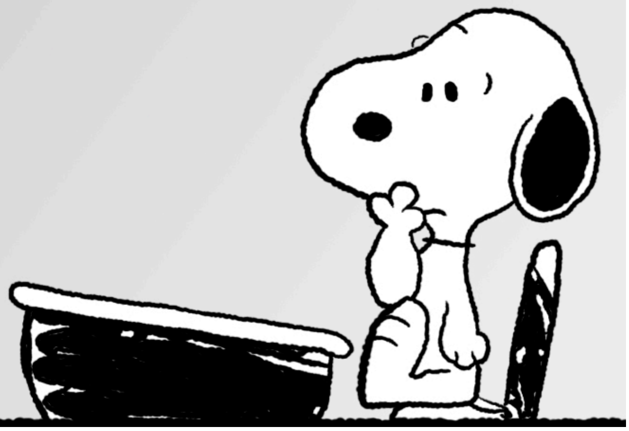
LG #15
Absolute Value
Part 2 - Equations



Agenda:

Topic 1

Example 1

Solve an Absolute Value Equation

Solve $|x-3| = 7$

Method 1: Use Algebra

Remember that an Absolute Value has Two Cases:

$$|x-3| = \begin{cases} x-3 & \text{positive case} \\ -(x-3) & \text{negative case} \end{cases}$$

Positive Case

$x-3 = 7$

$x = 10$

Negative Case

$-(x-3) = 7$

$x - 3 = -7$

$x = -4$

Verify the solutions by substitution

Left Side	Right Side
$= \|10 - 3\|$	7
$= \|7\|$	
$= 7$ ✓	

Left Side	Right Side
$= \|-4 - 3\|$	7
$= \|-7\|$	
$= 7$ ✓	

Method 2: Use a Graph

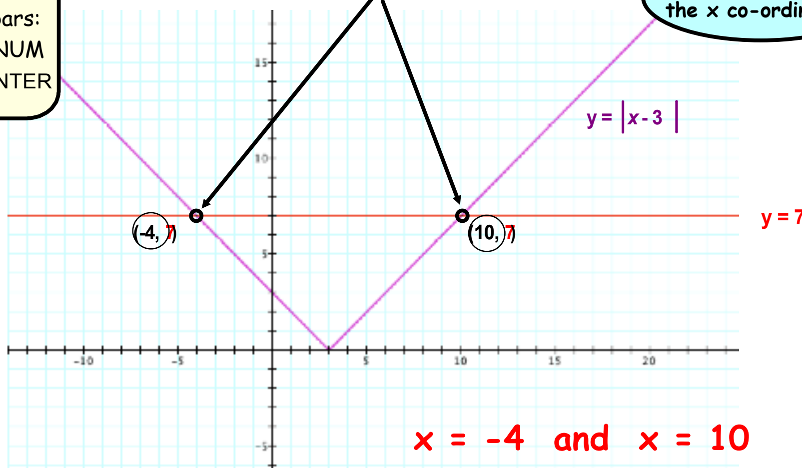
Using your Graphing Calculator

Type in: $Y_1 = |x - 3|$
$Y_2 = 7$

Where the two lines intersect are your solutions.

To get the Absolute bars: MATH > NUM 1 : abs (ENTER

Your answers are the x co-ordinates



x = -4 and x = 10

Example 2
Solve an Absolute Value Problem

A TV network receives a rating at the end of each month which reflects the viewers approval. For the month of January, NBC received a rating of 78.4% with an absolute error of ±5%. Write and solve an absolute value equation for the maximum and minimum rating, *r*, for NBC's January ratings.

Equation: $|r - 78.4| = 5$

positive case

$r - 78.4 = 5$

$r = 83.4\%$

negative case

$-(r - 78.4) = 5$

$r - 78.4 = -5$

$r = 73.4\%$

Example 3

Absolute Value Equation With an Extraneous Solution

Example: Solve by algebra $|x-4| = 2x+1$

positive case

x - 4 = 2x + 1

x = -5

Extraneous

negative case

-x + 4 = 2x + 1

-3x = -3

x = 1

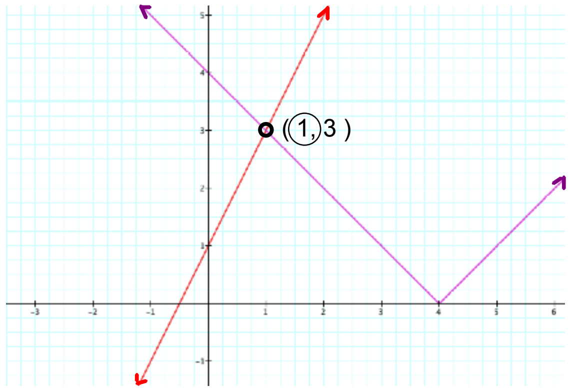
Verify the solutions by substitution

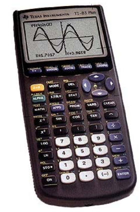
Left side *Right side*

$$x - 4 = 2x + 1$$
$$|(-5) - 4| = 2(-5) + 1$$
$$|-9| = -10 + 1$$
$$9 = -9 \quad \times$$

Left side *Right side*

$$x - 4 = 2x + 1$$
$$|(1) - 4| = 2(1) + 1$$
$$|-3| = 2 + 1$$
$$3 = 3 \quad \checkmark$$





When you check with your graphing calculator you will notice that the curve does not intersect at **-5** therefore $x = -5$ is **extraneous.** You must indicate this!

The only thing you will put in the solution box is

$x = 1$.

Example 4

Absolute Value Equation With No Solution

Example: Solve $\left| 3x - 4 \right| + 12 = 9$

1 st- Isolate the Radical $\left| 3x - 4 \right| = -3$

Since the absolute value of a number is always greater than or equal to zero, you can see that the left side will be zero or greater and the right side is negative. Thus, this equation has NO SOLUTION.

Example 5

Solve an Absolute Value Equation Involving a Quadratic Expression

Example: Solve $\left| x^2 - 3x \right| = 10$

Positive Case

$x^2 - 3x = 10$

$x^2 - 3x - 10 = 0$

Now factor or use your Quad. program.

x = -2 , x = 5

Check by Graphing Calculator

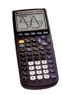
Negative Case

$-(x^2 - 3x) = 10$

$x^2 - 3x = -10$

$x^2 - 3x + 10 = 0$

Now factor or use your Quad. program.



Non-Real Numbers

Try:

1. Solve algebraically and check graphically $|6-x| = 2$

2. A computerized process controls the amount of fish that is packaged in a specific size of can. The computer program sets the ideal mass at 170 g but allows a tolerance of ± 6 g. Solve an absolute value equation for the maximum and minimum mass, m, of fish in this size of can.

3. Solve algebraically and check graphically $|x+5| = 4x-1$

4. Solve algebraically and check graphically $|x^2-3x| = 2$

Topic 2

Reciprocal Functions: $y = \frac{1}{f(x)}$

☆**Invariant points:** points that are the same on both $y = f(x)$ and $y = \frac{1}{f(x)}$

Example 1

Graph the Reciprocal of a Linear Function

Consider $f(x) = 2x + 5$.

a) Determine its reciprocal function $y = \frac{1}{f(x)}$

b) Determine the equation of the vertical asymptote of the reciprocal function.

c) Graph the function $y = f(x)$ and its reciprocal function $y = \frac{1}{f(x)}$.

Solution

$f(x) = 2x + 5$

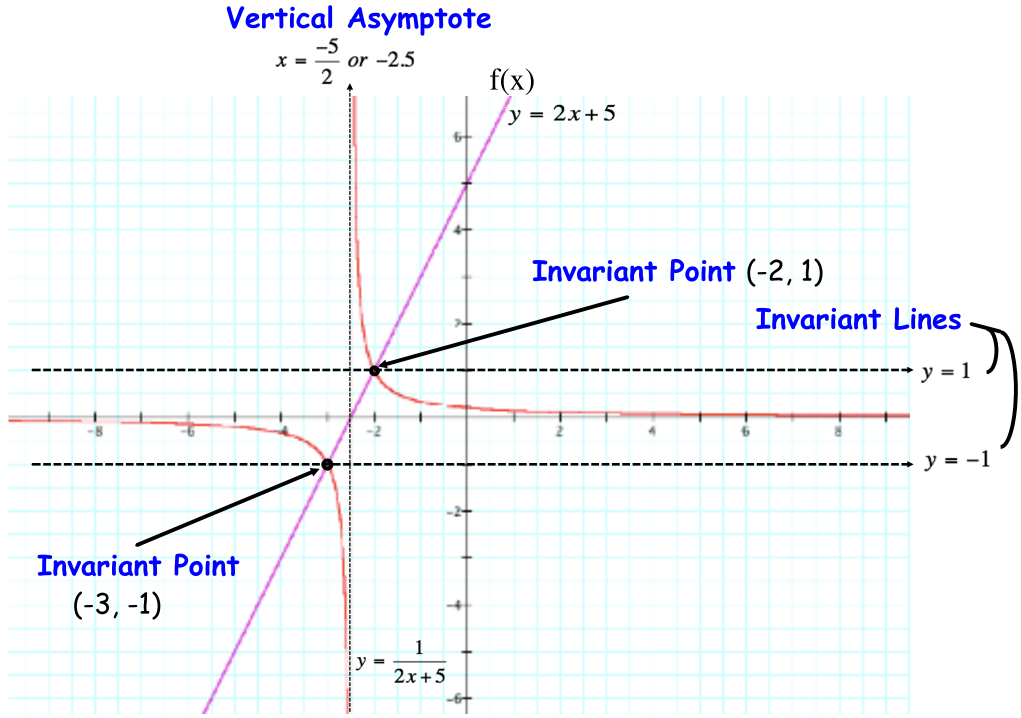
$a)\quad y = \dfrac{1}{2x + 5}$

$b)\quad 2x + 5 = 0$

$2x = -5$

$x = \dfrac{-5}{2}$ or -2.5 Vertical Asymptote

$c)$ *Graph* (see next page)



Vertical Asymptote
$x = \frac{-5}{2}$ or -2.5
f(x)
$y = 2x + 5$
Invariant Point (-2, 1)
Invariant Lines
$y = 1$
$y = -1$
Invariant Point
(-3, -1)
$y = \frac{1}{2x+5}$

Steps to Graphing:

$$y = \frac{1}{2x+5}$$

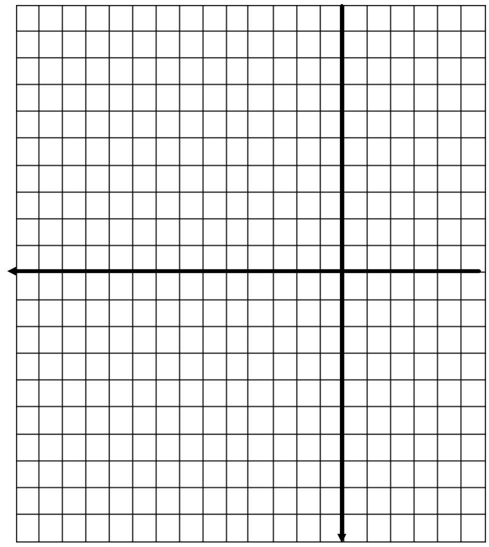
1st draw the $y = f(x)$ $y = 2x + 5$

2nd draw the vertical asymptote

3rd draw the invariant lines $y = 1$ and $y = -1$

4th mark the invariant points

5th from these invariant points "swing up to their reciprocal points



Try:

Consider: 1. $f(x) = 3x - 6$ and 2. $f(x) = x \quad + x - 6$

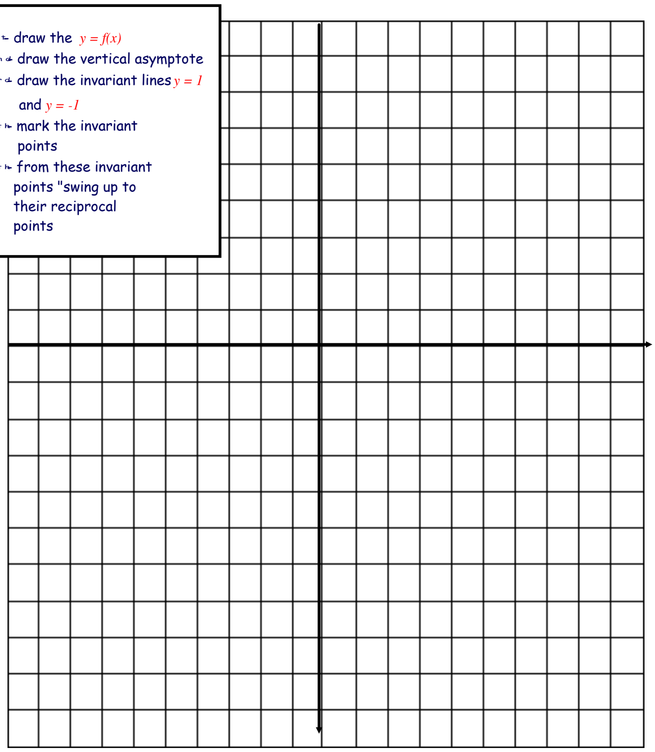
a) Determine its reciprocal function $y = \frac{1}{f(x)}$

b) Determine the equation of the vertical asymptote of the reciprocal function.

c) Graph the function $y = f(x)$ and its reciprocal function $y = \frac{1}{f(x)}$.

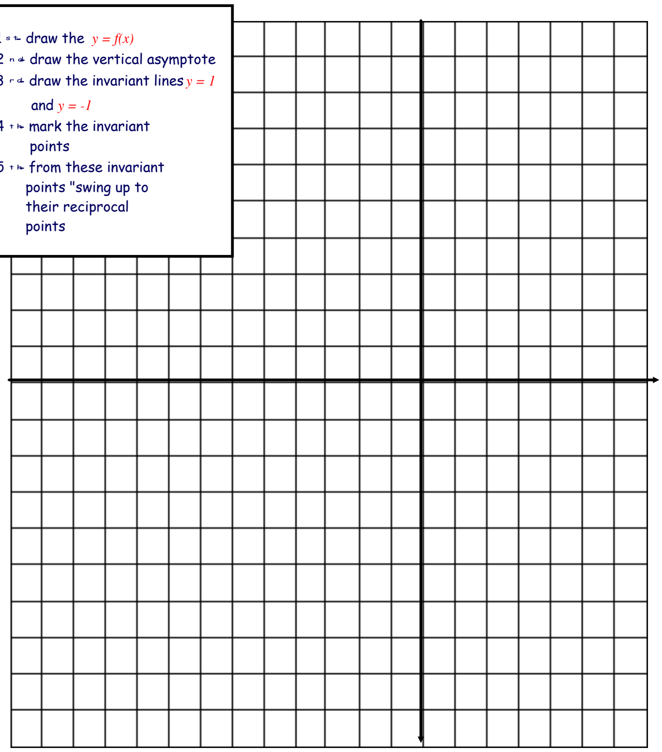
f(x) = 3x - 6

1st draw the $y = f(x)$
2nd draw the vertical asymptote
3rd draw the invariant lines $y = 1$ and $y = -1$
4th mark the invariant points
5th from these invariant points "swing up to their reciprocal points



$f(x) = x^2 + x - 6$

1st draw the $y = f(x)$
2nd draw the vertical asymptote
3rd draw the invariant lines $y = 1$ and $y = -1$
4th mark the invariant points
5th from these invariant points "swing up to their reciprocal points

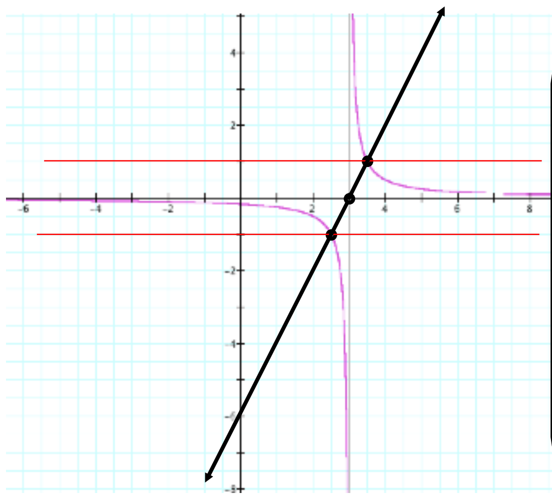


Example 2

Graph $y = f(x)$ Given the Graph of $y = \dfrac{1}{f(x)}$

a) Sketch the graph for $y = f(x)$.

b) Determine the equation for $y = f(x)$.



To Sketch:

1st - draw point at the vertical asymptote.

2nd - draw the two invariant lines($y = 1$; $y = -1$)

3rd - draw the point where the graph intersects the invariant lines.

4th - connect the points

To determine the equation:

- you can see the y-intercept = (-6)
- the slope $m = \frac{rise}{run} = \frac{6}{3}$ =2
- use the slope/y-intercept form **y = mx + b**

Solution: $y = \frac{1}{2x-6}$

Try:

Graph $y = f(x)$ **Given the Graph of** $y = \dfrac{1}{f(x)}$

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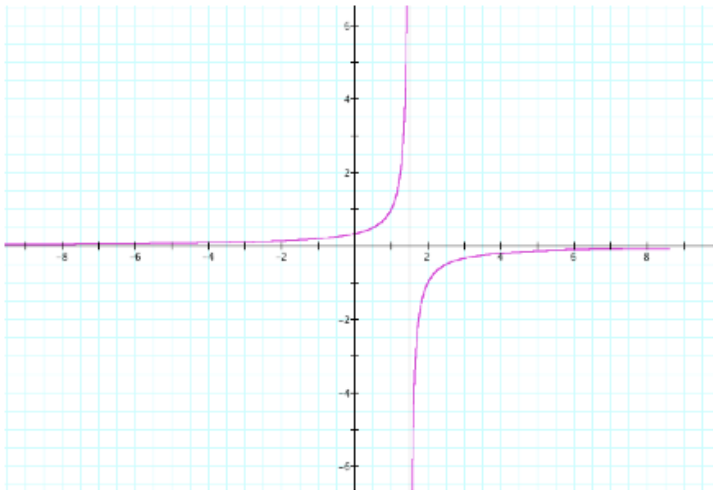

Try:

Graph $y = f(x)$ **Given the Graph of** $y = \dfrac{1}{f(x)}$

a) Sketch the graph for $y = f(x)$.

b) Determine the equation for $y = f(x)$.