

# LG 6 & 7 Factors and Products

## BUILDING ON

- determining factors and multiples of whole numbers to 100
- identifying prime and composite numbers
- determining square roots of rational numbers
- adding and subtracting polynomials
- multiplying and dividing polynomials by monomials

## NEW VOCABULARY

prime factorization  
greatest common factor  
least common multiple  
perfect cube, cube root  
radicand, radical, index  
factoring by decomposition  
perfect square trinomial  
difference of squares

## Factors and Multiples of Whole Numbers

### **Example 1** Determining the Prime Factors of a Whole Number

Write the prime factorization of 3300.

### **Example 2** Determining the Greatest Common Factor

Determine the greatest common factor of 138 and 198.

**Example 3** Determining the Least Common Multiple

Determine the least common multiple of 18, 20, and 30.

**Example 4** Solving Problems that Involve Greatest Common Factor and Least Common Multiple

- a) What is the side length of the smallest square that could be tiled with rectangles that measure 16 cm by 40 cm? Assume the rectangles cannot be cut. Sketch the square and rectangles.
- b) What is the side length of the largest square that could be used to tile a rectangle that measures 16 cm by 40 cm? Assume that the squares cannot be cut. Sketch the rectangle and squares.

## Perfect Squares, Perfect Cubes, and Their Roots

### **Example 1** Determining the Square Root of a Whole Number

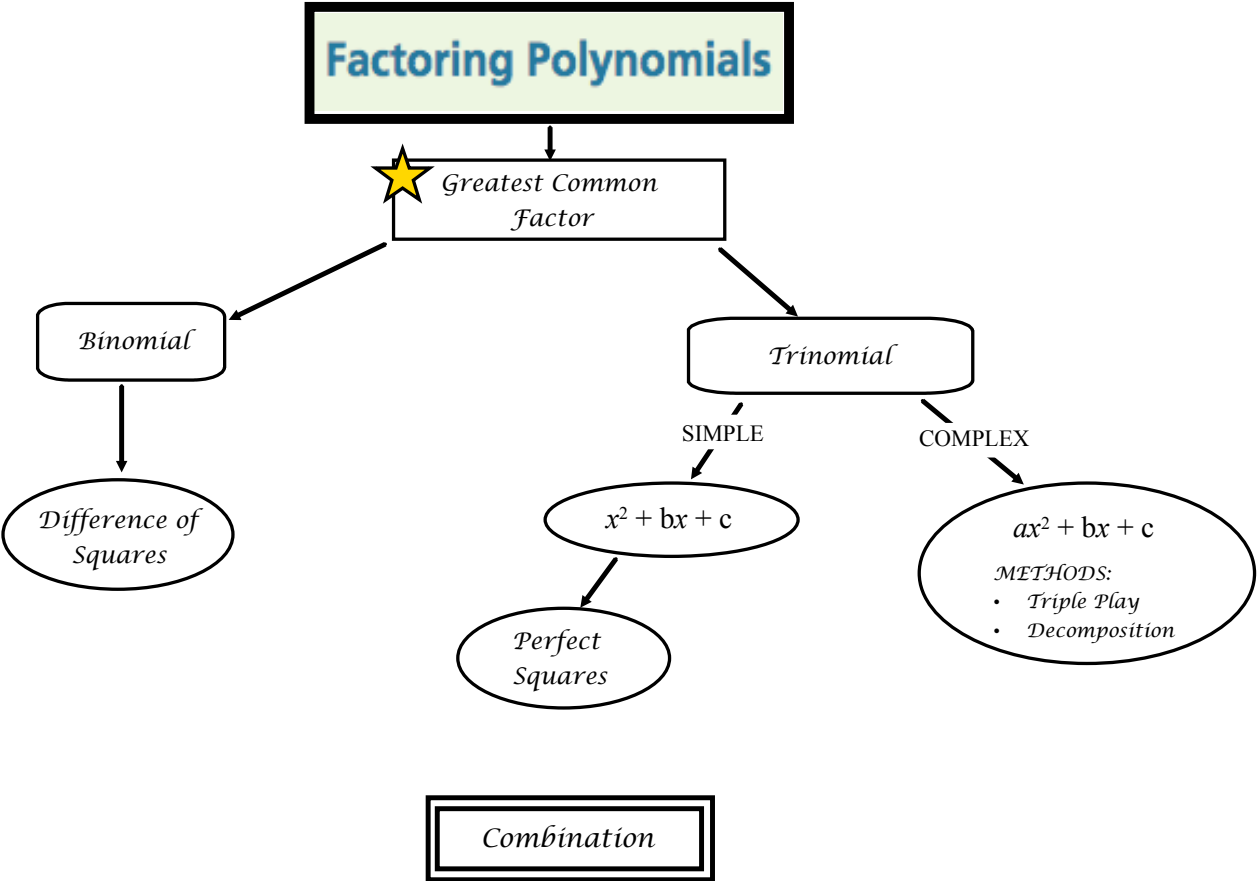
Determine the square root of 1296.

### **Example 2** Determining the Cube Root of a Whole Number

Determine the cube root of 1728.

### **Example 3** Using Roots to Solve a Problem

A cube has volume 4913 cubic inches. What is the surface area of the cube?



## Common Factors of a Polynomial

### Example 1 Using Algebra Tiles to Factor Binomials

Factor each binomial.

a)  $6n + 9$

b)  $6c + 4c^2$

#### CHECK YOUR UNDERSTANDING

1. Factor each binomial.

a)  $3g + 6$     b)  $8d + 12d^2$

[Answers: a)  $3(g + 2)$

b)  $4d(2 + 3d)$ ]

### Example 2 Factoring Trinomials

Factor the trinomial  $5 - 10z - 5z^2$ .

Verify that the factors are correct.

#### CHECK YOUR UNDERSTANDING

2. Factor the trinomial

$6 - 12z + 18z^2$

Verify that the factors are correct.

[Answer:  $6(1 - 2z + 3z^2)$ ]

**Example 3** Factoring Polynomials in More than One Variable

Factor the trinomial. Verify that the factors are correct.

$$-12x^3y - 20xy^2 - 16x^2y^2$$

**CHECK YOUR UNDERSTANDING**

3. Factor the trinomial. Verify that the factors are correct.

$$-20c^4d - 30c^3d^2 - 25cd$$

[Answer:  $-5cd(4c^3 + 6c^2d + 5)$ ]

## Polynomials of the Form $x^2 + bx + c$

### SIMPLE TRINOMIAL

#### Example 2 Factoring Trinomials

Factor each trinomial.

a)  $x^2 - 2x - 8$

b)  $z^2 - 12z + 35$

#### CHECK YOUR UNDERSTANDING

2. Factor each trinomial.

a)  $x^2 - 8x + 7$

b)  $a^2 + 7a - 18$

[Answers: a)  $(x - 7)(x - 1)$

b)  $(a + 9)(a - 2)$ ]

#### Example 3 Factoring a Trinomial Written in Ascending Order

Factor:  $-24 - 5d + d^2$

#### CHECK YOUR UNDERSTANDING

3. Factor:  $-30 + 7m + m^2$

[Answer:  $(-3 + m)(10 + m)$ ]



**TO CHECK IF YOU FACTORED CORRECTLY**

**Example 1** Multiplying Two Binomials

Expand and simplify.

a)  $(x - 4)(x + 2)$

b)  $(8 - b)(3 - b)$

NOW GO BACK TO THE PREVIOUS QUESTIONS AND CHECK

## Polynomials of the Form $ax^2 + bx + c$

### Example 3 Factoring a Trinomial

**COMPLEX  
TRINOMIAL**

Factor.

Method 1 → *TRIPLE PLAY*

a)  $6k^2 - 11k - 35$

PLAY 1 - *Set Up*

PLAY 2 - *"T Chart" of Product & Sum/Difference*

PLAY 3 - *Divide out Denominator*

**CHECK YOUR UNDERSTANDING**

**3.** Factor. *by TRIPLE PLAY*

a)  $4g^2 + 11g + 6$

## Polynomials of the Form $ax^2 + bx + c$

**COMPLEX  
TRINOMIAL**

### Example 3 Factoring a Trinomial

Factor.

a)  $6k^2 - 11k - 35$

#### Method 2

Use number sense and reasoning, with mental math.

The factors of  $6k^2$  are  $1k$  and  $6k$ , or  $2k$  and  $3k$ .

The coefficient of the middle term of the trinomial,  $-11$

*"T Chart" of Product & Sum/Difference*

Arrange the factor combinations vertically:

$$\begin{array}{r} 2k \quad 7 \\ \times \\ 3k \quad -5 \end{array}$$

$$-10k + 21k = 11k$$

$$\begin{array}{r} 2k \quad -7 \\ \times \\ 3k \quad 5 \end{array}$$

$$10k - 21k = -11k$$

Form the products, then add.

← This is the correct combination of factors.

$$6k^2 - 11k - 35 = (2k - 7)(3k + 5)$$

**CHECK YOUR UNDERSTANDING**

**3.** Factor. *by METHOD 2*

a)  $4g^2 + 11g + 6$

## Polynomials of the Form $ax^2 + bx + c$

**COMPLEX  
TRINOMIAL**

### Example 3 Factoring a Trinomial

Factor.

a)  $6k^2 - 11k - 35$

#### Method 3 $\rightarrow$ Decomposition

So, to factor  $6h^2 + 11h + 4$ , we *decompose* the  $h$ -term and write it as a sum of two terms whose coefficients have a product of 24.

Factors of 24	Sum of Factors
1, 24	$1 + 24 = 25$
2, 12	$2 + 12 = 14$
3, 8	$3 + 8 = 11$
4, 6	$4 + 6 = 10$

The two coefficients that have a sum of 11 are 3 and 8,  
so we write the trinomial  $6h^2 + 11h + 4$  as  $6h^2 + 3h + 8h + 4$ .

We remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$$6h^2 + 3h + 8h + 4 = 3h(2h + 1) + 4(2h + 1)$$

Each product has the common binomial factor  $2h + 1$ .

$$6h^2 + 11h + 4 = (2h + 1)(3h + 4)$$

**CHECK YOUR UNDERSTANDING**

**3.** Factor. *by Decomposition*

a)  $4g^2 + 11g + 6$

# Factoring Special Polynomials

## Example 1 Factoring a Perfect Square Trinomial

Factor each trinomial. Verify by multiplying the factors.

a)  $4x^2 + 12x + 9$       b)  $4 - 20x + 25x^2$

### CHECK YOUR UNDERSTANDING

1. Factor each trinomial. Verify by multiplying the factors.

a)  $36x^2 + 12x + 1$

b)  $16 - 56x + 49x^2$

[Answers: a)  $(6x + 1)^2$

b)  $(4 - 7x)^2$ ]



**Example 2** Factoring Trinomials in Two Variables

Factor each trinomial. Verify by multiplying the factors.

a)  $2a^2 - 7ab + 3b^2$     b)  $10c^2 - cd - 2d^2$

**CHECK YOUR UNDERSTANDING**

2. Factor each trinomial. Verify by multiplying the factors.

a)  $5c^2 - 13cd + 6d^2$

b)  $3p^2 - 5pq - 2q^2$

[Answers: a)  $(5c - 3d)(c - 2d)$

b)  $(3p + q)(p - 2q)$ ]

**Example 3** Factoring a Difference of Squares

Factor each binomial.

a)  $25 - 36x^2$       b)  $5x^4 - 80y^4$

**CHECK YOUR UNDERSTANDING**

3. Factor each binomial.

a)  $81m^2 - 49$

b)  $162v^4 - 2w^4$

[Answers: a)  $(9m - 7)(9m + 7)$

b)  $2(3v - w)(3v + w)(9v^2 + w^2)$ ]

**COMBINATION of Factoring****Example 4** Factoring a Trinomial with a Common Factor and Binomial Factors

Factor.

$$-4t^2 - 16t + 128$$

**CHECK YOUR UNDERSTANDING****4.** Factor.

$$-5h^2 - 20h + 60$$

[Answer:  $-5(h - 2)(h + 6)$ ]

## Multiplying Polynomials

### Example 1 Using the Distributive Property to Multiply Two Polynomials

Expand and simplify.

a)  $(2h + 5)(h^2 + 3h - 4)$     b)  $(-3f^2 + 3f - 2)(4f^2 - f - 6)$

### Example 2 Multiplying Polynomials in More than One Variable

Expand and simplify.

a)  $(2r + 5t)^2$     b)  $(3x - 2y)(4x - 3y + 5)$

**Example 3** Simplifying Sums and Differences of Polynomial Products

Expand and simplify.

a)  $(2c - 3)(c + 5) + 3(c - 3)(-3c + 1)$

b)  $(3x + y - 1)(2x - 4) - (3x + 2y)^2$