LG 8 & 9 Roots and Powers

BUILDING ON

- determining the square root of a positive rational number
- applying the exponent laws for powers with integral bases and whole number exponents

NEW VOCABULARY

irrational number
real number
entire radical
mixed radical

- Any number that can be written as the fraction $\frac{m}{n}$, $n \neq 0$, where m and n are integers, is rational.
- Exponents can be used to represent roots and reciprocals of rational numbers.
- The exponent laws can be extended to include powers with rational and variable bases, and rational exponents.

Estimating Roots

Make Connections

Since $3^2 = 9$, 3 is a square root of 9.

We write: $3 = \sqrt{9}$

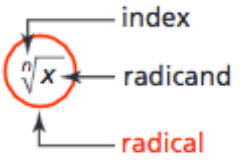
Since $3^3 = 27$, 3 is the cube root of 27.

We write: $3 = \sqrt[3]{27}$

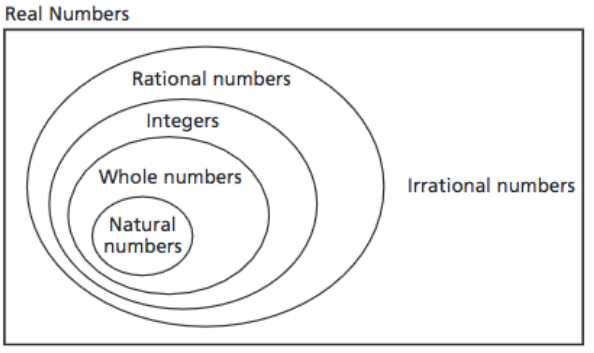
Since $3^4 = 81$, 3 is a fourth root of 81.

We write: $3 = \sqrt[4]{81}$

How would you write 5 as a square root? A cube root? A fourth root?



Irrational Numbers



Example 1 Classifying Numbers

Tell whether each number is rational or irrational. Explain how you know.

a) $-\frac{3}{5}$ **b)** $\sqrt{14}$ **c)** $\sqrt[3]{\frac{8}{27}}$

Example 2 Ordering Irrational Numbers on a Number Line

Use a number line to order these numbers from least to greatest.

$\sqrt[3]{13}, \sqrt{18}, \sqrt{9}, \sqrt[4]{27}, \sqrt[3]{-5}$

Mixed and Entire Radicals

Example 1 Simplifying Radicals Using Prime Factorization

Simplify each radical.

a) $\sqrt{80}$ **b)** $\sqrt[3]{144}$ **c)** $\sqrt[4]{162}$

Example 2 Writing Radicals in Simplest Form

Write each radical in simplest form, if possible.

a) $\sqrt[3]{40}$ **b)** $\sqrt{26}$ **c)** $\sqrt[4]{32}$

Example 3 Writing Mixed Radicals as Entire Radicals

Write each mixed radical as an entire radical.

a) $4\sqrt{3}$ **b)** $3\sqrt[3]{2}$ **c)** $2\sqrt[5]{2}$

Fractional Exponents and Radicals

Example 1 Evaluating Powers of the Form $a^{\frac{1}{n}}$

Evaluate each power without using a calculator.

a) $27^{\frac{1}{3}}$ **b)** $0.49^{\frac{1}{2}}$ **c)** $(-64)^{\frac{1}{3}}$ **d)** $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

Example 2 Rewriting Powers in Radical and Exponent Form

a) Write $40^{\frac{2}{3}}$ in radical form in 2 ways.

b) Write $\sqrt{3^5}$ and $\left(\sqrt[3]{25}\right)^2$ in exponent form.

Example 3 Evaluating Powers with Rational Exponents and Rational Bases

Evaluate.

a) $0.04^{\frac{3}{2}}$ **b)** $27^{\frac{4}{3}}$

c) $(-32)^{0.4}$ **d)** $1.8^{1.4}$

Example 4 Applying Rational Exponents

Biologists use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass, b kilograms, of a mammal with body mass m kilograms. Estimate the brain mass of each animal.

a) a husky with a body mass of 27 kg

b) a polar bear with a body mass of 200 kg

Negative Exponents and Reciprocals

Example 1 Evaluating Powers with Negative Integer Exponents

Evaluate each power.

a) 3^{-2} **b)** $\left(-\frac{3}{4}\right)^{-3}$ **c)** 0.3^{-4}

Example 2 Evaluating Powers with Negative Rational Exponents

Evaluate each power without using a calculator.

a) $8^{-\frac{2}{3}}$ **b)** $\left(\frac{9}{16}\right)^{-\frac{3}{2}}$

Applying the Exponent Laws

Recall the exponent laws for integer bases and whole number exponents.

Product of powers:	$a^m \cdot a^n = a^{m+n}$
Quotient of powers:	$a^m \div a^n = a^{m-n}, a \neq 0$
Power of a power:	$(a^m)^n = a^{mn}$
Power of a product:	$(ab)^m = a^m b^m$
Power of a quotient:	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$

Example 1 Simplifying Numerical Expressions with Rational Number Bases

Simplify by writing as a single power. Explain the reasoning.

a) $0.3^{-3} \cdot 0.3^5$

b) $\left[\left(-\frac{3}{2}\right)^{-4}\right]^2 \cdot \left[\left(-\frac{3}{2}\right)^2\right]^3$

c) $\frac{(1.4^3)(1.4^4)}{1.4^{-2}}$

d) $\left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}}\right)^6$

Example 2 Simplifying Algebraic Expressions with Integer Exponents

Simplify. Explain the reasoning.

a) $(x^3y^2)(x^2y^{-4})$ **b)** $\dfrac{10a^5b^3}{2a^2b^{-2}}$

Example 3 Simplifying Algebraic Expressions with Rational Exponents

Simplify. Explain the reasoning.

a) $(8a^3b^6)^{\frac{1}{3}}$ **b)** $(x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1})$

c) $\dfrac{4a^{-2}b^{\frac{2}{3}}}{2a^2b^{\frac{1}{3}}}$ **d)** $\left(\dfrac{100a}{25a^5b^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$

Example 4 Solving Problems Using the Exponent Laws

A sphere has volume 425 m^3.
What is the radius of the sphere to the nearest tenth of a metre?