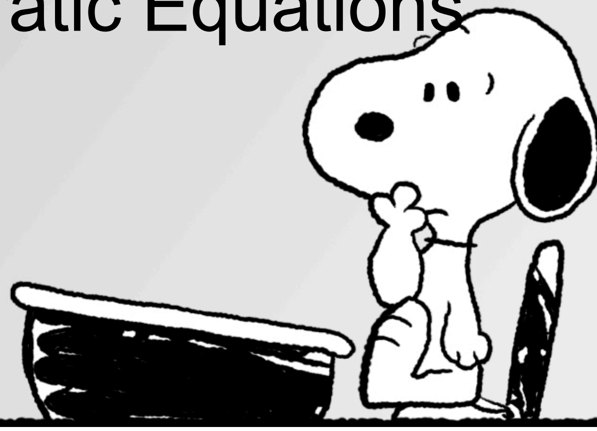
LG #7

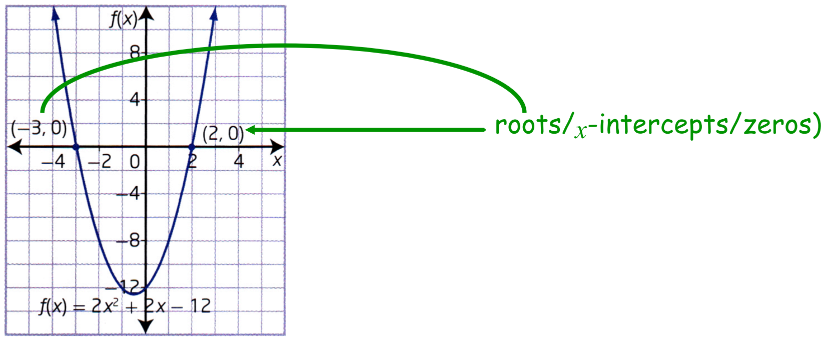
Quadratic Equations



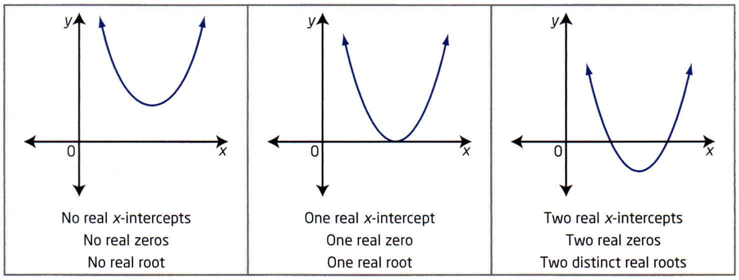
Agenda:

Topic 1 Quadratic Equations

Quadratic equations of the form $ax^2 + bx + c = 0$ can be solved by graphing the corresponding function, $f(x) = ax^2 + bx + c$. Solutions are called the **roots** (or x-intercepts or zeros)



Quadratic equations can have no solutions, one solution or two solutions.



Example 1

Solving Quadratics by Graphing

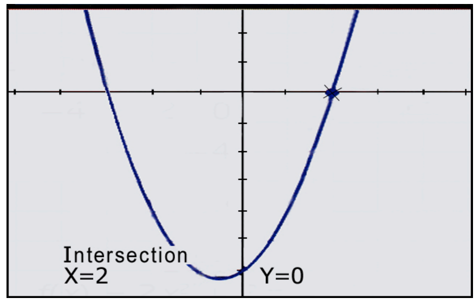
Example: What are the roots of the equation $2x^2 + 2x - 12 = 0$

1. Enter in the graphing calculator as $y = 2x^2 + 2x - 12$
2. Adjust window if necessary
3. Find one of the x-intercepts (2nd TRACE 5 and ENTER 3 times)

Example 1 (cont.)

4. Find the 2nd x-intercept as above, but make sure you move the cursor to the other side of the vertex before pressing enter

5. The roots are (2, 0) and (-3,0)



Check

1. Substitute $x = 2$ and $x = -3$ into the original equation

$$2x^2 + 2x - 12 = 0$$
$$2(2)^2 + 2(2) - 12 = 0$$
$$8 + 4 - 12 = 0$$
$$0 = 0$$

Both solutions are correct

$$2x^2 + 2x - 12 = 0$$
$$2(-3)^2 + 2(-3) - 12 = 0$$
$$18 - 6 - 12 = 0$$
$$0 = 0$$

Try: Determine the roots of each quadratic equation

a) $x^2 - 6x + 9 = 0$

b) $3x^2 - 7x + 6 = 0$

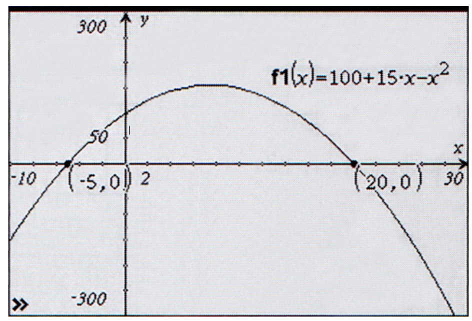
Example 2

Solving a problem with Quadratics

Example: The manager of a clothing store is investigating the effect that raising or lowering dress prices has on the daily revenue from dress sales. The function $R(x) = 100 + 15x - x^2$ gives the store's revenue R from dress sales, in dollars, where x is the price change in dollars. What price change will result in no revenue?

When there is no revenue, $R(x) = 0$. To answer the question, find the zeros.

1. Graph the equation. Adjust the window settings until you can see the vertex and the x-intercepts
2. Use the trace function to find the x-intercepts



Example 2 cont.

5. The roots are (-5, 0) and (20,0)

Check

Substitute $x = \text{-}5$ and $x = 20$ into the original equation

$$100 + 15x - x^2 = 0$$
$$100 + 15(-5) - (-5)^2 = 0$$
$$100 - 75 - 25 = 0$$
$$0 = 0$$

$$100 + 15x - x^2 = 0$$
$$100 + 15(20) - (20)^2 = 0$$
$$100 + 300 - 400 = 0$$
$$0 = 0$$

Both solutions are correct

A price decrease of $\$5$ or an increase of $\$20$ will both result in no revenue from dress sales

Try: The manager at Suzie's Fashions has determined that the function $R(x) = 600 - 6x^2$ models the weekly revenue, R, in dollars from sweatshirts as the price changes, where x, is the price change, in dollars. What price change will result in no revenue?

Topic 2 Factoring Quadratic Equations

Example 1

Factor Quadratic Expressions

Example: Factor each of the following:

a) $x^2 + 4x - 5$ factors of -5 add to 4

$(x+5)(x-1)$

5	-1

b) $2x^2 - 2x - 12$ Factor out GCF

$2(x^2 - x - 6)$ factors of -6 add to -1

$2(x-3)(x+2)$

-6	1
-3	2

Example 1 (cont.)

c) $2x^2 - x - 3$

factors of -3 | add to -1

-3	1

← Adds to -2 so doesn't work

multiply

$2x^2 - x - 3$

factors of -6 | add to -1

-6	1
-3	2

$$\frac{(2x-3)(2x+2)}{2}$$

$(2x-3)(x+1)$

d) $4x^2 - 81$ ← can square root both

$(2x+9)(2x-9)$

Try: Factor each of the following:

a) $3x^2 + 3x - 6$ b) $\frac{1}{2}x^2 - x - 4$ c) $0.49j^2 - 36k^2$

Example 2

Factor Polynomials of Quadratic Form

Example: Factor each polynomial

a) $12(x+2)^2+24(x+2)+9$

treat $x+2$ as a single variable
$r=x+2$

$12r^2+24r+9$

substitute r for $x+2$

$3(4r^2+8r+3)$

factor as before

$\dfrac{3(4r+6)(4r+2)}{4}$

$\dfrac{3(4r+6)(4r+2)}{2\times 2}$

$3(2r+3)(2r+1)$

Example 2 (cont.)

$3[2(x+2)+3][2(x+2)+1]$ substitute $x+2$ for r

$3(2x+4+3)(2x+4+1)$

$3(2x+7)(2x+5)$

b) $9(2t+1)^2 - 4(s-2)^2$

each term is a perfect square making it a difference of squares

$[3(2t+1)-2(s-2)][3(2t+1)+2(s-2)]$ factor like a difference of squares

$(6t+3-2s+4)(6t+3+2s-4)$

$(6t-2s+7)(6t+2s-1)$

Try: Factor each of the following:

a) $-2(n+3)^2+12(n+3)+14$

b) $4(x-2)^2-0.25(y-4)^2$

Example 3

Solve Quadratic Equations by Factoring

Example: Determine the roots of each quadratic equation. Verify your solutions.

a) $x^2 + 6x + 9 = 0$ — factor trinomial

$(x + 3)(x + 3) = 0$

$(x + 3) = 0$ or $(x + 3) = 0$ — for the quadratic equation to equal 0, one of the factors must equal 0

$x = -3$ $\quad$ $x = -3$

Check

Substitute $x = -3$ into the original equation

$x^2 + 6x + 9 = 0$

$(-3)^2 + 6(-3) + 9 = 0$

$9 - 18 + 9 = 0$

$0 = 0$

Example 3 (cont.)

a) $2x^2 - 9x - 5 = 0$ factor trinomial

$$\frac{(2x-10)(2x+1)}{2} = 0$$

$$(x-5)(2x+1) = 0$$

$x - 5 = 0$ or $2x + 1 = 0$

$x = 5$ $\quad$ $2x = -1$

$x = -\frac{1}{2}$

for the quadratic equation to equal 0, one of the factors must equal 0

Check

Substitute $x = 5$ and $x = -1/2$ into the original equation

$$2x^2 - 9x - 5 = 0$$
$$2(5)^2 - 9(5) - 5 = 0$$
$$50 - 45 - 5 = 0$$
$$0 = 0$$

$$2x^2 - 9x - 5 = 0$$
$$2\left(-\frac{1}{2}\right)^2 - 9\left(-\frac{1}{2}\right) - 5 = 0$$
$$\frac{1}{2} + \frac{9}{2} - 5 = 0$$
$$0 = 0$$

Example 3 cont.

a) Complete the Square (cont.)

$$y = 5\left[(x+3)^2 - 9\right] + 41$$

$$y = 5(x+3)^2 - 45 + 41$$

$$y = 5(x+3)^2 - 4$$

- vertex (-3, -4)
- graph opens up (a is positive)
- vertex is a minimum

b) Look back at the steps in a)

$y = 5x^2 + 30x + 41$ → $y = ax^2 + bx + 41$

$y = 5(x^2 + 6x) + 41$ ← b divided by a: 6 is $\frac{30}{5}$, or $\frac{b}{a}$

$y = 5(x + 3)^2 - 4$ ← 3 is half of 6, or half of $\frac{b}{a}$ or $\frac{b}{2a}$

$y = 5(x - p)^2 - 4$ ← $p = -\frac{b}{2a}$

Example 3 cont.

Quadratic equations of the form $ax^2 + bx + c = 0$ can be solved by factoring.

c) Determine the coordinates of the vertex

1. Find the x-coordinate using $x = -\frac{b}{2a}$

$$x = -\frac{30}{2(5)}$$

$$x = -\frac{30}{10}$$

$$x = -3$$

2. Find the y-coordinate by substitution

$$y = 5x^2 + 30x + 41$$

$$y = 5(-3)^2 + 30(-3) + 41$$

$$y = 45 - 90 + 41$$

$$y = -4$$

The vertex is (-3, -4)

**this is the same as the vertex we found in part a)

Try: For the function $y = 3x^2 + 30x + 41$

a) Complete the square to find the vertex

b) Use $x = -\frac{b}{2a}$ and the standard form to find the vertex. Compare your 2 answers.

Example 4

Write a Quadratic Model Function

Example: A rectangle field has dimensions $x + 4$ and $3x - 10$, where x is measured in metres. The area of the field is 4840 m^2.

a) Write a equation to model the situation.
b) Solve for x.

Example 5

Write a Quadratic Model Function

Example: Two whole numbers that differ by 5. The sum of their squares is 53.

a) What are the two numbers.