











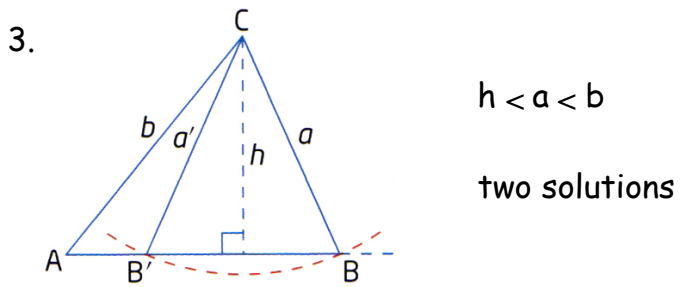
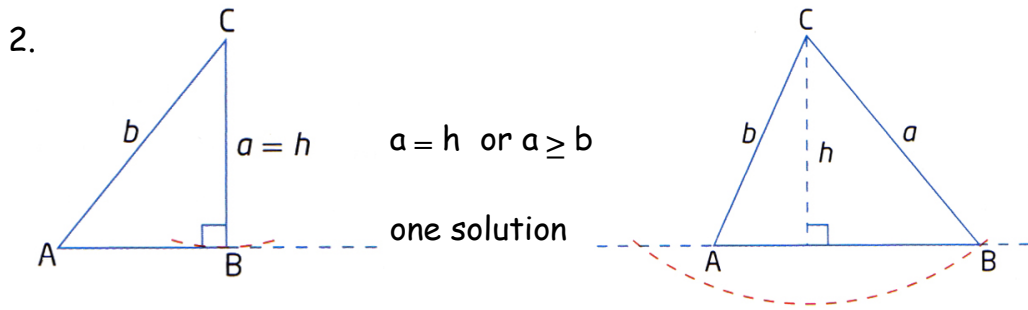








## The Ambiguous Case




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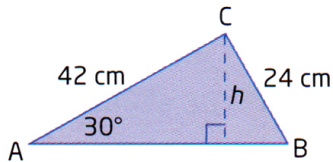
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## Example 3

### Sine Law in an Ambiguous Case

**Example:** In  $\triangle ABC$ ,  $\angle A = 30^\circ$ ,  $a = 24$  cm, and  $b = 42$  cm. Determine the measures of all other sides and angles



1. Sketch possible triangle

2. Find the height (h)

$$\sin A = \frac{h}{b}$$

$$h = b \sin A$$

$$h = 42 \sin 30^\circ$$

$$h = 21$$

$a > h$ , so there are 2  
possible triangles

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## Example 3 cont.

Triangle 1:

3. Solve for  $\angle B$  using sine law

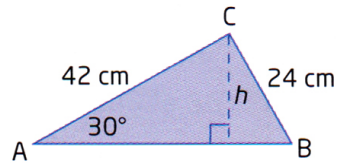
$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{42} = \frac{\sin 30^\circ}{24}$$

$$\sin B = \frac{42 \sin 30^\circ}{24}$$

$$\angle B = \sin^{-1}\left(\frac{42 \sin 30^\circ}{24}\right)$$

$$\angle B = 61.044\dots = 61^\circ$$



4. Find  $\angle C$  (sum of angles in a  $\Delta$ )

$$\angle C = 180^\circ - 61^\circ - 30^\circ = 89^\circ$$

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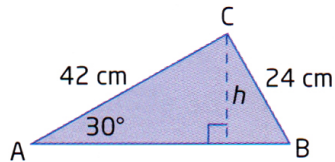
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### Example 3 cont.

5. Use sine law to find side  $c$

$$\frac{c}{\sin 89^\circ} = \frac{24}{\sin 30^\circ}$$
$$c = \frac{24 \sin 89^\circ}{\sin 30^\circ}$$
$$c = 47.992\dots = 48$$



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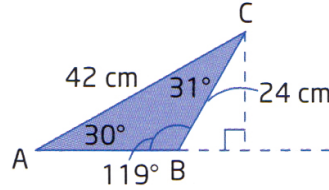
### Example 3 cont.

Triangle 2:

3. Solve for  $\angle B$  using  $61^\circ$  as the reference angle in quadrant II

$$\angle B = 180^\circ - 61^\circ$$

$$\angle B = 119^\circ$$



4. Find  $\angle C$  (sum of angles in a  $\Delta$ )

$$\angle C = 180^\circ - 119^\circ - 30^\circ$$

$$\angle C = 31^\circ$$

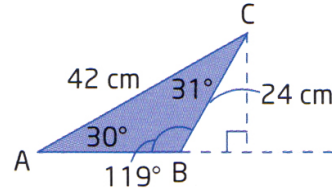
## Example 3 cont.

5. Use sine law to find side  $c$

$$\frac{c}{\sin 31^\circ} = \frac{24}{\sin 30^\circ}$$

$$c = \frac{24 \sin 31^\circ}{\sin 30^\circ}$$

$$c = 24.721\dots = 25$$




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**Try:** In triangle  $ABC$ ,  $\angle A = 21^\circ$ ,  $a = 12\text{m}$  and  $b = 17\text{m}$ .  
Determine the measures of all other sides and angles

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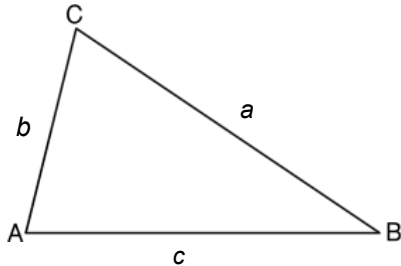
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**Topic 2****Cosine Law**

**Sine Law is the relationship between the cosine of an angle and the lengths of the three sides of any triangle.**



$$c^2 = a^2 + b^2 - 2bc \cos C$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

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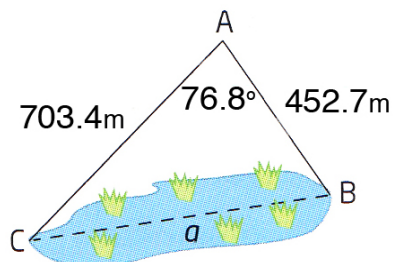
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## Example 1

### Determine an Distance (Side Length)

**Example:** A surveyor measures the distance to one end of a lake as 703.4m. The distance to the other end is 452.7m and the angle between the two is  $76.8^\circ$ . Find the length of a lake.



1. Sketch a diagram

2. Use cosine law

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = (703.4)^2 + (452.7)^2 - 2(703.4)(452.7)\cos 76.8^\circ$$

$$a^2 = 554281.6894$$

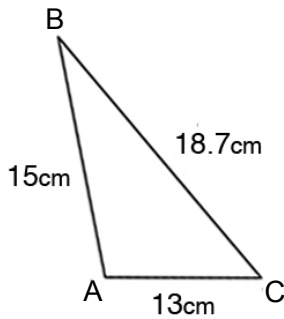
$$a = \sqrt{554281.6894} = 744.5009\dots = 744.5m$$



## Example 2

### Determine an Angle

**Example:** A triangular brace has side lengths of 15cm, 18.7cm and 13cm. Find the measure of the angle opposite the 15cm side.



1. Sketch a diagram
2. Use cosine law

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$(15)^2 = (18.7)^2 + (13)^2 - 2(18.7)(13)\cos C$$

$$225 - 349.69 - 169 = -486.2\cos C$$

$$-293.69 = -486.2\cos C$$

$$\frac{-293.69}{-486.2} = \cos C$$

$$\cos^{-1}\left(\frac{-293.69}{-486.2}\right) = \angle C$$

$$53^\circ = \angle C$$

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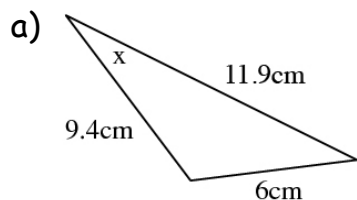
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Try: Find the measure of the indicated angle in each of the following triangles, to the nearest tenth.



b) In  $\triangle ABC$ ,  $a = 9\text{m}$ ,  $b = 18\text{m}$  and  $c = 21\text{m}$ . Find  $\angle A$ .

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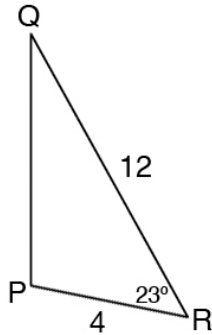
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## Example 3

### Solve a Triangle

**Example:** In  $\triangle PQR$ ,  $p = 12$ ,  $q = 4$ , and  $\angle R = 23^\circ$ . Find the length of the unknown side and the measure of the other 2 angles.



1. Sketch a diagram
2. Use cosine law to find  $r$

$$r^2 = p^2 + q^2 - 2pq\cos R$$

$$r^2 = 12^2 + 4^2 - 2(12)(4)\cos 23^\circ$$

$$r = \sqrt{71.6315\dots}$$

$$r = 8.46354\dots$$

3. Use cosine law to find  $\angle P$

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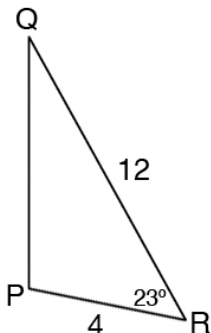
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### Example 3 (cont.)



$$\begin{aligned}
 3. \quad p^2 &= q^2 + r^2 - 2qr \cos P \\
 (12)^2 &= (4)^2 + (8.453)^2 - 2(4)(8.453)\cos P \\
 144 - 16 - 71.631 &= (-67.708)\cos P \\
 \frac{56.368}{-67.708} &= \cos P \\
 \cos^{-1}\left(\frac{56.368}{-67.708}\right) &= P \\
 146.4^\circ &= P
 \end{aligned}$$

4. Find  $\angle Q$  using sum of  $\angle$ 's in a triangle

$$\angle Q = 180^\circ - 146.4^\circ - 23^\circ$$

$$\angle Q = 10.6^\circ$$

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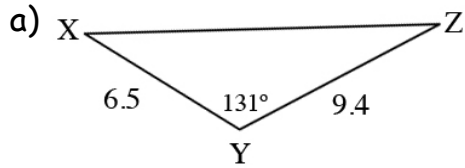
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Try: Solve the following triangles. Round your answers to the nearest tenth.



b) In  $\triangle ABC$ ,  $a = 9$ ,  $b = 7$  and  $\angle C = 33.6^\circ$

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