LG #6
Completing the Square

Agenda:

Topic 1   Example 1

Completing the Square

Example: Convert from Standard Form to Vertex Form.

$f(x) = x^2 + 6x + 5$

1. Group the 1st 2 terms

$$y = \left(x^2 + 6x\right) + 5$$

2. Add and subtract the square of half the coefficient of the x-term (inside the brackets)

$$y = \left(x^2 + 6x + 9 - 9\right) + 5$$

3. Group the trinomial

$$y = \left(x^2 + 6x + 9\right) - 9 + 5$$

4. Factor and Simplify

$$y = \left(x + 3\right)^2 - 9 + 5$$

$$y = \left(x + 3\right)^2 - 4$$

Try: Convert each of the following to vertex from standard form:

a) $f(x) = x^2 + 16x + 20$ b) $f(x) = 3x^2 - 12x - 9$

Example 2

Convert to Vertex Form and Verify

Example: Convert the function $y = 4x^2 - 28x - 23$ to vertex form and verify that the two forms are equivalent.

1. Complete the square to convert to vertex form (like example 1)

$$y = 4x^2 - 28x - 23$$

$$y = 4\left(x^2 - 7x\right) - 23 \qquad \text{Factor out 4}$$

$$y = 4\left(x^2 - 7x + \frac{49}{4} - \frac{49}{4}\right) - 23$$

$$y = 4\left[\left(x^2 - 7x + \frac{49}{4}\right) - \frac{49}{4}\right] - 23 \qquad \text{Group trinomial}$$

$$y = 4\left[\left(x - \frac{7}{2}\right)^2 - \frac{49}{4}\right] - 23 \qquad \text{Factor trinomial}$$

Example 2 cont.

Completing the square, cont.

$$y = 4\left(x - \frac{7}{2}\right)^2 - 4\left(\frac{49}{4}\right) - 23$$

$$y = 4\left(x - \frac{7}{2}\right)^2 - 49 - 23$$

$$y = 4\left(x - \frac{7}{2}\right)^2 - 72$$

2. Work backward to confirm (FOIL and simplify).

$$y = 4\left(x - \frac{7}{2}\right)^2 - 72$$

$$y = 4\left(x - \frac{7}{2}\right)\left(x - \frac{7}{2}\right) - 72 \qquad \text{FOIL trinomial}$$

$$y = 4\left(x^2 - \frac{7}{2}x - \frac{7}{2}x + \frac{49}{4}\right) - 72$$

Example 2 cont.

2. Work backward, cont.

$$y = 4\left(x^2 - 7x + \frac{49}{4}\right) - 72$$ collect like terms

$$y = 4x^2 - 28x + 49 - 72$$
   multiply coefficient through and simplify
$$y = 4x^2 - 28x - 23$$

result is the original function

Try: **Convert into vertex form and verify that the two forms are equivalent.**

a) $y = -3x^2 - 27x + 13$

Example 3

Determine the Vertex of a Quadratic Function by Completing the Square

Example: For the Function: $y = 5x^2 + 30x + 41$

a) Complete the Square to find the vertex

b) Verify the releationship between p and q and b

c) Use the relationship from part b) to find the vertex. Compare to the answer from part a)

a) Complete the Square to find the vertex

$$y = 5x^2 + 30x + 41$$

$$y = 5(x^2 + 6x) + 41$$

$$y = 5(x^2 + 6x + 9 - 9) + 41$$

$$y = 5[(x^2 + 6x + 9) - 9] + 41$$

Example 3 cont.

a) Complete the Square (cont.)

$$y = 5\left[(x+3)^2 - 9\right] + 41$$

$$y = 5(x+3)^2 - 45 + 41$$

$$y = 5(x+3)^2 - 4$$

- vertex (-3, -4)
- graph opens up (a is positive)
- vertex is a minimum

b) Look back at the steps in a)

$$y = 5x^2 + 30x + 41 \longrightarrow y = ax^2 + bx + 41$$

$$y = 5\left(x^2 + 6x\right) + 41$$

b divided by a

6 is $\dfrac{30}{5}$, or $\dfrac{b}{a}$

$$y = 5(x + 3)^2 - 4$$

3 is half of 6, or half of $\dfrac{b}{a}$ or $\dfrac{b}{2a}$

$$y = 5(x - p)^2 - 4$$

$$p = -\dfrac{b}{2a}$$

Example 3 cont.

c) Determine the coordinates of the vertex

1. Find the x- coordinate using $x = -\dfrac{b}{2a}$

$$x = -\dfrac{30}{2(5)}$$

$$x = -\dfrac{30}{10}$$

$$x = -3$$

2. Find the y- coordinate by substitution

$$y = 5x^2 + 30x + 41$$

$$y = 5(-3)^2 + 30(-3) + 41$$

$$y = 45 - 90 + 41$$

$$y = -4$$

The vertex is (-3, -4)

**this is the same as the vertex we found in part a)

Try: **For the function** $y = 3x^2 + 30x + 41$

    a) Complete the square to find the vertex

    b) Use $x = -\dfrac{b}{2a}$ **and the standard form to find**

       the vertex. Compare your 2 answers.

Example 4

Write a Quadratic Model Function

Example: Last year photo sessions were $10 and 400 sessions were booked. It is estimated that for every $1 increase in price, 20 fewer sessions will be booked.

a) Write a function to model the situation

let n = the number of price increases
let R = the expected revenue
$\Big\}$ Declare variables

$10 + 1n$ ←——this years price
$400 - 20n$ ← number of sessions this year
$\Big\}$ Write relationships

Revenue = (price)(number of sessions)
$R = (10 + n)(400 - 20n)$
$R = 4000 + 200n - 20n^2$
$R = -20n^2 + 200n + 4000$
$\Big\}$ Write equation

Example 4 cont.

b) Complete to find the maximum revenue and the price that brings that revenue

$R = -20n^2 + 200n + 4000$

$R = -20(n^2 + 10n) + 4000$

$R = -20(n^2 + 10n - 25 - 25) + 4000$

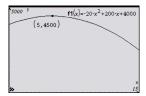$R = -20[(n^2 + 10n - 25) - 25] + 4000$

$R = -20[(n-5)^2 - 25] + 4000$

$R = -20(n-5)^2 + 500 + 4000$

$R = -20(n-5)^2 + 4500$

The vertex is (5, 4500). The maximum revenue will be $4500 when n = 5 (when there are 5 price increases of $1). So the price should go up $5 to $15.

Example 4 cont.

b) Verify the solution by graphing

1. Enter the function into y= and adjust the window. Then find the maximum like you did in learning guide 5

2. Press [2nd] [TRACE] 4 (maximum) move the cursor to the left side of the maximum and press [ENTER] then move the cursor to the right side of the maximum and press [ENTER]

3. The vertex is at (5, 4500). This verifies that the maximum revenue is $4500 with 5 price increases or a session fee of $15.

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Example 4 cont.

d) What assumptions were made in creating and using this model function?

It was assumed that:

· price affects revenue in a predictable way

Other factors could affect the revenue but were not considered, such as:

• advertising

• word of mouth

• ...

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Try: A sporting goods store sells water bottles for $8. At this price they sell about 100 bottles per week. Research says that for every $2 increase in price, they can expect to sell 5 fewer bottles.
a) Write a quadratic to represent this situation
b) Find the maximum revenue and the selling price
c) Verify the solution
d) What were your assumptions?