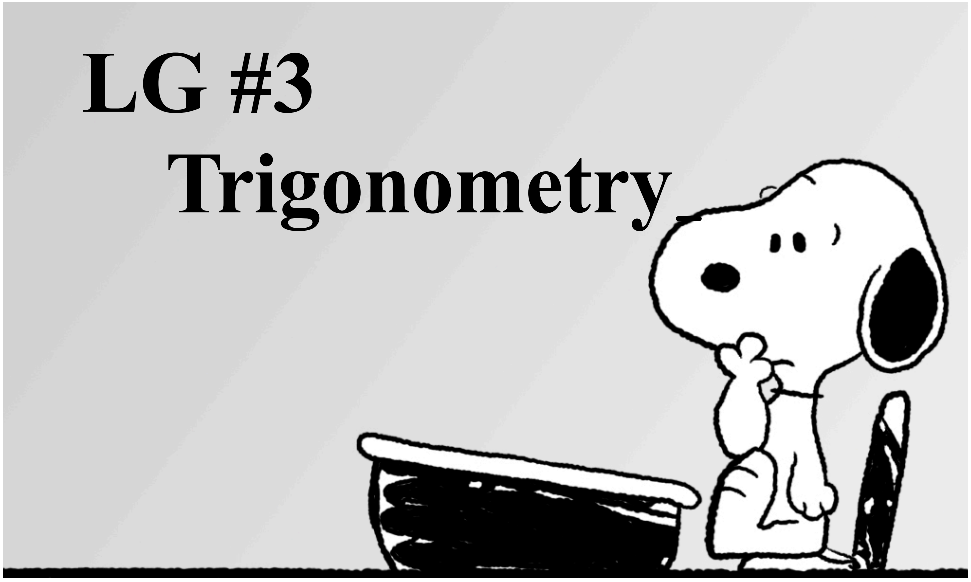


Agenda:

Trig. Review

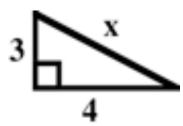
Basic Trigonometry

To find a side:

$$a^2 + b^2 = c^2$$

1. By Pythagorous

Example:



1. Write formula $a^2 + b^2 = c^2$ and substitute $3^2 + 4^2 = c^2$
2. Solve $9 + 16 = c^2$

$25 = c^2$

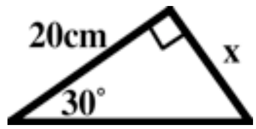
$\sqrt{25} = c$

$5 = c$

2. By Trigonometry

SOH CAH TOA

Example:



1. Determine the correct ratio (sin, cos or tan) - here we use tan because we are using the opposite and adjacent sides.

2. Write out ratio:

$$\tan = \frac{opp}{adj}$$

3. Substitute:

$$\tan 30^\circ = \frac{x}{20}$$

or

$$\frac{\tan 30^\circ}{1} = \frac{x}{20}$$

4. Cross multiply to solve:

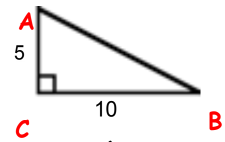
$$x = \tan 30^\circ \times 20 = 11.6 \text{ cm}$$

To find an Angle

1. By Trigonometry

SOH CAH TOA

Example: Find angle B



1. Determine the correct ratio (sin, cos or tan) - here we use tan because we are using the opposite and adjacent sides.

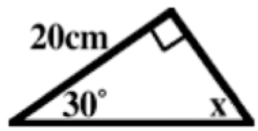
2. Write out ratio: $\tan = \dfrac{opp}{adj}$

3. Substitute: $x = \tan B = \dfrac{5}{10} = 0.5$

4. Solve using tan: $B = \tan^{-1}(0.5) = 26.6^\circ$

2. Using the Sum of Angles in a Triangle

Example: Find angle x

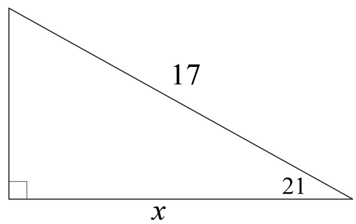


1. All 3 angles sum to 180° - we know 2 of the angles are 30° and 90°

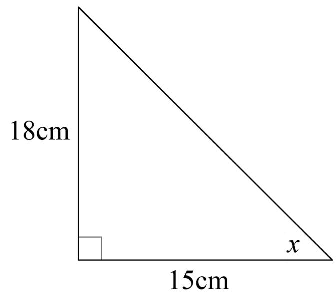
2. x = 180 - 90 - 30 = 60°

Try: Find the indicated side or angle

a)

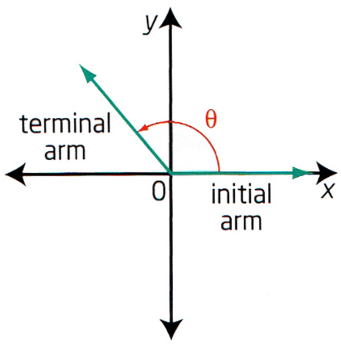


b)



Topic 1

Angles in Standard Position



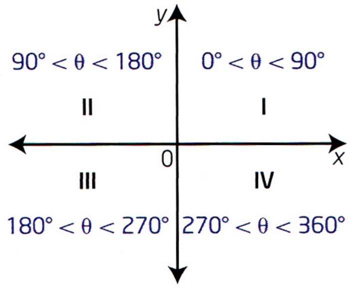
An angle is in standard position when:

1. it's vertex is at the origin

and

2. the initial arm is on the positive x-axis

*Angles in standard position are always measured counter-clockwise from the initial arm

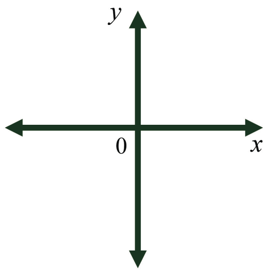


Angles in standard position are shown on the Cartesian plane
The x-axis and y-axis divide the plane into 4 quadrants

Example 1

Sketch an Angle in Standard Postion, $0^\circ \leq \theta \leq 360^\circ$

Example: a) 170°

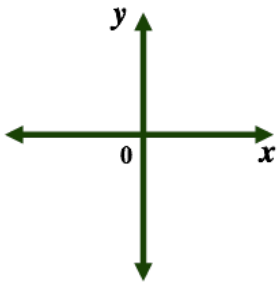


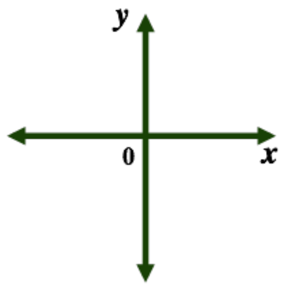
Try: Sketch each Angle in Standard Postion, and state the quadrant in which the terminal arm lies.

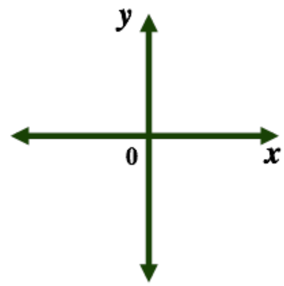
a) 37° b)

c) 320°

245°



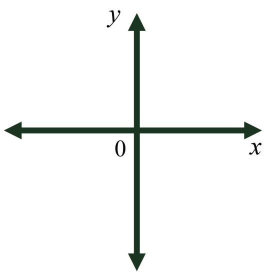




Example 1 - Part b

Directions: Are defined as a measure either east or west of north or south

Example: Show N40°W as an angle in standard position

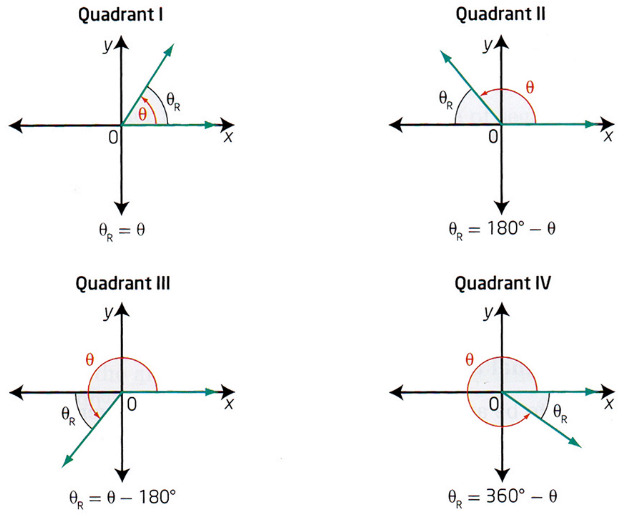


1. Start at North and go 40° toward west

Reference Angles

A Reference angle (θ_R) is:

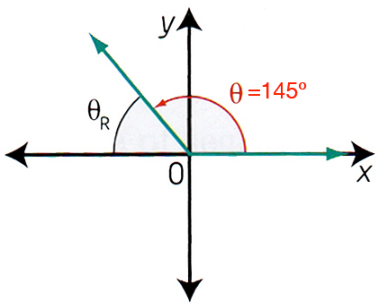
1. the acute ($\leq 90^{\circ}$) angle between the terminal arm and the x-axis
2. always positive



Example 2

Determine a Reference Angle

Example: $\theta = 145^\circ$



1. sketch angle
2. place the reference angle - find the shortest distance back to the x-axis
3. calculate reference angle

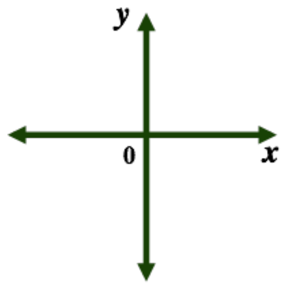
$$\theta_R = 180^\circ - \theta$$

$$\theta_R = 180^\circ - 145^\circ$$

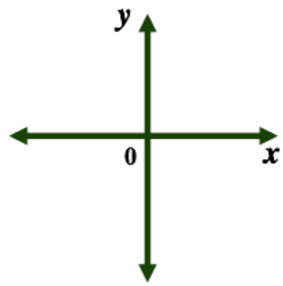
$$\theta_R = 35^\circ$$

Try: **Determine the Reference Angle for each angle in standard postition**

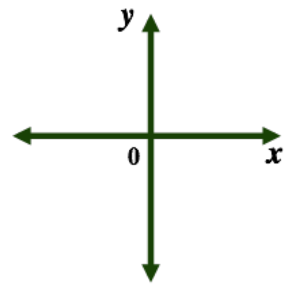
a) $79°$



b) $243°$



c) $317°$



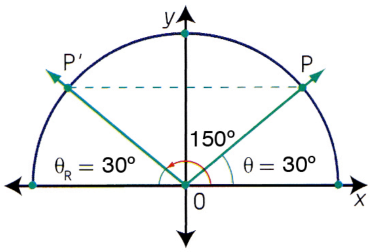
Example 3

Determine the Angle in Standard Position When Reflected

Example (part 1):

Determine the angle in standard position when an angle of 30° is reflected:

a) in the y -axis



1. the reference angle is the same as θ, 30°, but is across the y -axis
2. calculate the angle

$$\theta = 180^\circ - \theta_R$$

$$\theta = 180^\circ - 30^\circ$$

$$\theta = 150^\circ$$

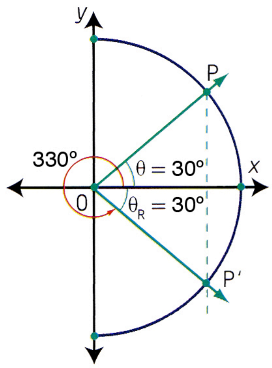
Example 3 cont.

Determine the Angle in Standard Position

Example (part 2):

Determine the angle in standard position when an angle of 30° is reflected:

b) in the x -axis



1. the reference angle is the same as θ, 30°, but is across the x -axis
2. calculate the angle

$$\theta = 360^\circ - \theta_R$$

$$\theta = 360^\circ - 30^\circ$$

$$\theta = 330^\circ$$

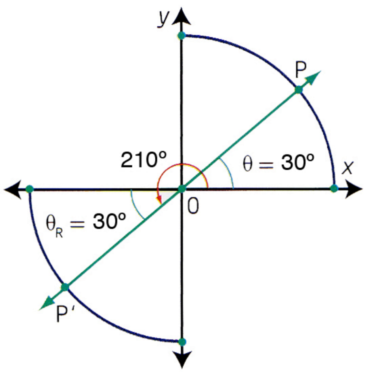
Example 3 cont.

Determine the Angle in Standard Position

Example (part 3):

Determine the angle in standard position when an angle of 30° is reflected:

c) in the y -axis & then in the x -axis



1. the reference angle is the same as θ, 30°, but is across both axes
2. calculate the angle

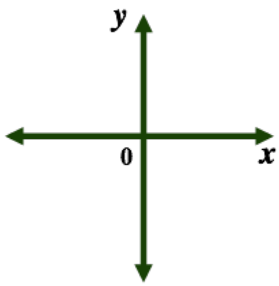
$$\theta = 180^{\circ} + \theta_R$$

$$\theta = 180^{\circ} + 30^{\circ}$$

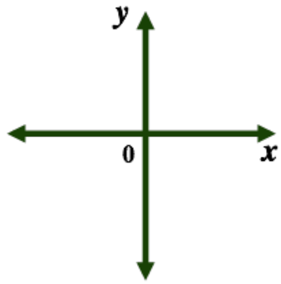
$$\theta = 210^{\circ}$$

Try: Determine the Angle in Standard Position When an angle of 45° is Reflected:

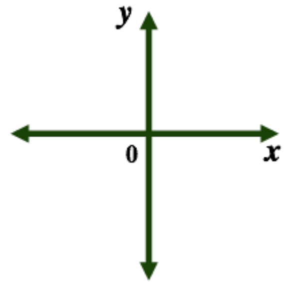
a) in the x-axis



b) in the y-axis

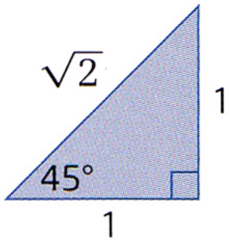


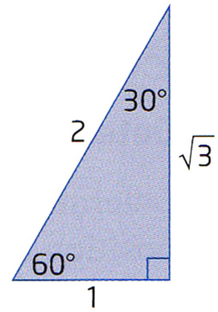
c) in the x-axis & the y-axis



Special Triangles

For right triangles of 30°, 45° and 60°, you can find exact values of trig. ratios using 2 special triangles





$$\sin 45° = \frac{1}{\sqrt{2}}$$

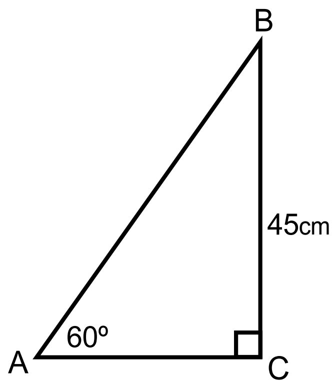
$$\cos 45° = \frac{1}{\sqrt{2}}$$

$$\tan 45° = \frac{1}{1} = 1$$

Example 4

Finding an Exact Distance

Example: Determine the exact length of side AC.



1. use the 30 , 60 , 90 special triangle
2. determine which ratio to use

$$\tan 60 = \frac{45}{AC}$$

3. substitute

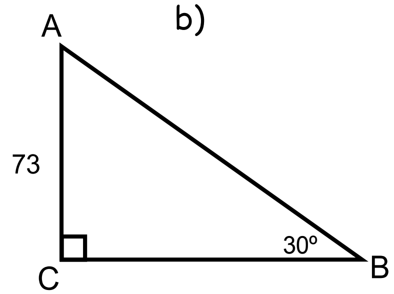
$$\frac{\sqrt{3}}{1} = \frac{45}{AC}$$

4. solve (cross multiply)

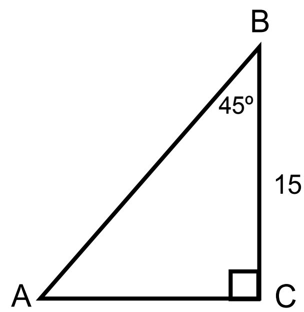
$$AC = \frac{45}{\sqrt{3}} = \frac{45\sqrt{3}}{3} = 15\sqrt{3}$$

Try: Find the exact length of AB in each of the triangles.

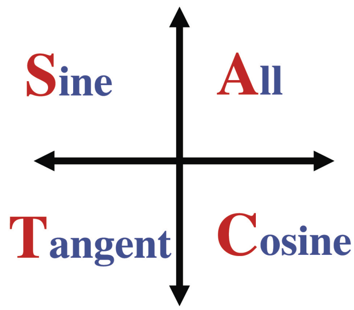
a)



b)



CAST Rule



The CAST rule tells you which trigonometric ratios are positive for each quadrant.

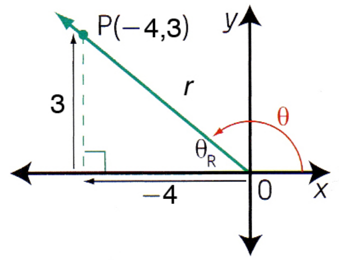
1. All (Sine, Cosine & Tangent) are positive in quadrant I

2. Only Sine is positive in quadrant II (so cosine and Tangent are negative) etc.

Topic 2 Example 1

Trigonometric Ratios of Any Angle

Example: The point P(-4, 3) lies on the terminal arm of an angle, θ, in standard position. Determine the exact ratios for sin θ, cos θ, and tan.



1. plot point & sketch triangle
2. use pythagorus to find

$a^2 + b^2 = c^2$

$3^2 + (-4)^2 = c^2$

$25 = c^2$

$5 = c$

3. write trig. ratios

$\sin\theta = \frac{3}{5}$ $\cos\theta = -\frac{4}{5}$ $\tan\theta = -\frac{3}{4}$

Try: The point P is on the terminal arm of an angle θ in standard position. Draw a diagram and calculate sinθ, tanθ, and cosθ.

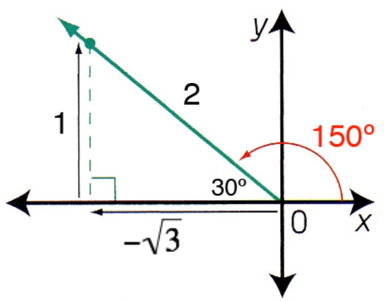
a) P (5, 2)

b) P (6, -5)

Example 2

Determine the Exact Value of a Trigonometric Ratio

Example: Determine the exact value of cos 150



1. sketch angle in standard pos.
2. calculate reference angle

$$\theta_R = 180^\circ - \theta$$

$$\theta_R = 180^\circ - 150^\circ$$

$$\theta_R = 30^\circ$$

3. sketch special triangle
4. write cos ratio

$$\cos\theta = -\frac{\sqrt{3}}{2}$$

Try: Determine the exact value of each of the following:

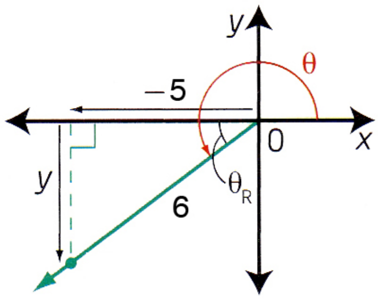
a) $\cos 45°$

b) $\tan 315°$

Example 3

Determine Trigonometric Ratios

Example: θ is an angle in standard position with terminal arm in quadrant III and $\cos\theta = -\frac{5}{6}$. Find the exact values of $\sin\theta$ and $\tan\theta$.



1. sketch a diagram
2. find y using pythagorus

$$x^2 + y^2 = r^2$$

$$(-5)^2 + y^2 = 6^2$$

$$25 + y^2 = 36$$

$$y = \sqrt{36-25} = \sqrt{11}$$

3. write ratios

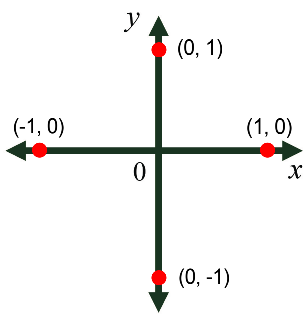
$$\sin\theta = -\frac{\sqrt{11}}{6} \qquad \tan\theta = \frac{\sqrt{11}}{5}$$

Try: θ is an angle in standard position with terminal arm in quadrant II and $\sin\theta = 3/5$. Find the exact value of $\cos\theta$ and $\tan\theta$.

Example 4

Determine Trigonometric Ratios of Quadrantal Angles

Example: Determine the values of sin θ, cos θ, and tan θ for quadrantal angles of 0°, 90°, 180° and 270°



1. Sketch a diagram
2. Place 4 points, each 1 unit from the origin
3. The x-coordinate tells you the cosine & the y-coordinate the sine
4. Find the tangent using Tan = Sin/Cos

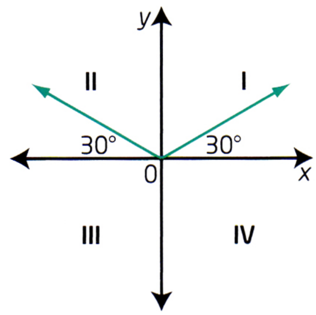
Example 5

Find Angles Given the Exact Sine, Cosine or Tangent

Example: Solve for θ

a) $\sin\theta = 0.5,\ 0^\circ \le \theta < 360^\circ$

1. Sketch a diagram - since sin θ is positive the terminal arm is quadrant I or II



2. Determine the angle in quadrant I

$\sin\theta = 0.5$

$\theta = \sin^{-1}(0.5)$

$\theta = 30^\circ \qquad \theta_R = 30^\circ$

3. Determine the angle in quadrant II

$\theta = 180^\circ - \theta_R$

$\theta = 180^\circ - 30^\circ$

$\theta = 150^\circ$

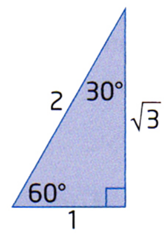
The two solutions are θ =30° or θ =150°

Example 5 cont.

b) $\cos\theta = -\dfrac{\sqrt{3}}{2}$, $0^\circ \le \theta < 180^\circ$

1. Since $\cos\theta$ is negative the terminal arm is in quadrant II or III. Because the angle is $< 180°$, it must be in quadrant II.

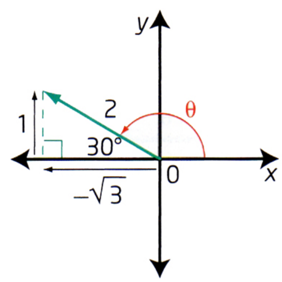
2. Use a 30°, 60°, 90° triangle to find θ_R



$\cos\theta_R = \dfrac{\sqrt{3}}{2}$

$\theta_R = 30^\circ$

3. Find the angle using a 30° reference angle in quadrant II



$\theta = 180^\circ - \theta_R$

$\theta = 180^\circ - 30^\circ$

$\theta = 150^\circ$

Try: Solve $\sin\theta = -\dfrac{1}{\sqrt{2}}, \quad 0^\circ \leq \theta < 360^\circ$

Example 6

Find Angles Given Approximate Sine, Cosine or Tangent

Example: Given cosθ = -0.6753, where $0^\circ \le \theta < 360^\circ$ determine the measure of θ, to the nearest tenth of a degree.

1. Since cosθ is negative, the angles the terminal arm is quadrant II or III

2. Determine the reference angle

$$\cos\theta_R = 0.6753$$

$$\theta_R = \cos^{-1}(0.6753)$$

$$\theta_R \approx 47.5$$

3. Determine the angles in quadrants II & III

quadrant II	quadrant III
$\theta = 180^\circ - \theta_R$	$\theta = 180^\circ + \theta_R$
$\theta = 180^\circ - 47.5^\circ$	$\theta = 180^\circ + 47.5^\circ$
$\theta = 132.5^\circ$	$\theta = 227.5^\circ$

Try: Determine the measure of θ, to the nearest degree, given $\sin\theta$ = -0.8090, where $0^{\circ} \leq \theta < 360^{\circ}$