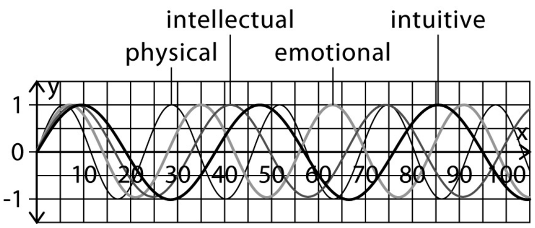
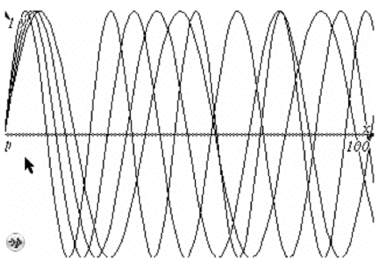
b) The graph of $y = 2 \sin x$ first crosses the midline while increasing at $x = 0°$. Therefore, it has not shifted. The graph of $y = -2 \sin x$ first crosses the midline while increasing at $x = 180°$. So, either graph could have a translation of π or 180° to the left or right in order to match the other graph.
c) Yes, e.g., graphs of functions of the form $y = a \sin x$ and $y = -a \sin x$ are horizontal translations of each other by 180° or π.

Math in Action, page 562



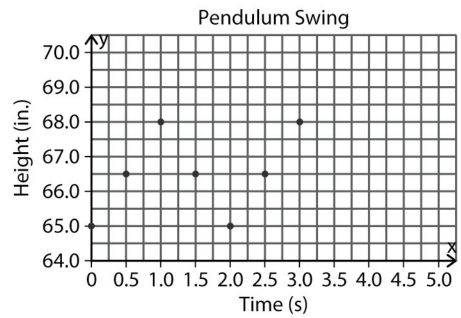


• All four cycles will again be at zero at 201 894 days. The person will be about 553 years old. A point at which all the cycles are at zero together is the least common multiple of the periods:
$23 \cdot (2^2 \cdot 7) \cdot (3 \cdot 11) \cdot 19 = 403\ 788$
However, since sine functions intersect the x-axis halfway through their period, the first point at which all the cycles will be zero is half the number above, or 201 894.

• The first maximum for the physical cycle will occur at 5.75 days and every 23 days after (28.75, 51.75, and so on). The first maximum for the emotional cycle will be at 7 days and then every 28 days after (35, 63, 91, and so on). These two maximum values will never coincide, because one is never a whole number and the other is always a whole number. The same is true for the minimum values.
• Today, I am 6239 days old. According to my equations, my physical and intuitive states should be strong, my intellectual state is increasing, and my emotional state is in decline. No, these charts do not match how I feel today. Today, I feel my intellectual and emotional states are strong and my physical state is average. My intuitive state is in decline.

Lesson 8.5: Modelling Data with Sinusoidal Functions, page 571

1. a)



b) The equation of the sinusoidal regression function is
$y = 1.5 \sin (3.141\ldots x - 1.570\ldots) + 66.5$.

2. a)

Revolution	Time (s)	Height (ft)
0	0	11
$\frac{1}{4}$	0.5	6
$\frac{1}{2}$	1	1
$\frac{3}{4}$	1.5	6
1	2	11
$1\frac{1}{4}$	2.5	6
$1\frac{1}{2}$	3	1
$1\frac{3}{4}$	3.5	6
2	4	11

b) The equation of the sinusoidal regression function is
$y = 5 \sin (3.141\ldots x + 1.570\ldots) + 6$.

c)
$$\begin{aligned}
y &= 5\sin(3.141...x + 1.570...) + 6 \\
y &= 5\sin(3.141...(1.15) + 1.570) + 6 \\
y &= 5\sin(3.612... + 1.570...) + 6 \\
y &= 5\sin(5.183...) + 6 \\
y &= 5(-0.891...) + 6 \\
y &= -4.455... + 6 \\
y &= 1.544...\text{ft}
\end{aligned}$$

Convert the decimal feet to feet and inches.
$$\begin{aligned}
y &= 1.544...\text{ft} \\
y &= 1\text{ ft } (0.544...\cdot 12)\text{ in.} \\
y &= 1\text{ ft } 6.539...\text{ in.}
\end{aligned}$$

The height at 1.15 s is 1 ft 7 in. high.

d) $y = 5\sin(3.141...x + 1.570...) + 6$

$y = 5\sin(3.141...(7.2) + 1.570) + 6$

$y = 5\sin(22.619... + 1.570...) + 6$

$y = 5\sin(24.190...) + 6$

$y = 5(-0.809...) + 6$

$y = -4.045... + 6$

$y = 1.954...\text{ft}$

Convert the decimal feet to feet and inches.

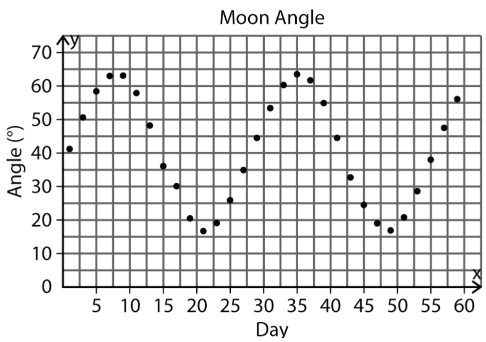
$y = 1.954...\text{ft}$

$y = 1\text{ ft }\left(0.954...\cdot 12\right)\text{ in.}$

$y = 1\text{ ft }11.458...\text{ in.}$

The height at 7.2 s is 1 ft 11 in. high.

3. a) The period is about 28 days.



b) e.g., The data appears to be sinusoidal, because the data appears to follow a sinusoidal pattern.

c) The equation of the sinusoidal regression function is

y = 22.954… sin (0.230… x – 0.208…) + 40.284…

d) The maximum altitude will be $a + d$, or 63.2°.

e) $y = 22.954...\sin(0.230...x - 0.208...) + 40.284...$

$y = 22.954...\sin(0.230...(100) - 0.208...) + 40.284...$

$y = 22.954...\sin(23.034... - 0.208) + 40.284...$

$y = 22.954...\sin(22.825...) + 40.284...$

$y = 22.954...(-0.740...) + 40.284...$

$y = -17.006... + 40.284...$

$y = 23.277...$

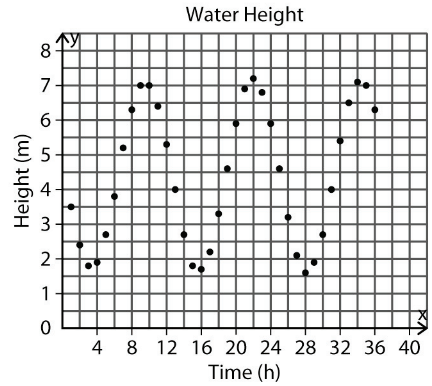
On day 100, the altitude of the Moon will be about 23.3°.

f) Since day 1 represents April 1, the days represented by May are 31-61. The first day in this domain of numbers that the Moon's altitude will drop below 30° is day 44, or May 14.

4. a) e.g., Tide heights could be sinusoidal, because the tide moves in a cyclical pattern and the data follows a sinusoidal pattern.

b) The equation of the sinusoidal regression function is

y = 2.726… sin (0.505… x + 3.016…) + 4.426…, which matches the data fairly closely.



c) Maximum = $d + a$

Maximum = 4.426… + 2.726…

Maximum = 7.152…

Minimum = $d - a$

Minimum = 4.426… – 2.726…

Minimum = 1.700…

The height at high tide is about 7.2 m. The height at low tide is about 1.7 m.

d) $\text{Period} = \dfrac{2\pi}{b}$

$\text{Period} = \dfrac{2\pi}{0.505...}$

$\text{Period} = 12.418...h$

$\text{Period} = 12\text{ h, }25.084...\text{ min}$

It takes the tide 12 h, 25 min to cycle from high tide to low tide and back again.

e) Substituting $x = 50$:

$y = 2.726...\sin(0.505...x + 3.016...) + 4.426...$

$y = 2.726...\sin(0.505...(50) + 3.016) + 4.426...$

$y = 2.726...\sin(25.298... + 3.016) + 4.426...$

$y = 2.726...\sin(28.315...) + 4.426...$

$y = 2.726...(-0.040...) + 4.426...$

$y = -0.111... + 4.426...$

$y = 4.314...$

At hour 50, the tide will be 4.3 m high.

5. At t = 6.145… h, the tide is coming in, and is 4 m above the seabed. At t = 12.975…, the tide is going out, and is 4 m above the seabed. The length of time it is safe to sail is

12.975… **–** 6.145… = 6.83 hours, or 6 hours 50 minutes.

6. a) The equation of the sinusoidal regression function for the average high temperatures is

y = 19.581… sin (0.462…x – 1.662…) + 6.238….

b) The equation of the sinusoidal regression function for the average low temperatures is

y = 16.992… sin (0.470…x – 1.751) – 4.738…

c) The equation of the sinusoidal regression function for the average temperatures is
y = 18.388… sin (0.464…x – 1.694…) + 0.686….
d) e.g., The amplitudes, phase shifts and periods are similar; the equations of the midlines are different.
e) e.g., Yes, but it is more difficult to fit a curve to record high data.
f) If January 15 to December 15 is represented by the numbers 1 to 12, then October 1 will be represented by 9.5.

$$y = 19.581\ldots\sin(0.462\ldots x - 1.662\ldots) + 6.238\ldots$$
$$y = 19.581...\sin(0.462...(9.5) - 1.662...) + 6.238...$$
$$y = 19.581...\sin(4.391... - 1.662...) + 6.238...$$
$$y = 19.581...\sin(2.728...) + 6.238...$$
$$y = 19.581...(0.401...) + 6.238...$$
$$y = 7.853... + 6.238...$$
$$y = 14.092...$$

On October 1, the average high temperature will be about 14.1 °C.

7. a) The equation of the sinusoidal regression function for the average temperatures is
y = 20.341… sin (0.536… x – 2.332…) – 2.706….
b) The maximum value of the regression is $d + a$, or 17.635…°C or approximately 17.6 °C. The minimum value of the regression is $d - a$, or –23.048…°C or approximately 23.0 °C. The error in the maximum and minimum shows that while the sinusoidal regression function approximates the relation very closely, it is not 100% accurate. However, the function is reasonably accurate.
c) June next year is 6 months away from December, the last data point. The month number for June next year is 30.

$$y = 20.341\ldots\sin(0.536\ldots x - 2.332\ldots) - 2.706\ldots$$
$$y = 20.341...\sin(0.536...(30) - 2.332...) - 2.706...$$
$$y = 20.341...\sin(16.083... - 2.332...) - 2.706...$$
$$y = 20.341...\sin(13.751...) - 2.706...$$
$$y = 20.341...(0.926...) - 2.706...$$
$$y = 18.847... - 2.706$$
$$y = 16.140...$$

The average temperature in June of next year will be 16.1 °C.

8. a) The equation of the sinusoidal regression function is
y = 4.207… sin (0.017…x – 1.398…) + 12.411….
b) The midline is y = 12.411… because $y = d$.
The amplitude is 4.207… since a is equal to the amplitude.
Maximum = $d + a$
Maximum = 12.411… + 4.207…
Maximum = 16.619…
Convert from decimal hours to hours and minutes.
Maximum = 16 h (0.619 · 60) min
Maximum = 16 h 37 min
Minimum = $d - a$
Minimum = 12.411… – 4.207…
Minimum = 8.204…
Convert from decimal hours to hours and minutes.
Minimum = 8 h (0.204… · 60) min
Minimum = 8 h 12 min
The range of the data is from 8 h 12 min to 16 h 37 min.
c) The day with the most hours of sunlight is the x value for the maximum value. The day with most hours of sunlight is day 171.
d) $y = 4.207...\sin(0.017...x - 1.398...) + 12.411...$

$$y = 4.207...\sin(0.017...(30) - 1.398...) + 12.411...$$
$$y = 4.207...\sin(0.520... - 1.398...) + 12.411...$$
$$y = 4.207...\sin(-0.878...) + 12.411...$$
$$y = 4.207...(0.769...) + 12.411...$$
$$y = -3.238... + 12.411...$$
$$y = 9.173...$$
$$y = 9 \text{ h, } 10.409... \text{ min}$$

On day 30, Regina will get 9 h 10 min of daylight.
e) To determine which days get 15 h of sunlight, graph the horizontal line y = 15. This line crosses the graph twice, at day 119 and at day 224.
Days 119 and 224 will get 15 h of sunlight.

9. a) e.g., I predict the average temperature next January to be –12.9°C.
I determined a sinusoidal regression function that models the data, substituting numbers for months, and extrapolated the predicted value for the 13th month.
b) e.g., I predict the average precipitation next January to be 39 mm. I determined a sinusoidal regression function that models the data, substituting numbers for months, and extrapolated the predicted value for the 13th month.
c) e.g., The answer with temperature is more reliable, because the curve of best fit created by the regression function fits the data closely. The answer with precipitation is less reliable, because the data does not match the regression function as closely. There are several points that appear far away from the graph. This is reasonable, since temperature depends mainly on the time of year. Precipitation can be affected by other factors. Thus, temperature is more reliable since precipitation is more likely to vary.

10. a) e.g.

Time (s)	Height (in.)
0	70
0.25	55
0.5	40
0.75	55
1	70
1.25	55
1.5	40
1.75	55
2	70
2.25	55
2.5	40
2.75	55
3	70

b) $y = 15 \sin (2\pi x + 0.5\pi) + 55$

c) e.g., $y = 15\sin(2\pi x - 0.5\pi) + 55.$

$$y = 15\sin(2\pi(7.25) - 0.5\pi) + 55$$
$$y = 15\sin(14.5\pi - 0.5\pi) + 55$$
$$y = 15\sin(14\pi) + 55$$
$$y = 15(0) + 55$$
$$y = 55$$

After 7.25 s, the pendulum will be 55 cm high.

11. a) The equation of the sinusoidal function that models the data is
$y = 6 \sin (\pi x - 1.570\ldots) + 11$.

b) At 0.75 s:

$$y = 6\sin(\pi x - 1.570...) + 11$$
$$y = 6\sin(\pi(0.75) - 1.570...) + 11$$
$$y = 6\sin(2.356... - 1.570...) + 11$$
$$y = 6\sin(0.785...) + 11$$
$$y = 6(0.707...) + 11$$
$$y = 4.242... + 11$$
$$y = 15.242...$$

At 5.3 s:

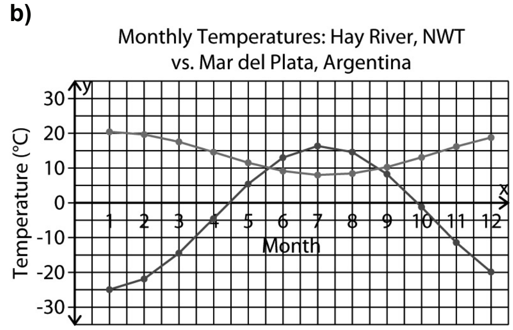
$$y = 6\sin(\pi x - 1.570...) + 11$$
$$y = 6\sin(\pi(5.3) - 1.570...) + 11$$
$$y = 6\sin(16.650... - 1.570...) + 11$$
$$y = 6\sin(15.079...) + 11$$
$$y = 6(0.587...) + 11$$
$$y = 3.526... + 11$$
$$y = 14.526...$$

After 0.75 s and 5.3 s, the spring will be 15.2 in. and 14.5 in. above the table, respectively.

12. e.g., No, because the regression function takes into account the effects of other months. The temperature likely will be close to –1.6 °C, but it could be that that temperature is an outlier. If you do a sine regression, to see how regular the pattern is, the results should be more reliable. The data was graphed using 1 for January, 2 for February, and so on. For November, that is, when $x = 11$,
$y = –0.695…°$ To the nearest tenth, this is –0.7 °C. This is close to –1.6 °C, but not exact.

13. a) The sinusoidal regression function for Hay River is
$y = 20.681\ldots \sin (0.501\ldots x – 2.018\ldots) – 4.284\ldots$,
and for Mar del Plata is
$y = 6.221\ldots \sin (0.504\ldots x +1.059\ldots) + 14.188\ldots$.
In both cases, the sinusoidal regression function fits the data closely.

b)



e.g., The graphs have very similar periods; the amplitude is much less for Mar del Plata, but the midline is much greater; the maximum is only slightly greater for Mar del Plata; the maximums and minimums of the graphs are almost exactly opposite.

c) e.g., Their locations (north or south of the equator) affect when the temperature is high or low. Their distance from the equator affect the magnitude of the temperature changes. When a location is north of the equator, it will have warm temperatures in the middle of year, and cold temperatures at the beginning and end of the year. However, when a location is south of the equator, it will have cold temperatures in the middle of the year, and warm temperatures at the beginning and end of the year.

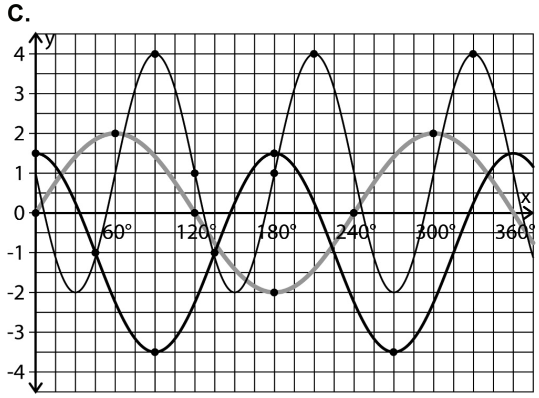
d) e.g., These communities will have the same temperatures in late May and late August since this when the graphs intersect. At both these times, one city's temperatures are increasing, while the other city's temperatures are decreasing.

e) e.g., south, as January and February temperatures are warmer than July and August temperatures.

14. a) e.g., I think the sidereal period is shorter than the synodic period because there are more cycles in a given period. The sidereal period involves 13 cycles, while the synodic involves 12 for a period of one year.
b) The period of the two graphs will be different (with the synodic being longer), but all other attributes will be the same.

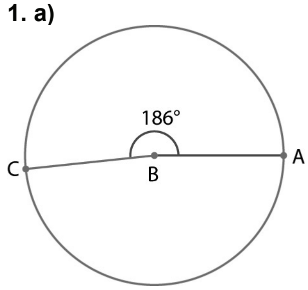
Applying Problem-Solving Strategies, page 578

A. The *x*-axis has points at 0°, 120°, and 240°. They are evenly spaced, so they could be on the midline of a graph.
B. There are points at (60°, 2), (180°, –2) and (300°, 2). 60° is midway between 0° and 120°, 180° is midway between 120° and 240°, and 300° is 60° past 240°. These points are either 2 above or 2 below the *x*-axis, so they could all belong to the same graph. This graph has an amplitude of 2, a midline at $y = 0$, and a period of 240°.
C.



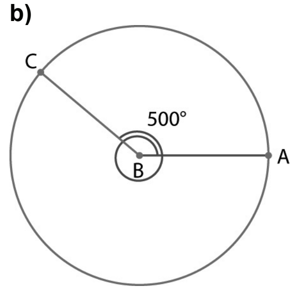
Chapter Self-Test, page 579

1. a)



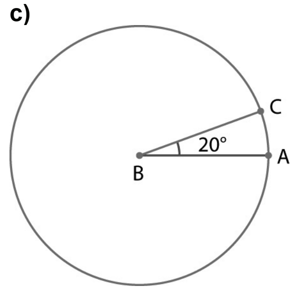
e.g., 186° = 180° + 6°
180° is about 3.2 radians.
6° is one tenth of 60°, which is about 1 radian.
One tenth of 1 radian is 0.1.
3.2 + 0.1 = 3.3 radians
Therefore the measure is about 3.3 radians.

b)



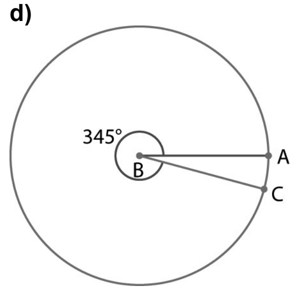
e.g., 500° is slightly less than 360° + 180° – 30°, or 510°.
360° is about 6.3 radians.
180° is about 3.2 radians.
30° is one half of 60°, which is about 1 radian.
30° is about 0.5 radians.
6.3 + 3.2 – 0.5 = 9.0 radians
510° is about 9.0 radians.
Therefore the measure is about 8.8 radians.

c)



e.g., 20° is one third of 60°.
60° is about 1 radian.
$\frac{1}{3} = 0.333...$
Therefore the measure is about 0.3 radians.

d)



e.g., 345° = 360° – 15°
360° is about 6.3 radians.
15° is one quarter of 60°.
60° is about 1 radian.
One quarter of 1 radian is 0.25 radians.
6.3 – 0.25 = 6.05.
Therefore the measure is about 6.1 radians.