

1G 15 & 16

Systems of Linear Equations

7.1 Developing Systems of Linear Equations

Which system of equations is *not* a linear system?

a) $2x + y = 11$
 $x = 13 + y$

b) $2x = 11 - y$
 $4x - y = 13$

c) $-\frac{1}{2}x - y = \frac{3}{4}$
 $\frac{3}{2}x + 2 = -\frac{7}{8}$

d) $-x^2 + y = 10$
 $x + y = 5$

Which linear systems have the solution $x = -1$ and $y = 2$?

a) $3x + 2y = -1$
 $2x - y = 1$

b) $3x - y = -1$
 $-x - y = -1$

c) $-3x + 5y = 13$
 $4x - 3y = -10$

Example 1 Using a Diagram to Model a Situation

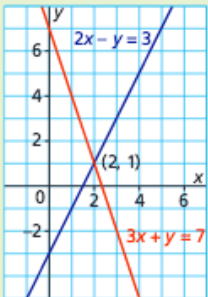
- a) Create a linear system to model this situation:
The perimeter of a Nunavut flag is 16 ft.
Its length is 2 ft. longer than its width.



- b) Denise has determined that the Nunavut flag is 5 ft. long and 3 ft. wide.
Use the linear system from part a to verify that Denise is correct.

Example 2 Using a Table to Create a Linear System to Model a Situation

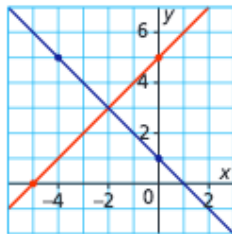
- a) Create a linear system to model this situation:
In Calgary, a school raised \$195 by collecting 3000 items for recycling.
The school received 5¢ for each pop can and 20¢ for each large plastic bottle.
- b) The school collected 2700 pop cans and 300 plastic bottles.
Use the linear system to verify these numbers.

Linear System		
$3x + y = 7$ $2x - y = 3$		
<p>Solve by graphing</p> <p>Use graphing software, a graphing calculator, or grid paper.</p> 	<p>Solve by substitution</p> $3x + y = 7 \quad \textcircled{1}$ $2x - y = 3 \quad \textcircled{2}$ <p>Solve equation $\textcircled{1}$ for y.</p> $y = -3x + 7$ <p>Substitute for y in equation $\textcircled{2}$.</p> $2x - (-3x + 7) = 3$ $5x - 7 = 3$ $5x = 10$ $x = 2$ <p>Substitute for x in equation $\textcircled{1}$.</p> $3(2) + y = 7$ $6 + y = 7$ $y = 1$ <p>Solution: $x = 2$ and $y = 1$</p>	<p>Solve by elimination</p> $3x + y = 7 \quad \textcircled{1}$ $2x - y = 3 \quad \textcircled{2}$ <p>Add the equations to eliminate y.</p> $5x = 10$ $x = 2$ <p>Substitute for x in equation $\textcircled{1}$.</p> $3(2) + y = 7$ $6 + y = 7$ $y = 1$ <p>Solution: $x = 2$ and $y = 1$</p>

7.2 Solving a System of Linear Equations Graphically

Make Connections

Two equations in a linear system are graphed on the same grid.



What are the equations of the graphs? Explain your reasoning.

What are the coordinates of the point of intersection of the two lines?

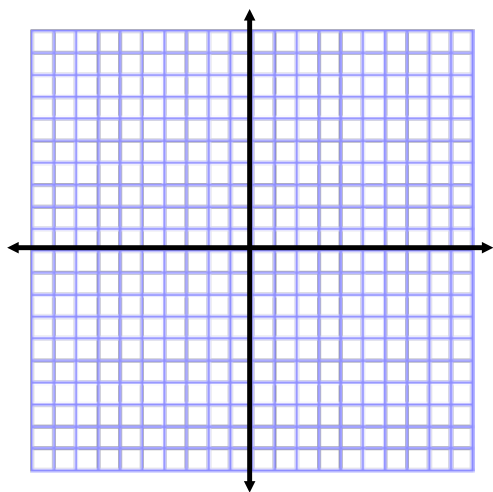
Explain why these coordinates are the solution of the linear system.

Example 1 Solving a Linear System by Graphing

Solve this linear system.

$$x + y = 8$$

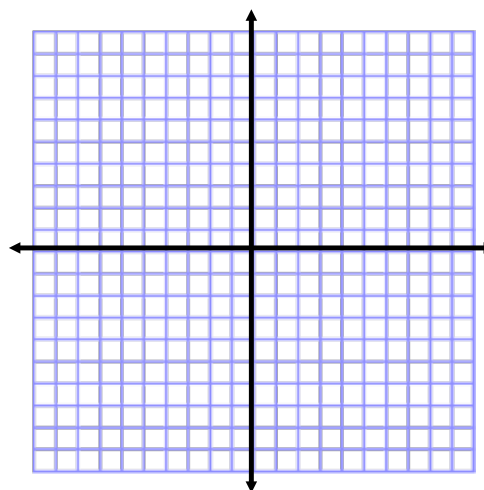
$$3x - 2y = 14$$

**CHECK YOUR UNDERSTANDING**

1. Solve this linear system.

$$2x + 3y = 3$$

$$x - y = 4$$



Example 2 Solving a Problem by Graphing a Linear System

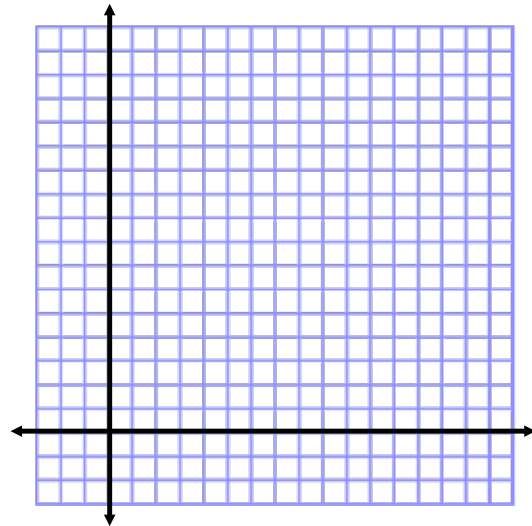
One plane left Regina at noon to travel 1400 mi. to Ottawa at an average speed of 400 mph. Another plane left Ottawa at the same time to travel to Regina at an average speed of 350 mph. A linear system that models this situation is:

$$d = 1400 - 400t$$

$$d = 350t$$

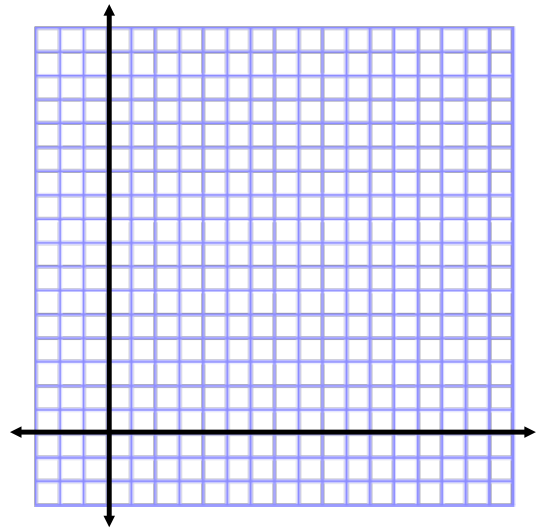
where d is the distance in miles from Ottawa and t is the time in hours since the planes took off

- Graph the linear system above.
- Use the graph to solve this problem: When do the planes pass each other and how far are they from Ottawa?



Example 3 Solving a Problem by Writing then Graphing a Linear System

- a) Write a linear system to model this situation:
To visit the Head-Smashed-In Buffalo Jump interpretive centre near Fort Macleod, Alberta, the admission fee is \$5 for a student and \$9 for an adult. In one hour, 32 people entered the centre and a total of \$180 in admission fees was collected.
- b) Graph the linear system then solve this problem: How many students and how many adults visited the centre during this time?



7.4 Using a Substitution Strategy to Solve a System of Linear Equations

Example 1 Solving a Linear System by Substitution

Solve this linear system.

$$2x - 4y = 7$$

$$4x + y = 5$$

Example 2 Using a Linear System to Solve a Problem

- a) Create a linear system to model this situation:
Nuri invested \$2000, part at an annual interest rate of 8% and the rest at an annual interest rate of 10%. After one year, the total interest was \$190.
- b) Solve this problem: How much money did Nuri invest at each rate?

Example 3 Using a Linear System to Solve a Problem

- a) Write a linear system to model this situation:
An alloy is a mixture of metals. An artist was commissioned to make a 100-g bracelet with a 50% silver alloy. He has a 60% silver alloy and a 35% silver alloy.



- b) Solve this problem:
What is the mass of each alloy needed to produce the desired alloy?

CHECK YOUR UNDERSTANDING

3. a) Write a linear system to model this situation:
An artist was commissioned to make a 625-g statue of a raven with a 40% silver alloy. She has a 50% silver alloy and a 25% silver alloy.
- b) Solve this problem: What is the mass of each alloy needed to produce the desired alloy?

[Answers: a) $f + t = 625$;
 $0.50f + 0.25t = 250$
b) 375 g of the 50% alloy; 250 g of the 25% alloy]

7.5 Using an Elimination Strategy to Solve a System of Linear Equations

Example 1

Solve this linear system by elimination.

$$3x - 4y = 7$$

$$5x - 6y = 8$$

CHECK YOUR UNDERSTANDING

1. Solve this linear system by elimination.

$$2x + 7y = 24$$

$$3x - 2y = -4$$

Example 2

Use an elimination strategy to solve this linear system.

$$\frac{2}{3}x - \frac{1}{2}y = 4$$

$$\frac{1}{2}x + \frac{1}{4}y = \frac{5}{2}$$

CHECK YOUR UNDERSTANDING

2. Use an elimination strategy to solve this linear system.

$$\frac{3}{4}x - y = 2$$

$$\frac{1}{8}x + \frac{1}{4}y = 2$$

7.6 Properties of Systems of Linear Equations

LESSON FOCUS

Determine the numbers of solutions of different types of linear systems.

Work in a group of 3.

Each linear system below contains the equation: $-2x + y = 2$
Solve each linear system by graphing.

System 1	System 2	System 3
$-2x + y = 2$	$-2x + y = 2$	$-2x + y = 2$
$2x + y = 2$	$-2x + y = 4$	$-4x + 2y = 4$

Share your results with your group.

How many solutions does each linear system have?

Example 1 Determining the Number of Solutions of a Linear System

Determine the number of solutions of each linear system.

- a) $x + y = -2$
 $-2x - 2y = 4$
- b) $4x + 6y = -10$
 $-2x - y = -1$
- c) $3x + y = -1$
 $-6x - 2y = 12$

CHECK YOUR UNDERSTANDING

1. Determine the number of solutions of each linear system.
- a) $x + y = 3$
 $-2x - y = -2$
- b) $4x + 6y = -10$
 $-2x - 3y = 5$
- c) $2x - 4y = -1$
 $3x - 6y = 2$

[Answers: a) one solution
b) infinite solutions
c) no solution]

Example 2 Creating a Linear System with 0, 1, or Infinite Solutions

Given the equation $-2x + y = 4$, write another linear equation that will form a linear system with:

- a) exactly one solution
- b) no solution
- c) infinite solutions

CHECK YOUR UNDERSTANDING

2. Given the equation $-6x + y = 3$, write another linear equation that will form a linear system with:
- a) exactly one solution
 - b) no solution
 - c) infinite solutions

[Sample Answers: a) $y = 2x + 4$
b) $y = 6x - 3$ c) $-12x + 2y = 6$]

