LG #2
Geometric Sequence & Series

Agenda:

Sequences

Arithmetic   Geometric

Has a Common Difference   **Has a Common Ratio**

$d = t_2 - t_1$

$where:$

$t_2 - t_1 = t_3 - t_2$

$r = \dfrac{t_2}{t_1}$

$where: \quad \dfrac{t_2}{t_1} = \dfrac{t_3}{t_2}$

PART 2. Geometric Sequences

☞ The general term of a geometric sequence is:

$$t_n = t_1 r^{n-1}$$

This is on your Formula Sheet

Topic 1   **Example 1**
Write a General Term

a) **Write a general term t   for:**
    4, 20, 100, 500, ...

b) Write a general term t for:
    1, -3, 9, -27, ...

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Topic 2   *Example 2*
Find a Specific Term

a)   Write a general term t_n for $1, 5,$
$25, 125, ...t_{10}$

b)   Write a general term t_n for $2, 4,$
$8, 16, ...$
What is term t_{20}?

c) Suppose the smallest reduction a photo copier could make is 60% of the original. What is the shortest possible length after 8 reductions of a photograph that is originally 42 cm long?

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Example 3
Find the Term Number

a)   For 3, 6, 12, 24, ...which term is
3072?

b)   For 2, 8, 32, 128, ... , which
term is 32768?

Topic 4

Example 4
Finding Specific Term(s)

a) What are the 2 geometric means between 2 and 250?

b) In a geometric sequence,
$t_3 = -12$ and $t_8 = -384$,
Find the first two terms.

C) In a geometric sequence, the second term is 28 and the fifth term is 1792.

Determine the value of t_1 and r, and list the first three terms of the sequence.

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The **SUM** of a Geometric series can be determined using the formulas:

$$S_n = \frac{t_1(r^n - 1)}{r - 1} \quad , \quad r \neq 1$$

$$S_n = \frac{rt_n - t_1}{r - 1} \quad , \quad r \neq 1$$

used if t is known

where: **t** is first term

n is the number of terms

r is the common ratio

S is the sum of the first n terms

Topic 5

Finding the Sum of a Geometric Series
Example 1

Determine the sum of the first 12 terms of each geometric series

    a) $5 + 10 + 20 + 40 + \ldots$ $(S)_2$

Example 2

Given: $t = \dfrac{1}{12^1}$, r = -3, n = 8. Find the S

Express answer as exact values in fraction form.

Example 3

In a badminton tournament the winner advances
until a final match which determines the
champion, and the loser is out of the tournament
(single elimination). If the tournament has 512
players, how many matches will be played?

Ask yourself:
What do I know
here?

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The SUM of an Infinite Geometric series can be determined using the formula:

$$S_\infty = \frac{t_1}{1-r}$$

☆ **Important to check to see that the Ratio is between -1 and 1.** ☆

Since -1 < r < 1, the series is convergent.

where: **t** is first term

r is the common ratio

S is the sum of the first n terms

Infinite Geometric Series **will** have a sum if the series is **Convergent.**   **where: -1 < r < 1**

Infinite Geometric Series will not have a sum if the series is **Divergent.**   **where: r < -1 and r > 1**

State if the series is convergent or divergent.

a) $t = 7, \quad r = \dfrac{2}{3}$

b) $t = {}_1 3, \quad r = 5$

c) -5, -2.5, -1.25, ...

d) $\dfrac{1}{3}, \dfrac{-2}{3}, \dfrac{4}{3}, ...$

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Topic 6

Finding the Sum of an Infinite Series

Example 1

Determine the *sum to infinity*.

i) $\quad 5 + \dfrac{5}{4} + \dfrac{5}{16} + \ \ldots$

ii) $\ t =_1 4, \ r = -\dfrac{1}{3}$

Repeating Decimals

1. Express 0.23232323 .... as an infinite series. Then find it's sum.

    1st create a series > .23 + .0023 + .000023 + ... Then find ratio $t \div t$

    2nd use your Sum of an Infinite formula

    3rd convert to a fraction

Example 2

Express $0.\overline{45}$ as an infinite geometric series, then determine it's sum.

The sum of an infinite geometric series:

$$t_1 + t_1(r) + t_1(r) + t_1(r) + t_1(r) + ...$$

Example 3

What is the sum of the following infinite geometric series?

$$3 + 3\left(\frac{1}{3}\right) + 3\left(\frac{1}{3}\right)^2 + 3\left(\frac{1}{3}\right)^3 + ...$$