## LG \#4

Sine \& Cosine Laws


Agenda:

## Topic 1 <br> Sine Law

Sine Law is a relationship between the sides and angles of any triangle.


$$
\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}
$$

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## Example 1

## Determine an Unknown Side Length

Example: Find the length of side $b$.


1. Calculate $\angle C$
$\angle C=180^{\circ}-83^{\circ}-54^{\circ}$
$\angle C=43^{\circ}$
2. $\frac{\sin C}{c}=\frac{\sin B}{b}$
$\frac{\sin 43^{\circ}}{17}=\frac{\sin 54^{\circ}}{b}$
$b=\frac{\sin 54^{\circ} \times 17}{\sin 43^{\circ}}=20.166 \ldots$
$b=20.2 \mathrm{~cm}$

Try: Find the length of side $c$ in each of the following triangles.
a)

b)


## Example 2

## Determine an Unknown Angle Measure

Example: In $\triangle P Q R, \angle P=33^{\circ}, p=19.7 \mathrm{~m}$, and $q=28.4 \mathrm{~m}$. Find the measure of $\angle R$, to the nearest degree.


1. Find $\angle Q$ using sine law

$$
\begin{aligned}
& \frac{\sin 33^{\circ}}{19.7}=\frac{\sin Q}{28.4} \\
& \sin Q=\frac{\sin 33^{\circ} \times 28.4}{19.7} \\
& \angle Q=\sin ^{-1}\left(\frac{\sin 33^{\circ} \times 28.4}{19.7}\right)=51.73 \ldots \approx 52^{\circ}
\end{aligned}
$$

2. Find $\angle R$ using sum of $\angle ' s$ in a triangle

$$
\angle R=180^{\circ}-33^{\circ}-54^{\circ}=95^{\circ}
$$

Try: Find the measure of $\angle A$, to the nearest degree, in each of the following triangles.

b) In $\triangle A B C, \angle B=63^{\circ}, b=25.5 \mathrm{~cm}$ and $c=17.3 \mathrm{~cm}$

## The Ambiguous Case

If you are given two sides and an angle opposite one of those sides (ASS), the ambiguous case may occur. There are 3 possibilities:

1. no triangle exists with the given measures - NO SOLUTION
2. one triangle exists with the given measures - 1 SOLUTION
3. two distinct triangles exist-2 SOLUTIONS

$a<h$
no solution - the sides don't meet

## The Ambiguous Case

If you are given information that is an "ASS"

$$
\text { Angle (acute } \angle \text {, Side, Side) }
$$

$1^{\text {st }}$ - Ambiguous Triangle Template:


## The Ambiguous Case


3.


$$
h<a<b
$$

two solutions

## Example 3

Sine Law in an Ambiguous Case
Example: In $\triangle A B C, \angle A=30^{\circ}, a=24 \mathrm{~cm}$, and $b=42 \mathrm{~cm}$. Determine the measures of all other sides and angles


1. Sketch possible triangle
2. Find the height ( $h$ )

$$
\begin{aligned}
& \sin A=\frac{h}{b} \\
& h=b \sin A \\
& h=42 \sin 30^{\circ} \\
& h=21
\end{aligned}
$$

$$
a>h, \text { so there are } 2
$$

possible triangles

## Example 3 cont.

Triangle 1:
3. Solve for $\angle B$ using sine law
$\frac{\sin B}{b}=\frac{\operatorname{Sin} A}{a}$
$\frac{\sin B}{42}=\frac{\operatorname{Sin} 30^{\circ}}{24}$

$\sin B=\frac{42 \operatorname{Sin} 30^{\circ}}{24}$
$\angle B=\sin ^{-1}\left(\frac{42 \operatorname{Sin} 30^{\circ}}{24}\right)$
$\angle B=61.044 \ldots=61^{\circ}$
4. Find $\angle C$ (sum of angles in a $\Delta$ )
$\angle C=180^{\circ}-61^{\circ}-30^{\circ}=89^{\circ}$

## Example 3 cont.

5. Use sine law to find side $c$

$$
\begin{aligned}
& \frac{c}{\sin 89^{\circ}}=\frac{24}{\sin 30^{\circ}} \\
& c=\frac{24 \sin 89^{\circ}}{\sin 30^{\circ}} \\
& c=47.992 \ldots=48
\end{aligned}
$$


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## Example 3 cont.

Triangle 2:
3. Solve for $\angle B$ using $61^{\circ}$ as the reference angle in quadrant II

$$
\begin{aligned}
& \angle B=180^{\circ}-61^{\circ} \\
& \angle B=119^{\circ}
\end{aligned}
$$


4. Find $\angle C$ (sum of angles in a $\Delta$ )

$$
\angle C=180^{\circ}-119^{\circ}-30^{\circ}
$$

$$
\angle C=31^{\circ}
$$

## Example 3 cont.

5. Use sine law to find side $c$

$$
\begin{aligned}
& \frac{c}{\sin 31^{\circ}}=\frac{24}{\sin 30^{\circ}} \\
& c=\frac{24 \sin 31^{\circ}}{\sin 30^{\circ}} \\
& c=24.721 \ldots=25
\end{aligned}
$$


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Try: In triangle $A B C, \angle A=21^{\circ}, a=12 \mathrm{~m}$ and $b=17 \mathrm{~m}$. Determine the measures of all other sides and angles

## Topic 2 <br> Cosine Law

Sine Law is the relationship between the cosine of an angle and the lengths of the three sides of any triangle.


$$
c^{2}=a^{2}+b^{2}-2 b c \cos C
$$

$$
b^{2}=a^{2}+c^{2}-2 a c \cos B
$$

$$
a^{2}=b^{2}+c^{2}-2 b c \cos A
$$

## Example 1

## Determine an Distance (Side Length)

Example: A surveyor measures the distance to one end of a lake as 703.4 m . The distance to the other end is 452.7 m and the angle between the two is $76.8^{\circ}$. Find the length of a lake.


1. Sketch a diagram
2. Use cosine law

$$
\begin{aligned}
& a^{2}=b^{2}+c^{2}-2 b c \cos A \\
& a^{2}=(703.4)^{2}+(452.7)^{2}-2(703.4)(452.7) \cos 76.8^{\circ} \\
& a^{2}=554281.6894 \\
& a=\sqrt{554281.6894}=744.5009 \ldots=744.5 \mathrm{~m}
\end{aligned}
$$

Try: Find the length of the indicated side in each of the following triangles, to the nearest tenth.
a)

b) In $\triangle A B C, \angle B=115^{\circ}, \quad a=9 \mathrm{~cm}$ and $c=8 \mathrm{~cm}$. Find $b$.

## Example 2

## Determine an Angle

Example: A triangular brace has side lengths of $15 \mathrm{~cm}, 18.7 \mathrm{~cm}$ and 13 cm . Find the measure of the angle opposite the 15 cm side.


1. Sketch a diagram
2. Use cosine law

$$
c^{2}=a^{2}+b^{2}-2 a b \cos C
$$

$$
(15)^{2}=(18.7)^{2}+(13)^{2}-2(18.7)(13) \cos C
$$

$$
225-349.69-169=-486.2 \cos C
$$

$$
-293.69=-486.2 \cos C
$$

$$
\frac{-293.69}{-486.2}=\cos C
$$

$$
\cos ^{-1}\left(\frac{-293.69}{-486.2}\right)=\angle C
$$

$$
53^{\circ}=\angle C
$$

Try: Find the measure of the indicated angle in each of the following triangles, to the nearest tenth.
a)

b) In $\triangle A B C, a=9 \mathrm{~m}, b=18 \mathrm{~m}$ and $c=21 \mathrm{~m}$. Find $\angle A$.

## Example 3

## Solve a Triangle

Example: In $\triangle P Q R, p=12, q=4$, and $\angle R=23^{\circ}$. Find the length of the unknown side and the measure of the other 2 angles.


1. Sketch a diagram
2. Use cosine law to find $r$
$r^{2}=p^{2}+q^{2}-2 p q \cos R$
$r^{2}=12^{2}+4^{2}-2(12)(4) \cos 23^{\circ}$
$r=\sqrt{71.6315 \ldots}$
$r=8.46354$...
3. Use cosine law to find $\angle P$

## Example 3 (cont.)


3. $p^{2}=q^{2}+r^{2}-2 q r \cos P$
$(12)^{2}=(4)^{2}+(8.453)^{2}-2(4)(8.453) \cos P$
$144-16-71.631=(-67.708) \cos P$

$$
\begin{aligned}
& \frac{56.368}{-67.708}=\cos P \\
& \cos ^{-1}\left(\frac{56.368}{-67.708}\right)=P
\end{aligned}
$$

$$
146.4^{\circ}=P
$$

4. Find $\angle Q$ using sum of $\angle ' s$ in a triangle

$$
\begin{aligned}
& \angle Q=180^{\circ}-146.4^{\circ}-23^{\circ} \\
& \angle Q=10.6^{\circ}
\end{aligned}
$$

Try: Solve the following triangles. Round your answers to the nearest tenth.
a)

b) In $\triangle A B C, a=9, b=7$ and $\angle C=33.6^{\circ}$

