## LG 10\&11 RELATIONS \& FUNCTIONS

## Example 1



3 Ways you can represent a Relation:

1. Table

| Fruit | Colour |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

2. Ordered Pairs
3. Arrow Diagram


## Example 2

Mean Heights of Different Breeds of Dogs


Different breeds of dogs can be associated with their mean heights.
Consider the relation represented by this graph. Represent the relation:
a) as a table
b) as an arrow diagram

| Breed of Dog | Mean Height (cm) |
| :--- | :---: |
| Afghan hound | 75 |
| Chihuahua | 20 |
| Corgi | 30 |
| Golden retriever | 60 |
| German shepherd | 60 |
| Malamute | 65 |



| Input | Output |
| :---: | :---: |
| 1 | 5 |
| 2 | 7 |
|  | 9 |
| 4 |  |
|  | 13 |

## Make Connections

$\mathrm{y}=2 \mathrm{x}-1$

| $x$ | $y$ |
| :---: | :---: |
|  |  |
|  |  |
|  |  |



## Identifying Functions

## What's a Functions or Not a Function?

## Exercises

## A

4. Which arrow diagrams represent functions? a)

b) $\xrightarrow{\text { is greater than }}$

c)


# Graphs of Relations and Functions 

A relation that is not a function has two or more ordered pairs with the same first coordinate. So, when the ordered pairs of the relation are plotted on a grid, a vertical line can be drawn to pass through more than one point.


A function has ordered pairs with different first coordinates. So, when the ordered pairs of the function are plotted on a grid, any vertical line drawn will always pass through no more than one point.


## Vertical Line Test for a Function

A graph represents a function when no two points on the graph lie on the same vertical line.
Place a ruler vertically on a graph, then slide the ruler across the graph. If one edge of the ruler always intersects the graph at no more than one point, the graph represents a function.

## Identifying whether a Graph Represents a Function

Which of these graphs represents a function? Justify the answer.
a)

b) World Population


In the workplace, a person's gross pay, $P$ dollars, often depends on the number of hours worked, $h$.
So, we say $P$ is the dependent variable. Since the number of hours worked, $h$, does not depend on the gross pay, $P$, we say that $h$ is the independent variable.


| Number of Marbles, | Mass of Marbles, $\boldsymbol{m}$ <br> $\boldsymbol{n}$ |
| :---: | :---: |
| 1 | 1.27 |
| 2 | 2.54 |
| 3 | 3.81 |
| 4 | 5.08 |
| 5 | 6.35 |
| 6 | 7.62 |

# Number of Marbles, Mass of Marbles, $\boldsymbol{m}$ 

1 1.27
$2 \quad 2.54$
$3 \quad 3.81$
$4 \quad 5.08$
$5 \quad 6.35$
$6 \quad 7.62$
a) Why is this relation also a function?
b) Identify the independent variable and the dependent variable. Justify the choices.
c) Write the domain and range.
5. Which sets of ordered pairs represent functions? Identify the domain and range of each set of ordered pairs.
a) $\{(1,3),(2,6),(3,9),(4,12)\}$
b) $\{(1,0),(0,1),(-1,0),(0,-1)\}$
c) $\{(2,3),(4,5),(6,7),(8,9)\}$
d) $\{(0,1),(0,2),(1,2),(0,3),(1,3),(2,3)\}$

## Writing in Function Notation

6. Write in function notation.
a) $C=20 n+8$
b) $P=n-3$
c) $t=5 d$
d) $y=-x$
7. Write as an equation in two variables.
a) $d(t)=3 t-5$
b) $f(x)=-6 x+4$
c) $C(n)=5 n$
d) $P(n)=2 n-7$
8. Given $\mathbf{g}(\mathbf{n})=\mathbf{5 n - 1}$ find each of the following:
g(4)
g(0)
$\mathrm{g}(-2)$

$$
\begin{aligned}
\mathrm{g}(4) & =5(4)-1 \\
& =20-1 \\
& =29---->(4,19)
\end{aligned}
$$

13. Given $h(x)=3 x+4$ find the value of $x$ when:

$$
\begin{array}{rlr}
\mathbf{h}(\mathbf{x}) & =\mathbf{7}(\mathbf{x})=\mathbf{- 8} & \mathbf{h}(\mathbf{x})=\mathbf{5} \\
& \\
\mathrm{h}(\mathrm{x}) & =3 \mathrm{x}+4 \\
7 & =3 \mathrm{x}+4 \\
3 & =3 \mathrm{x} & \\
\mathbf{1} & =\mathbf{x} &
\end{array}
$$

14. If $f(x)=2 x+5$ and $g(x)=3 x$ find $f(4)-g(2)$

## Interpreting and Sketching Graphs

## Example 1

In math, a graph provides much information.
This graph shows the depth of a scuba diver as a function of time.
A Scuba Diver's Dive


How many minutes did the dive last?
At what times did the diver stop her descent?
What was the greatest depth the diver reached? For how many minutes was the diver at that depth?
5. To raise a flag, Sepideh pulls the rope steadily with both hands for a short time, then moves both hands up the rope and pulls again. She does this until the flag has been raised. Which graph best represents the height of the flag? Give reasons for your choice.

Graph A


Graph C


Time (s)

Graph B


Graph D

9. a) Describe what is happening for each line segment in this graph.

Volume of Gas in a Snowmobile

b) How much gas was in the tank at the start of the journey? Was the tank full at this time? Explain.
17. For each graph, choose a dependent variable and an independent variable, and suggest a possible situation that it represents. Describe the significance of any key points or changes in the graph.
a)

b)

c)


## Determining the Domain and Range of the Graph of a Function

Determine the domain and range of the graph of each function.

|  | iii) Discrete |
| :---: | :---: |
| c) |  |


b)


## 3 ways you could be asked

|  | i) Set Notation | ii) Interval Notation |
| :--- | :--- | :--- |
| a) |  |  |
| b) |  |  |

This is a graph of the function $f(x)=2 x-1$.

a) Determine the range value when the domain value is 3 .
b) Determine the domain value when the range value is $\mathbf{- 1}$.

### 5.6 Properties of Linear Relations

 is the cost.We can identify that this is a linear relation in different ways.

- a table of values

Independent variable $\longrightarrow$ Distance (km) Cost (\$) $\longleftarrow$ Dependent variable

|  | 0 | 60 | $\bigcirc+2$ |
| :---: | :---: | :---: | :---: |
|  | 100 | 80 |  |
|  | 200 | 100 | + |
| +100 | 300 | 120 | 20 |
| +100 | 400 | 140 | $\int+20$ |

- a set of ordered pairs

- a graph


## Car Rental Cost



## a graph

Car Rental Cost


We can use each representation above to calculate the rate of change.
The rate of change can be expressed as a fraction:
$\frac{\text { change in dependent variable }}{\text { change in independent variable }}=\frac{\$ 20}{100 \mathrm{~km}}$

$$
=\$ 0.20 / \mathrm{km}
$$

The rate of change is $\$ 0.20 / \mathrm{km}$; that is, for each additional 1 km driven, the rental cost increases by $20 \Varangle$. The rate of change is constant for a linear relation.

We can determine the rate of change from the equation that represents the linear function.
Let the cost be $C$ dollars and the distance driven be $d$ kilometres.
An equation for this linear function is:

|  |
| :---: |
|  |  |

Which table of values represents a linear relation? Justify the answer.

| a) The relation between temperature in degrees Celsius, $C$, and temperature in degrees Fahrenheit, $F$ |  | b) The relation between the current, $I \mathrm{amps}$, and power, $P$ watts, in an electrical circuit |  |
| :---: | :---: | :---: | :---: |
| C | $F$ | I | $P$ |
| 0 | 32 | 0 | 0 |
| 5 | 41 | 5 | 75 |
| 10 | 50 | 10 | 300 |
| 15 | 59 | 15 | 675 |
| 20 | 68 | 20 | 1200 |

## Example 2 Determining whether an Equation Represents a Linear Relation

a) Graph each equation.
i) $y=-3 x+25$
ii) $y=2 x^{2}+5$
iii) $y=5$
iv) $x=1$
b) Which equations in part a represent linear relations? How do you know?


## Example 4 <br> Determining the Rate of Change of a Linear Relation from Its Graph

A water tank on a farm near Swift Current, Saskatchewan, holds 6000 L .
Graph A represents the tank being filled at a constant rate.
Graph B represents the tank being emptied at a constant rate.

Graph A
Filling a Water Tank


Graph B
Emptying a Water Tank

a) Identify the independent and dependent variables.
b) Determine the rate of change of each relation, then describe what it represents.

### 5.7 Interpreting Graphs of Linear Functions

Float planes fly into remote lakes in Canada's Northern wilderness areas for ecotourism. This graph shows the height of a float plane above a lake as the plane descends to land.


Where does the graph intersect the vertical axis? What does this point represent?

Where does the graph intersect the horizontal axis?
What does this point represent?
What is the rate of change for this graph? What does it represent?

## Determining Intercepts, Domain, and Range of the Graph of

 a Linear FunctionThis graph shows the fuel consumption of a scooter with a full tank of gas at the beginning of a journey.

Volume of Gas in a Scooter

a) Write the coordinates of the points where the graph intersects the axes. Determine the vertical and horizontal intercepts. Describe what the points of intersection represent.
b) What are the domain and range of this function?
c) Find the rate of change?

## Example 3 Matching a Graph to a Given Rate of Change and Vertical Intercept

Which graph has a rate of change of $\frac{1}{2}$ and a vertical intercept of 6 ? Justify the answer.
a)

b)


## Example 4 Solving a Problem Involving a Linear Function

This graph shows the cost of publishing a school yearbook for Collège Louis-Riel in Winnipeg.

Cost of Publishing a Yearbook


The budget for publishing costs is $\$ 4200$. What is the maximum number of books that can be printed?

