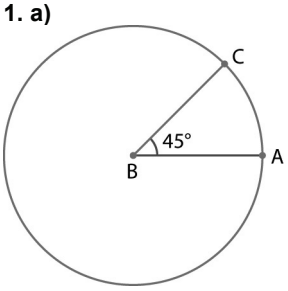
Chapter 8: Sinusoidal Functions

Lesson 8.1: Understanding Angles, page 519

1. a)

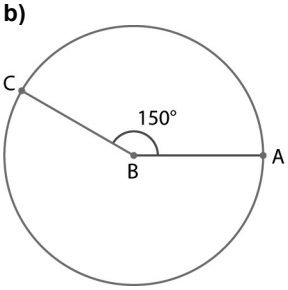


e.g., 45° is $\frac{1}{4}$ of 180°.

180° is slightly less than 3.2 radians.

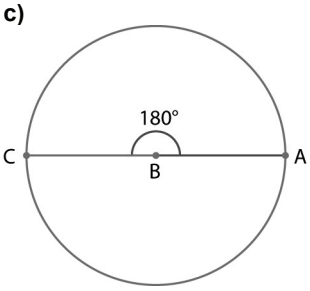
I can estimate 45° as $\frac{3.2}{4}$, or 0.8 radians.

b)



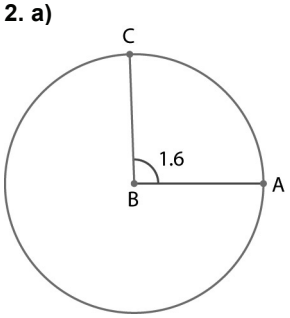
e.g., 150° is 30° less than 180°.
180° is slightly less than 3 radians.
30° is half of 60°, which is slightly more than 1 radian. So I can estimate 30° as $\frac{1}{2}$, or 0.5 radians.
I can estimate 150° as 3 – 0.5, or 2.5 radians.

c)



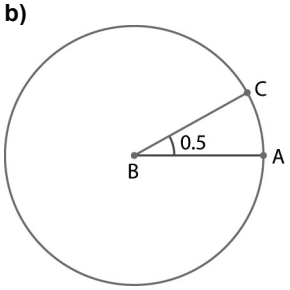
e.g., 180° is equivalent to π radians. So 180° is about 3.0 radians.

2. a)



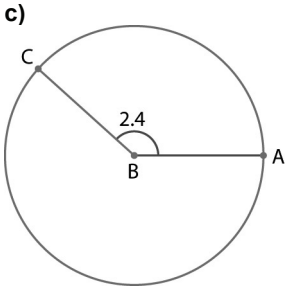
e.g., 1.6 radians is one half of 3.2 radians.
3.2 radians is approximately 180°. One half of 180° is 90°. Therefore I estimate that 1.6 radians is about 90°.

b)



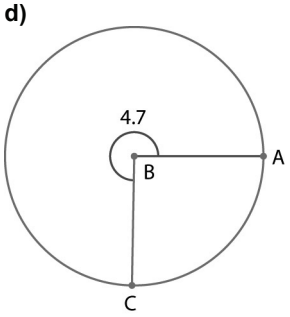
e.g., 0.5 radians is one half of 1 radian. 1 radian is about 60°. One half of 60° is 30°. Therefore I estimate that 0.5 radians is about 30°.

c)



e.g., 2.4 radians is halfway in between 1.6 radians and 3.2 radians. 1.6 radians is about 90° and 3.2 radians is about 180°. Halfway between 90° and 180° is 135°. Therefore I estimate that 2.4 radians is about 135°.

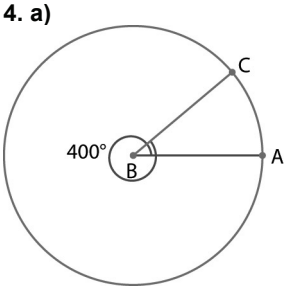
d)



e.g., 4.7 radians is about halfway in between 3.2 radians and 6.3 radians. 3.2 radians is about 180° and 6.3 radians is about 360°. Halfway between 180° and 360° is 270°. Therefore I estimate that 4.7 radians is about 270°.

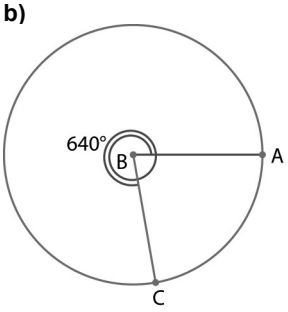
3. a) It is 60°. It is an angle that is greater than 45° but smaller than 90°.
b) It is 235°. It is sum of 180° of 55°.
c) It is 390°. It is the sum of 360° and 30°.

4. a)



e.g., 400° is in between 360° + 30°, or 390°, and 360° + 45°, 405°. 360° is about 6.3 radians. 30° is about 0.5 radians. 45° is about 0.8 radians. So 400° is in between 6.3 + 0.5, or 6.8 radians, and 6.3 + 0.8, or 7.1 radians. Since 400° is closer to 405° than 390°, I estimate that 400° is about 7.0 radians.

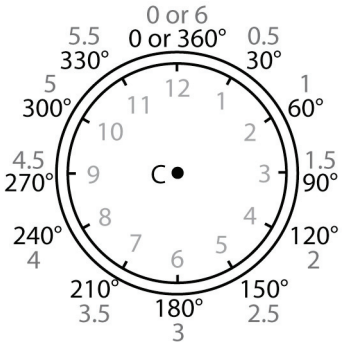
b)



640° is slightly above 360° + 270°, or 630°. 360° is about 6.3 radians and 270° is about 4.7 radians. So 630° is about 6.3 + 4.7, or 11.0 radians. Since 640° is close 630°, I estimate that 640° is about 11.0 radians.

5. a) e.g., 8.1 is about 6.3 + 2.0
6.3 radians is about 360°.
2 radians is about 90°.
360° + 90° = 450°
Therefore I estimate that 8.1 radians is about 450°.
b) e.g., 10.5 = 6.3 + 3.2 + 1
6.3 radians is about 360°.
3.2 radians is about 180°.
1 radian is about 60°.
360° + 180° + 60° = 600°
Therefore I estimate that 10.5 radians is about 600°.

6. a) A clock has 60 minutes in a full circle. That means that each minute is $\frac{360°}{60 \text{ min}} = 6°$.



i) $\frac{120°}{6°}$ = 20 minutes. It will be 9:20 am.

ii) $\frac{330°}{6°}$ = 55 minutes. It will be 9:55 am.

iii) $\frac{690°}{6°}$ = 115 minutes. It will be 10:55 am.

b) i) e.g., 120° = 180° – 60°
180° is about 3.1 radians.
60° is about 1 radian.
3.1 – 1 = 2.1 radians
Therefore I estimate that 120° is about 2.1 radians.
ii) e.g., 330° = 360° – 30°
360° is about 6.3 radians.
30° is about 0.5 radians.
6.3 – 0.5 = 5.8 radians
Therefore I estimate that 330° is about 5.8 radians.
iii) e.g., 690° = 360° + 360° – 30°
360° is about 6.3 radians.
30° is about 0.6 radians.
6.3 + 6.3 – 0.6 = 12.0 radians
Therefore I estimate that 690° is about 12.0 radians.

7. a) The formula for the circumference of a circle is $C = 2\pi r$. Here, the radius is 3 m:
$C = 2\pi 3$
$C = 18.849\ldots$ m
To determine the distance travelled, determine a third of the circumference:

$$\frac{C}{3} = \frac{18.849...}{3}$$

$$\frac{C}{3} = 6.283...$$

I travelled about 6.28 m.
b) A third of the circle is 360°/3 = 120°.
120° = 180° – 60°
180° is about 3.2 radians.
60° is about 1 radian.
3.2 – 1 = 2.2 radians
Therefore 120° is about 2.0 radians.
The central angle created was 120° or 2.0 radians.

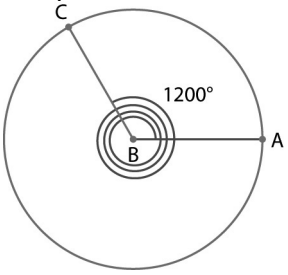
8. a) 2 = 1 + 1
1 radian is about 60°.
60° + 60° = 120°
2 radians is about 120°.
Therefore 2 radians is greater than 100°.
b) 0.5 is one half of 1.
1 radian is about 60°.
One half of 60° is 30°.
0.5 radians is about 30°.
Therefore 45° is greater than 0.5 radians.
c) 5 = (5)(1)
1 radian is about 60°.
(5)(60°) = 300°
5 radians is about 300°.
Therefore 5 radians is greater than 280°.
d) 6.5 = (6)(1) + 0.5
1 radian is about 60°.
0.5 radians is about 30°.
(6)(60°) + 30° = 390°
6.5 radians is about 390°.
Therefore 400° is greater than 6.5 radians.

9. Disagree, e.g., the measure of an angle independent of the circle radius. The central angle, measured in radians, in a circle with a radius of 5 m will be the same as an equivalent angle in a circle with a radius of 10 m. For example, 180° is equal to π radians for a circle of any radius.

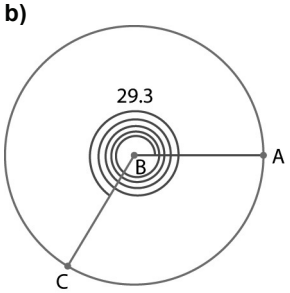
10. e.g., Use benchmarks to estimate the radian measure equivalent of angles greater than 360°, where 1 radian is about 60°, 3.2 radians is about 180° and 6.3 radians is about 360°. For example, determine the radian measure of
i) 480° **ii)** 525° **iii)** 650°
i) 480° = 8 · 60°, and 8 · 1 = 8
OR 480° = 360° + 90° + 30°, and
6.3 + 1.6 + 0.5 = 8.4
ii) 525° < 540°
540° = 9 · 60°, and 9 · 1 = 9
OR 540° = 360° + 180°, and 6.3 + 3.2 = 9.5
ii) 650° < 660°
660° = 11 · 60°, and 11 ·1 = 11
OR 660° = 360° + 180° + 120°, and

$$2\pi + \pi + \frac{\pi}{3} = \frac{10\pi}{3}$$

11. a)



b)



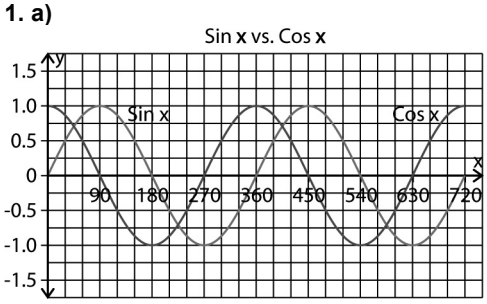
12. a) The estimate is 4.0 radians. This is because the sum of 3.2 radians (180°) and 0.8 radians (45°) is 4.0 radians or 225°. The measured angle is 227°.

b) The estimate is 3.0 radians or about 170°. This is because it looks slightly less than 3.2 radians. The measured angle is 170°.

c) This estimate is 5.9 radians because the section of the circle that is not measured looks to be approximately a sixteenth of the circle. 6.3 divided by 16 is about 0.4. 6.3 subtracted by 0.4 is 5.9 radians. This is approximately 337° in degree. The measured angle is 342°.

Lesson 8.2: Exploring Graphs of Periodic Functions, page 524

1. a)



b) When the value of sin *x* is a maximum, cos *x* is at zero. When the value of cos *x* is a minimum, *x* is equal to 180° and 540°.
c) When the value of cos *x* is a maximum, sin *x* is at zero. The value of sin *x* is a minimum when *x* is equal to 270° and 630°.
d)