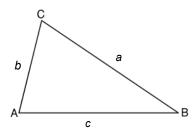


Agenda:

Topic 1

Sine Law

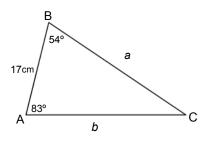
Sine Law is a relationship between the sides and angles of any triangle.



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Determine an Unknown Side Length

Example: Find the length of side b.



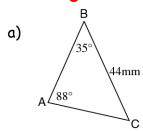
1. Calculate
$$\angle C$$

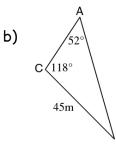
 $\angle C = 180^{\circ} - 83^{\circ} - 54^{\circ}$
 $\angle C = 43^{\circ}$

2.
$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

 $\frac{\sin 43^{\circ}}{17} = \frac{\sin 54^{\circ}}{b}$
 $b = \frac{\sin 54^{\circ} \times 17}{\sin 43^{\circ}} = 20.166...$
 $b = 20.2 \text{ cm}$

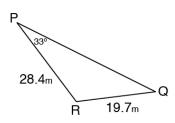
Try: Find the length of side c in each of the following triangles.





Determine an Unknown Angle Measure

Example: In $\triangle PQR$, $\angle P=33^{\circ}$, $p=19.7_{\rm m}$, and $q=28.4_{\rm m}$. Find the measure of $\angle R$, to the nearest degree.



1. Find ∠Q using sine law

$$\frac{\sin 33^{\circ}}{19.7} = \frac{\sin Q}{28.4}$$

$$\sin Q = \frac{\sin 33^{\circ} \times 28.4}{19.7}$$

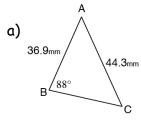
$$\angle Q = \sin^{-1} \left(\frac{\sin 33^{\circ} \times 28.4}{19.7} \right) = 51.73... \approx 52^{\circ}$$

2. Find $\angle R$ using sum of $\angle l$'s in a triangle

$$\angle R = 180^{\circ} - 33^{\circ} - 54^{\circ} = 95^{\circ}$$

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Try: Find the measure of $\angle A$, to the nearest degree, in each of the following triangles.

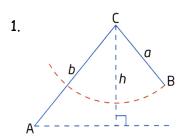


b) In
$$\triangle ABC$$
, $\angle B = 63^{\circ}$, $b = 25.5$ cm and $c = 17.3$ cm

The Ambiguous Case

If you are given two sides and an angle opposite one of those sides (ASS), the ambiguous case may occur. There are 3 possibilities:

- 1. no triangle exists with the given measures NO SOLUTION
- 2. one triangle exists with the given measures 1 SOLUTION
- 3. two distinct triangles exist 2 SOLUTIONS



a < h

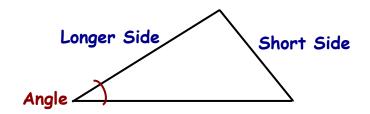
no solution - the sides don't meet

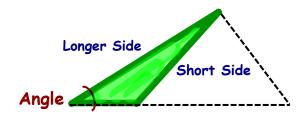
The Ambiguous Case

If you are given information that is an "ASS"

Angle (acute∠, Side, Side)

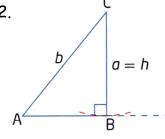
1st - Ambiguous Triangle Template:





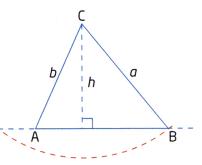
The Ambiguous Case

2.

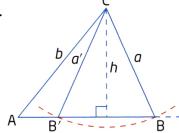


 $a=h \ or \ a\geq b$

one solution



3.

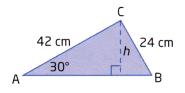


 $h < \alpha < b \\$

two solutions

Sine Law in an Ambiguous Case

Example: In $\triangle ABC$, $\angle A=30^\circ$, a=24 cm, and b=42 cm. Determine the measures of all other sides and angles



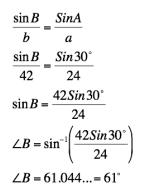
- 1. Sketch possible triangle
- 2. Find the height (h)

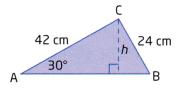
$$\sin A = \frac{h}{b}$$
 $h = b \sin A$
 $h = 42 \sin 30^{\circ}$
 $h = 21$

a > h, so there are 2
possible triangles

Triangle 1:

3. Solve for $\angle B$ using sine law





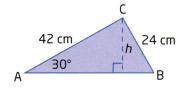
4. Find $\angle C$ (sum of angles in a Δ)

$$\angle C = 180^{\circ} - 61^{\circ} - 30^{\circ} = 89^{\circ}$$

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5. Use sine law to find side c

$$\frac{c}{\sin 89^{\circ}} = \frac{24}{\sin 30^{\circ}}$$
$$c = \frac{24 \sin 89^{\circ}}{\sin 30^{\circ}}$$
$$c = 47.992... = 48$$

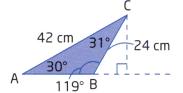


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Triangle 2:

3. Solve for $\angle B$ using 61° as the reference angle in quadrant II

$$\angle B = 180^{\circ} - 61^{\circ}$$
$$\angle B = 119^{\circ}$$



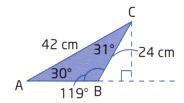
4. Find $\angle C$ (sum of angles in a Δ)

$$\angle C = 180^{\circ} - 119^{\circ} - 30^{\circ}$$

$$\angle C = 31^{\circ}$$

5. Use sine law to find side c

$$\frac{c}{\sin 31^{\circ}} = \frac{24}{\sin 30^{\circ}}$$
$$c = \frac{24 \sin 31^{\circ}}{\sin 30^{\circ}}$$
$$c = 24.721... = 25$$

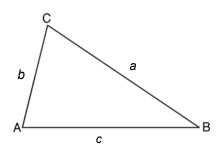


Try: In triangle	$ABC. \angle A = 21$	a = 12m	and $b=17$ m.
Determine	the measures	of all other	sides and
angles			

Topic 2

Cosine Law

Sine Law is the relationship between the cosine of an angle and the lengths of the three sides of any triangle.



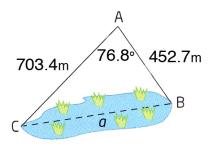
$$c^2 = a^2 + b^2 - 2bc \cos C$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$

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Determine an Distance (Side Length)

Example: A surveyor measures the distance to one end of a lake as 703.4m. The distance to the other end is 452.7m and the angle between the two is 76.8°. Find the length of a lake.



- 1. Sketch a diagram
- 2. Use cosine law

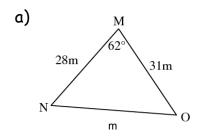
$$a^{2} = b^{2} + c^{2} - 2bc \cos A$$

$$a^{2} = (703.4)^{2} + (452.7)^{2} - 2(703.4)(452.7)\cos 76.8^{\circ}$$

$$a^{2} = 554281.6894$$

$$a = \sqrt{554281.6894} = 744.5009... = 744.5m$$

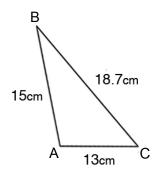
Try: Find the length of the indicated side in each of the following triangles, to the nearest tenth.



b) In $\triangle ABC$, $\angle B = 115^{\circ}$, a = 9cm and c = 8cm. Find b.

Determine an Angle

Example: A triangular brace has side lengths of 15cm, 18.7cm and 13cm. Find the measure of the angle opposite the 15cm side.



- 1. Sketch a diagram
- 2. Use cosine law

$$c^{2} = a^{2} + b^{2} - 2ab\cos C$$

$$(15)^{2} = (18.7)^{2} + (13)^{2} - 2(18.7)(13)\cos C$$

$$225 - 349.69 - 169 = -486.2\cos C$$

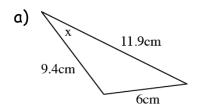
$$-293.69 = -486.2\cos C$$

$$\frac{-293.69}{-486.2} = \cos C$$

$$\cos^{-1}\left(\frac{-293.69}{-486.2}\right) = \angle C$$

$$53^{\circ} = \angle C$$

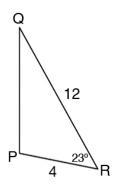
Try: Find the measure of the indicated angle in each of the following triangles, to the nearest tenth.



b) In $\triangle ABC$, $a = 9_{\text{m}}$, $b = 18_{\text{m}}$ and $c = 21_{\text{m}}$. Find $\angle A$.

Solve a Triangle

Example: In $\triangle PQR$, p=12, q=4, and $\angle R=23^{\circ}$. Find the length of the unknown side and the measure of the other 2 angles.



- 1. Sketch a diagram
- 2. Use cosine law to find r

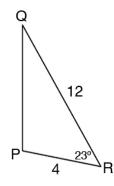
$$r^{2} = p^{2} + q^{2} - 2pq\cos R$$

$$r^{2} = 12^{2} + 4^{2} - 2(12)(4)\cos 23^{\circ}$$

$$r = \sqrt{71.6315...}$$

$$r = 8.46354...$$

3. Use cosine law to find $\angle P$



3.
$$p^2 = q^2 + r^2 - 2qr\cos P$$

 $(12)^2 = (4)^2 + (8.453)^2 - 2(4)(8.453)\cos P$
 $144 - 16 - 71.631 = (-67.708)\cos P$
 $\frac{56.368}{-67.708} = \cos P$
 $\cos^{-1}\left(\frac{56.368}{-67.708}\right) = P$
 $146.4^\circ = P$

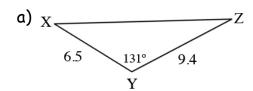
4. Find $\angle Q$ using sum of $\angle 's$ in a triangle

$$\angle Q = 180^{\circ} - 146.4^{\circ} - 23^{\circ}$$

 $\angle Q = 10.6^{\circ}$

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Try: Solve the following triangles. Round your answers to the nearest tenth.



b) In $\triangle ABC$, a = 9, b = 7 and $\angle C = 33.6^{\circ}$

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