = 2$
The function is decreasing because the base is less than 1.
c) The y-intercept is 10 and the function is increasing.
$y = 10(1.5)^x$
$y = 10(1.5)^{(0)}$
$y = 10(1.5)$
$y = 10$
The function is increasing because the base is greater than 1.
d) The y-intercept is 1 and the function is decreasing.
$y = (0.4)^x$
$y = (0.4)^{(0)}$
$y = 1$
The function is decreasing because the base is less than 1.

5. a) i) Yes, an exponential equation can be used to model the function because the rate of change in y-values doubles for each unit increase in x.
ii) y-intercept: $y = 1$, the function is increasing.
b) i) No, y increases by 2 as x increases by 1.
ii) y-intercept: $y = 3$, the function is increasing.
c) i) Yes, an exponential equation can be used to model the function because the rate of change in y-values get divided by 4 for each unit increase in x.
ii) y-intercept: $y = 64$, the function is decreasing.
d) i) No. e.g., y decreases, then increases, then decreases again.
ii) y-intercept: $y = 1$, the function is first decreasing than increasing as it reaches the y-intercept.

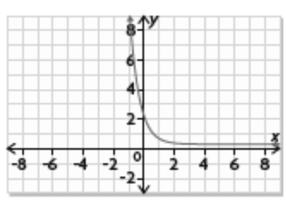
6. a) Number of x-intercepts: 0
y-intercept: $y = 3$; Domain: $\{x \mid x \in \mathrm{R}\}$
Range: $\{y \mid y > 0, y \in \mathrm{R}\}$
End Behaviour: QII to QI



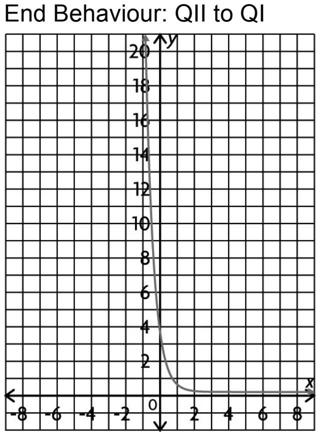
b) Number of x-intercepts: 0; y-intercept: $y = 4$
Domain: $\{x \mid x \in \mathrm{R}\}$; Range: $\{y \mid y > 0, y \in \mathrm{R}\}$
End Behaviour: QII to QI



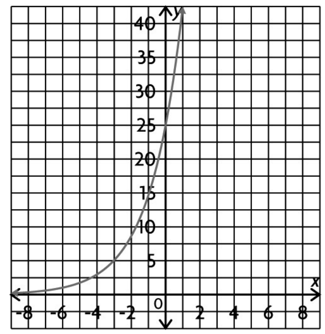
c) Number of x-intercepts: 0; y-intercept: $y = 2$
Domain: $\{x \mid x \in \mathrm{R}\}$; Range: $\{y \mid y > 0, y \in \mathrm{R}\}$
End Behaviour: QII to QI



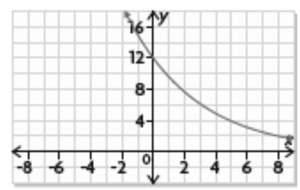
d) Number of x-intercepts: 0; y-intercept: $y = 3.5$
Domain: $\{x \mid x \in \mathrm{R}\}$; Range: $\{y \mid y > 0, y \in \mathrm{R}\}$
End Behaviour: QII to QI



e) Number of x-intercepts: 0; y-intercept: $y = 25$
Domain: $\{x \mid x \in \mathrm{R}\}$; Range: $\{y \mid y > 0, y \in \mathrm{R}\}$
End Behaviour: QII to QI



f) Number of x-intercepts: 0; y-intercept: $y = 12$
Domain: $\{x \mid x \in \mathrm{R}\}$; Range: $\{y \mid y > 0, y \in \mathrm{R}\}$
End Behaviour: QII to QI

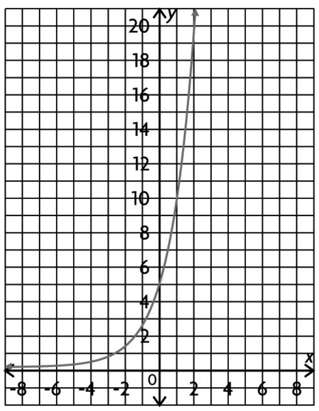


7. a) The y-intercept is positive and since the base is 8, it must mean the function is increasing because the base is larger than 1.
b) The y-intercept is positive and since the base is 0.6, it must mean the function is decreasing because the base is less than 1.
c) The y-intercept is positive and since the base is e, it must mean the function is increasing because e is greater than 1.

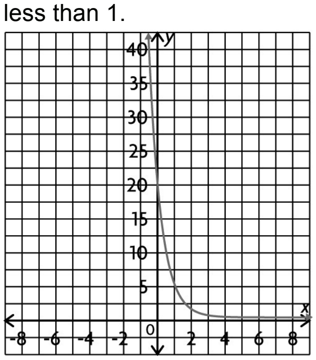
8. a) Number of x-intercepts: 0; y-intercept: $y = 4$
Domain: $\{x \mid x \in \mathrm{R}\}$; Range: $\{y \mid y > 0, y \in \mathrm{R}\}$
End Behaviour: QII to QI
b) Number of x-intercepts: 0; y-intercept: $y = 8$
Domain: $\{x \mid x \in \mathrm{R}\}$; Range: $\{y \mid y > 0, y \in \mathrm{R}\}$
End Behaviour: QII to QI
c) Number of x-intercepts: 0; y-intercept: $y = 3$
Domain: $\{x \mid x \in \mathrm{R}\}$; Range: $\{y \mid y > 0, y \in \mathrm{R}\}$
End Behaviour: QII to QI
d) Number of x-intercepts: 0; y-intercept: $y = 10$
Domain: $\{x \mid x \in \mathrm{R}\}$; Range: $\{y \mid y > 0, y \in \mathrm{R}\}$
End Behaviour: QII to QI
e) Number of x-intercepts: 0; y-intercept: $y = 30$
Domain: $\{x \mid x \in \mathrm{R}\}$; Range: $\{y \mid y > 0, y \in \mathrm{R}\}$
End Behaviour: QII to QI
f) Number of x-intercepts: 0; y-intercept: $y = 1$
Domain: $\{x \mid x \in \mathrm{R}\}$; Range: $\{y \mid y > 0, y \in \mathrm{R}\}$
End Behaviour: QII to QI
g) Number of x-intercepts: 0; y-intercept: $y = 3$
Domain: $\{x \mid x \in \mathrm{R}\}$; Range: $\{y \mid y > 0, y \in \mathrm{R}\}$
End Behaviour: QII to QI
h) Number of x-intercepts: 0; y-intercept: $y = 45$
Domain: $\{x \mid x \in \mathrm{R}\}$; Range: $\{y \mid y > 0, y \in \mathrm{R}\}$
End Behaviour: QII to QI

9. a) It is a decreasing exponential function because it is a function with an exponent of *x* and the base is less than 1.
b) It is not a decreasing exponential function because although it has an exponent of *x*, the base is greater than 1.
c) It is a decreasing exponential function because it is a function with an exponent of *x* and the base is less than 1.
d) It is not a decreasing exponential function because although it has an exponent of *x*, the base is greater than 1.

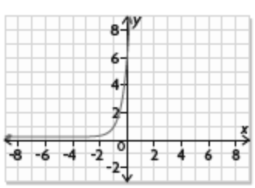
10. a) It is an increasing function because its base is more than 1.



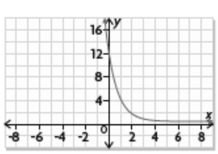
b) It is a decreasing function because its base is less than 1.



c) It is an increasing function because its base is more than 1.

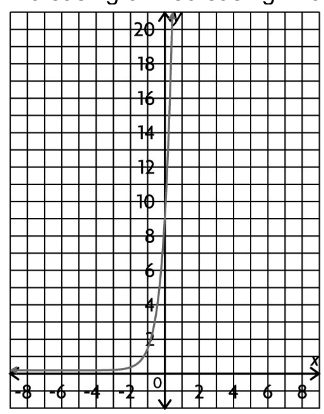


d) It is a decreasing function because its base is less than 1.

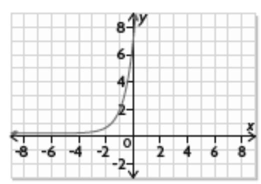


11. a) It is an increasing function with a *y*-intercept of (0,6). It matches with iii).
b) It is a decreasing function with a *y*-intercept of (0, 4). It matches with i).
c) It is a decreasing function with a *y*-intercept of (0, 2). It matches with ii).
d) It is an increasing function with a *y*-intercept of (0, 3). It matches with iv).

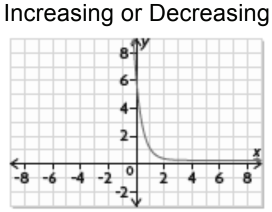
12. a) *y*-intercept: $y = 9$; Base: 7
Domain: $\{x \mid x \in \mathrm{R}\}$; Range: $\{y \mid y > 0, y \in \mathrm{R}\}$
Increasing or Decreasing: Increasing



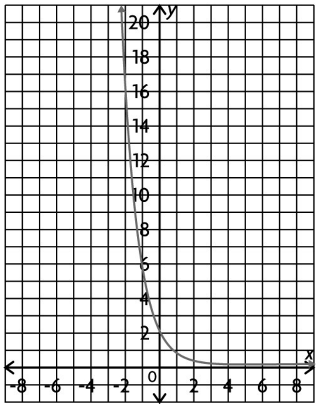
b) *y*-intercept: $y = 7$; Base: 4
Domain: $\{x \mid x \in \mathrm{R}\}$; Range: $\{y \mid y > 0, y \in \mathrm{R}\}$
Increasing or Decreasing: Increasing



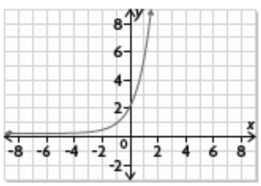
c) *y*-intercept: $y = 6$; Base: $\frac{1}{7}$
Domain: $\{x \mid x \in \mathrm{R}\}$; Range: $\{y \mid y > 0, y \in \mathrm{R}\}$
Increasing or Decreasing: Decreasing



d) y-intercept: $y = 2$; Base: 0.35
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
Increasing or Decreasing: Decreasing



e) y-intercept: $y = 2$; Base: e
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
Increasing or Decreasing: Increasing



13. a) i) Range: $\{y \mid y > 0, y \in R\}$ a: 2 b: 0.5
This is a decreasing function.
ii) Range: $\{y \mid y > 0, y \in R\}$ a: 1 b: 3
This is an increasing function.
iii) Range: $\{y \mid y > 0, y \in R\}$ a: 3 b: 0.5
This is a decreasing function.
iv) Range: $\{y \mid y > 0, y \in R\}$ a: 2 b: 4
This is an increasing function.

b) i) This function matches with the graph B. because it is a decreasing function with a y-intercept of 2.
ii) This function matches with graph D because it is an increasing function with a y-intercept of 1.
iii) This function matches with the graph A. because it is a decreasing function with a y-intercept of 3.
iv) This function matches with the graph C. because it is an increasing function with a y-intercept of 2.

14. a) e.g. An example of an increasing exponential function with a y-intercept of 5 would be $y = 5(2)^x$ while an example of a decreasing exponential function with a y-intercept of 5 would be $y = 5(0.25)^x$.
b) e.g. Same: number of x-intercepts, y-intercepts, and behaviour, domain, and range. Different: rate of change (increasing vs. decreasing function.

15. Yes, because all exponential functions have the form $y=a(b)^x$. Polynomial functions are characterized by having a numerical exponent while exponential functions have variable exponents.

16.

	$f(x) = ax + b$	$f(x) = a(b)^x$
Number of x-Intercepts	1	0
y-Intercept	b	a
End Behaviour	QIII to QI or QII to QIV	QII to QI
Domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
Range	$\{y \mid y \in R\}$	$\{y \mid y > 0, y \in R\}$
Increasing or Decreasing	Either increasing ($a > 0$) or decreasing ($a < 0$)	Either increasing ($b > 1$) or decreasing ($b < 1$)

17. The number of x-intercepts, the end behaviour, the domain, and the range are common to all exponential functions. The y-intercept and whether the function increases or decreases are unique to the function.

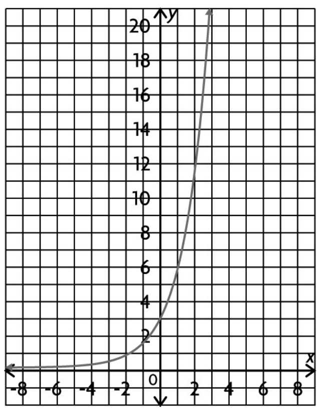
18. a) Student A:
y-intercept: $y = 80$
domain: $\{x \mid x \geq 0, x \in R\}$
range: $\{y \mid 0 < y \leq 8, y \in R\}$
Student B:
y-intercept: $y = 100$
domain: $\{x \mid x \geq 0, x \in R\}$
range: $\{y \mid 0 < y \leq 100, y \in R\}$
b) e.g., Concentration of caffeine in blood naturally decreases over time as the kidneys filter it from the blood into the urine.
c) Student B consumed more caffeine. Student B processed the caffeine more quickly. This can be seen from how much they initially started with and how much they consumed in four hours. Student A had 80 mg of caffeine at the start and consumed 60 mg after four hours. Student B started with 100 mg of caffeine and within the same time period, had consumed 80 mg.
d) Both Student A and B had about 20 mg of caffeine in their body after four hours.
e) The y-intercepts are the values representing the initial amount of caffeine in the students' bodies. The values are different because the students drank different amounts of the same drink, or they drank different drinks that did not have the same concentration of caffeine.

19. a)

Hour	Students who are Told
0	3
1	6
2	12
3	24
4	48
5	96

b) Yes. e.g., After each hour, the number of students who are told in that hour about the presentation doubles.
c) $y = 3(2)^x$; domain: $\{x \mid x \geq 0, x \in \mathrm{R}\}$
range: $\{y \mid y \geq 3, y \in \mathrm{R}\}$

d) After 8 h, 768 students would be told.

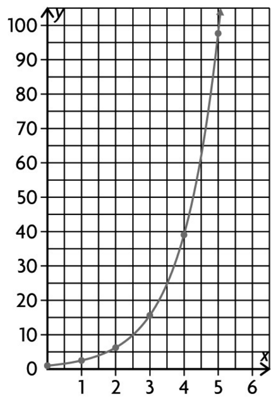


20. a) Yes. e.g., Any data with a constant doubling time can be expressed with an exponential function.
b) $y = 4(1.26)^x$; *a* represents the initial number of requests, *b* represents the rate of growth of the number of requests, *x* represents the amount of time in hours since the news broke, *y* represents the total number of interview requests.
c) domain: $\{x \mid x > 0, x \in \mathrm{N}\}$
range: $\{y \mid y > 4, y \in \mathrm{N}\}$
d)

Time	*x*	*y*
9:00 a.m.	0	4.0
10:00 a.m.	1	5.0
11:00 a.m.	2	6.4
12:00 p.m.	3	8.0
1:00 p.m.	4	10.1
2:00 p.m.	5	12.7
3:00 p.m.	6	16.0
4:00 p.m.	7	20.2

Lesson 7.3: Modelling Data Using Exponential Functions, page 461

1. a) This data set does not involve exponential growth or decay because the differences are constant. It is a linear function.
b) This data set does involve exponential growth because it has constant ratios between the *y*-values with each consequent unit value of *x*. The function is $y = (2.5)^x$.



c) This data set does not involve exponential growth or decay because the differences are not constant, or a constant ratio.
d) This data set does not involve exponential growth or decay the differences are not constant, or a constant ratio.

2. a) Using a graphing calculator, the exponential regression function for the data is $y = 10.097\ldots(0.200\ldots)^x$.
Domain: $\{x \mid x \in \mathrm{R}\}$; Range: $\{y \mid y > 0, y \in \mathrm{R}\}$
y-intercept: $y = 10.097\ldots$;
End Behaviour: QII to QI
This function is decreasing and shows exponential decay.
b) Using a graphing calculator, the exponential regression function for the data is $y = 2.780\ldots(1.054\ldots)^x$.
Domain: $\{x \mid x \in \mathrm{R}\}$; Range: $\{y \mid y > 0, y \in \mathrm{R}\}$
y-intercept: $y = 2.780\ldots$; End Behaviour: QII to QI
This function is increasing and shows exponential growth.

3. a)

Years since Retirement	Rent ($)	Ratios
0	9600	
1	9960	1.037…
2	10 344	1.038…
3	10 752	1.039…

Yes, an exponential model can represent the data since the ratio of consecutive pairs of *y*-values are close.