LG \#13
Rationals Part 2

Agenda:


## Topic 1 Example 1

Add or Subtract Rational Expressions With Common Denominators
Determine each sum or difference. Express each answer in simplest form. Identify all non-permissible values.
a) $\frac{2 a}{b}-\frac{a-1}{b} \nabla_{\text {. }}$.
b) $\frac{2 x}{x+4}+\frac{8}{x+4}$
$=\frac{2 a-(a-1)}{b} \Rightarrow \begin{aligned} & \text { don't forget } \\ & \text { bracket }\end{aligned}$
$=\frac{2 x+8}{x+4} 4$ factor
$=\frac{2 a-a+1}{b}$ collect like terms
$=\frac{2(x+4)}{x+4}$
$=\frac{a+1}{b} ; b \neq 0$
$=2 ; x \neq-4$

## Try:

a) $\frac{2}{m-2}+\frac{-7}{m-2}$
b) $\frac{4 x}{x+5}-\frac{-2 x}{x+5}$

## Try:

Determine each sum or difference. Express each answer in simplest form. Identify all non-permissible values.

$$
\frac{2 w^{2}-w}{(w-3)(w+1)}+\frac{3-2 w}{(w-3)(w+1)}-\frac{8}{(w-3)(w+1)}
$$

## Example 2

## Add or Subtract Rational Expressions With Unlike Denominators

Simplify. Express each answer in simplest form.
a) $\frac{2 x}{x y}+\frac{4}{x^{2}}-3$

1 s ffind LCD $=x^{2} y$, then multiply each numerator by the missing term.
$=\frac{2 x(x)}{x y(x)}+\frac{4(y)}{x^{2}(y)}-\frac{3\left(x^{2} y\right)}{1\left(x^{2} y\right)}$
$=\frac{2 x^{2}+4 y-3 x^{2} y}{x^{2} y}$
b) $\frac{1-\frac{1}{x}}{x-\frac{1}{x}}\left[\begin{array}{l}\text { this is a complex } \\ \text { fraction } \\ \text { a fraction in the } \\ \text { numerator/denominator }\end{array}\right]$

Find LCD in the Complex Fraction $x$ then multiply LCD by every term.
$=\frac{(x) 1-\frac{1}{x}(x)}{(x) x-\frac{1}{x}(x)} \begin{aligned} & \text { This simplifies } \\ & \text { a complex } \\ & \text { fraction. }\end{aligned}$
$=\frac{x-1}{x^{2}-1}=\frac{{ }^{1} x-1}{(x-1)(x+1)}$

$$
=\frac{1}{x+1}
$$

Try: Simplify. What are the non-permissible values.
a) $\frac{4}{p^{2}-1}+\frac{3}{p+1}$
b) $\frac{2-\frac{4}{n}}{n-\frac{4}{n}}$
c) $\frac{5}{2 x-8}-\frac{3}{x-4}$
d) $\frac{-1}{c+1}+\frac{3 c+1}{c^{2}-2 c-15}$

Example 1

## Solve a Rational Equation

Solve the following equation. What values are nonpermissible.

$$
\frac{2}{a^{2}-4}+\frac{10}{6 a+12}=\frac{1}{a-2}
$$

$\frac{2}{a^{2}-4}+\frac{10}{6 a+12}=\frac{1}{a-2}$
1 s Factor denominator to find LCD
$=\frac{2}{(a-2)(a+2)}+\frac{10}{6(a+2)}=\frac{1}{a-2}$
2 ndTo clear fractions multiply each
$L C D=6(a-2)(a+2)$
From the factors, the nonpermissible values are:

$$
x \neq-2,2
$$

$$
={ }^{6(a-2)(a+2)}\left[\frac{2}{(a-2)(a+2)}\right]+{ }^{6(a-2)(a+2)}\left[\frac{10}{6(a+2)}\right]^{6(a-2)(a+2)}\left[\frac{1}{a-2}\right]
$$

3 rasolve equation

$$
\begin{aligned}
12+10 a-20 & =6 a+12 \\
4 a & =20 \\
a & =5 \quad \text { Check your answer: }
\end{aligned}
$$

## Check:

Substitute $a=5$ into the original equation.

$$
\begin{array}{rlr} 
& \text { Left Side } & \text { Right Side } \\
& \frac{2}{a^{2}-4}+\frac{10}{6 a+12} & \\
= & \frac{2}{a-2} \\
=\frac{10}{(5)^{2}-4}+\frac{1}{6(5)+12} & & \frac{1}{(5)-2} \\
= & \frac{2}{21}+\frac{10}{42} & \\
= & \frac{1}{3} &
\end{array}
$$

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$\qquad$ $\longrightarrow$

Try: Solve the following equation. What values are non-permissible.

$$
\frac{9}{x-3}-\frac{4}{x-6}=\frac{18}{x^{2}-9 x+18}
$$

Example 2
Solve a Rational Equation With an Extraneous Root

Solve the following equation. What values are non-permissible.

$$
\frac{2 x}{x+3}+\frac{x}{x-3}=\frac{18}{x^{2}-9}
$$

$$
\frac{2 x}{x+3}+\frac{x}{x-3}=\frac{18}{x^{2}-9}
$$

$1 s \leftarrow$ Find RESTRICTION , $x \neq 3,-3$ Then find LCD $=(x+3)(x-3)$
2 na- multiply each rational expression by LCD (this eliminates denominator)

$$
\begin{aligned}
& \left(\frac{2 x}{x+3}\right)+{ }^{(x+3)(x-3)(x-3)}\left(\frac{x}{x-3}\right)={ }^{(x+3)(x-3)}\left(\frac{18}{x^{2}-9}\right) \underset{x}{x} \text { factor } \\
& (x+3)(x-3) \\
& 2 x(x-3)+x(x+3)=18 \quad \text { (expand) } \\
& 2 x^{2}-6 x+x^{2}+3 x=18 \quad \text { (collect like terms) } \\
& 3 x^{2}-3 x-18=0 \quad \text { (factor) } \\
& 3\left(x^{2}-x-6\right)=0 \\
& 3(x-3)(x+2)=0 \\
& x=3, x=-2 \\
& \text { CHECK FOR EXTRANEOUS } \\
& x=-2, \quad x=3 \text { is extraneous } \\
& \text { Solution } \\
& \text { EXTRANEOUS is when } \\
& \text { a Solution equals a } \\
& \text { Restriction }
\end{aligned}
$$

Try: Solve the following equation. What values are non-permissible.

$$
\frac{3 x}{x+2}-\frac{5}{x-3}=\frac{-25}{x^{2}-x-6}
$$

## Example 3

## Use a Rational Equation to Solve a Problem

Two brothers share in cutting their lawn. Tim can cut the lawn in 40 min . Jim can cut the same lawn in 50 min . How long will it take to cut the lawn if they work together?

1 sMake a Table

|  | Time to cut <br> lawn (min) | Fraction of <br> work in 1 min | Fraction of work <br> done ir minutes |
| :--- | :---: | :---: | :---: |
| Tim | 40 | $\frac{1}{40}$ | $\frac{t}{40}$ |
| Jim | 50 | $\frac{1}{50}$ | $\frac{t}{50}$ |
| Together | $t$ | $\frac{1}{2}$ | 1 |

From the table, the equation for Tim and Jim to cut the lawn together is:

$$
\frac{t}{40}+\frac{t}{50}=1 \begin{gathered}
\text { The } 1 \text { lawn they } \\
\text { cut together } \\
\text { Fraction of } \\
\text { Time }
\end{gathered}
$$

Find the $L C D=200$, now multiply each term by $L C D$ 5

$$
\begin{aligned}
&{ }^{5} 200 \\
&\left.\left(\frac{t}{40}\right)\right)^{4}+\left(\frac{t}{50}\right) \stackrel{200}{=}(1) \\
& 5 t+4 t=200 \\
& 9 t=200 \\
& t=\frac{200}{9} \text { or } 22.2 \mathrm{~min} .
\end{aligned}
$$

Don't forget to do a Check!

Try: Mary takes $4 h$ to paint a room. It takes Sue 3 h to paint the same area. How long
will the paint job take if they together?

## Example 4

## Use a Rational Equation to Solve a Problem

A bike race goes from Victoria to Chemainus and back. The total distance was 140 km. Conditions were excellent on the way from Victoria to Chemainus. However, bad weather caused the winner's average speed to decrease by $6 \mathrm{~km} / \mathrm{h}$ on the return trip. The total time for the trip was 8.5 h . What was the winning rider's average speed on the way to Chemainus?

- Use the formula distance $=$ rate $\times$ time, or time $=\frac{\text { dist. }}{\text { rate }}$ ) - Let $x$ represent the average speed, in $\mathrm{km} / \mathrm{h}$, on the trip from Victoria to Chemainus.


## Make a Table

|  | Distance (km) | Rate (km/h) | Time (h) |
| :--- | :---: | :---: | :---: |
| Trip to <br> Chemainus | 70 | $x$ | $\frac{70}{x}$ |
| Return from <br> Chemainus | 70 | $x-6$ | $\frac{70}{x-6}$ |
|  |  | Total | $8 \frac{8}{2}$ or $\frac{17}{2}$ |

$$
\frac{70}{x}+\frac{70}{x-6}=\frac{17}{2}
$$

Find LCD $=2(x)(x-6)$, then multiply each term by LCD

$$
{ }^{2(x)(x-6)}\left(\frac{70}{x}\right)+{ }^{2(x)(x-6)}\left(\frac{70}{x-6}\right)=^{2(x)(x-6)}\left(\frac{17}{x}\right)
$$

$$
\begin{aligned}
& 2(x-7)(70)+2(x)(70)=(x)(x-6)(17) \\
& 140 x-840+140 x=17 x^{2}-102 x \\
& 0=17 x^{2}-382 x+840
\end{aligned}
$$

Use Quadratic Program on Calculator.

$$
x=20 \mathrm{~km} / \mathrm{h} \quad \text { or } \quad x=2.471 \mathrm{~km} / \mathrm{h}
$$

(i)

This answer will not work because you will get a negative answer.

## TRY: <br> $\dot{W}$

Simon Whitfield won a Gold medal in the 2000 Summer Olympics in Triathlon. He swam 1.5 km , biked 40 km and ran 10 km . He ran at an average speed of, biked at an average speed of $2 x$, and swam at an average speed of $\frac{x}{4}$, where is in km per
hours.
a) Determine an expression for the total time taken to complete the race.
b) Simon can swim at $5 \mathrm{~km} / \mathrm{hr}$. How long will it take him to complete the race?
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