LG #8
The Quadratic Formula

Agenda:

Topic 1 Solving Quadratic Equations Without Factoring

Example 1

Solve a Quadratic Equation by Completing the Square

Example: Solve $x^2 - 21 = -10x$ by completing the square. Express your answers to the nearest tenth.

$x^2 - 21 = -10x$

$x^2 + 10x = 21$ isolate the variable terms to one side

$x^2 + 10x + 25 = 21 + 25$ complete the square

$(x+5)^2 = 46$

$\sqrt{(x+5)^2} = \sqrt{46}$ square root both sides

$x + 5 = \pm\sqrt{46}$ isolate the variable

$x = -5 \pm \sqrt{46}$ ← exact solution

Example 1 cont.

$x = -5x + \sqrt{46}$ or $x = -5x - \sqrt{46}$

$x = 1.7823...$ $x = -11.7823...$

$x = 1.8$ $x = -11.8$

approximate solution
(decimal answer)

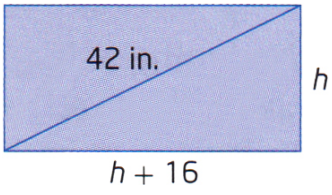
Try: Solve $p^2 - 4p = 11$ by completing the square.
Express your answers to the nearest tenth.

Example 2

Write and Solve a Quadratic Equation by Taking the Square Root

Example: A wide-screen T.V. has a diagonal measure of 42 in. The width of the screen is 16 in. more than the height. Determine the dimensions to the nearest tenth of an inch.

1. Draw a diagram.



let h = height
then $h + 16$ = length

Example 2 cont.

2. Use Pythagorean Theorem

$a^2 + b^2 = c^2$

$h^2 + (h+16)^2 = 42^2$ substitute

$h^2 + (h^2 + 32h + 256) = 1764$

$2h^2 + 32h + 256 = 1764$

$2h^2 + 32h = 1508$

$h^2 + 16h = 754$ simplify

$h^2 + 16h + 64 = 754 + 64$ solve by completing the square

$(h+8)^2 = 818$ isolate the square

$\sqrt{(h+8)^2} = \sqrt{818}$ square root both sides

$h + 8 = \pm\sqrt{818}$ isolate the variable

$h = -8 \pm \sqrt{818}$

$h = -8 + \sqrt{818}$ and $h = -8 - \sqrt{818}$

$h \approx 20.6$ $h \approx -36.6$

can't have a negative height - reject as extraneous

Try: A Toonie consists of an aluminium and bronze core and a nickel outer ring. If the radius of the inner core is 0.84 cm and the area of the circular face is $1.96\pi\ \text{cm}^2$ what is the width of the outer ring?

Example 3

Solve a Quadratic When $a \neq 1$

Example: Determine the roots of $-2x^2 - 3x + 7 = 0$, to the nearest hundredth. Verify your answer with your calculator.

$-2x^2 - 3x + 7 = 0$

$\frac{2}{2}x^2 + \frac{3}{2}x - \frac{7}{2} = 0$ Divide both sides by 2

$x^2 + \frac{3}{2}x = \frac{7}{2}$ Isolate the variable terms to the left

$x^2 + \frac{3}{2}x + \frac{9}{16} = \frac{7}{2} + \frac{9}{16}$

$\left(x + \frac{3}{4}\right)^2 = \frac{65}{16}$

} Complete the square

Example 3 cont.

$$\sqrt{\left(x+\frac{3}{4}\right)^2}=\sqrt{\frac{65}{16}}$$ square root both sides

$$x+\frac{3}{4}=\pm\sqrt{\frac{65}{16}}$$

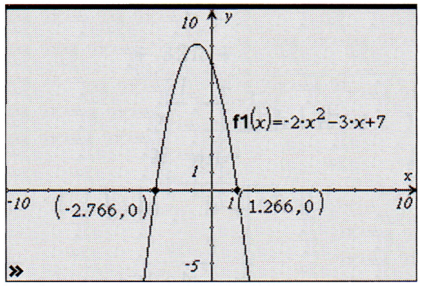
$$x=-\frac{3}{4}\pm\frac{\sqrt{65}}{4}$$ isolate the variable

$$x=\frac{-3\pm\sqrt{65}}{4}$$ ← exact solution

$x=1.27$ and $x=-2.77$

Check

Graphing and finding the x-intercepts confirms the solution



Try: Determine the roots of the equation $-2x^2 - 5x + 2 = 0$ to the nearest hundredth. Verify by graphing.

Topic 2

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The quadratic formula is used to find the roots of quadratic equations of the form $ax^2 + bx + c = 0$

Example 1

Use the Quadratic Formula to Solve Quadratic Equations

Example: Use the quadratic formula to solve $9x^2 + 12x = -4$

1. Write $9x^2 + 12x = -4$ in standard form, $ax^2 + bx + c = 0$

   $9x^2 + 12x + 4 = 0$ $\quad a = 9,\ b = 12,$ and $c = 4$

Example 1 (cont.)

2. Substitute the values for a, b, and c into the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-12 \pm \sqrt{12^2 - 4(9)(4)}}{2(9)}$$

$$x = \frac{-12 \pm \sqrt{144 - 144}}{18}$$

$$x = \frac{-12 \pm \sqrt{0}}{18}$$

$$x = \frac{-12}{18}$$

$$x = -\frac{2}{3}$$

Check

Substitute $x = -\frac{2}{3}$ into the original equation

$$9x^2 + 12x = -4$$

$$9\left(-\frac{2}{3}\right)^2 + 12\left(-\frac{2}{3}\right) = -4$$

$$4 - 8 = -4$$

$$-4 = -4$$

Try: Determine the roots of the each equation to the nearest hundredth and verify your solution.

a) $3x^2 + 5x - 2 = 0$

b) $\frac{t^2}{2} - t - \frac{5}{2} = 0$

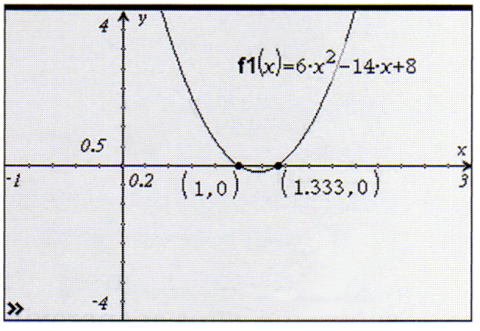
Example 2

Select a Strategy to Solve a Quadratic Equation

Example: Solve $6x^2 - 14x + 8 = 0$ by:

a) graphing
b) factoring
c) completing the square
d) using the quadratic formula

a) Graphing



1. Graph the related function, $y = 6x^2 - 14x + 8$
2. Remember to graph $Y_2 = 0$
3. The x-intercepts are (1, 0) and about (1.33, 0)

Example 2 (cont.)

b) Factoring

$$6x^2 - 14x + 8 = 0$$

$$3x^2 - 7x + 4 = 0$$

$$(3x - 4)(x - 1) = 0$$

$$(3x - 4) = 0 \qquad (x - 1) = 0$$

$$3x = 4$$

$$x = \frac{4}{3} \quad \text{or} \quad x = -1$$

c) Completing the Square

$$6x^2 - 14x + 8 = 0$$

$$x^2 - \frac{7}{3}x + \frac{4}{3} = 0$$

$$x^2 - \frac{7}{3}x = -\frac{4}{3}$$

Example 2 (cont.)

c) Completing the Square (cont.)

$$x^2 - \frac{7}{3}x + \frac{49}{36} = -\frac{4}{3} + \frac{49}{36}$$

$$\left(x - \frac{7}{6}\right)^2 = \frac{1}{36}$$

$$\sqrt{\left(x - \frac{7}{6}\right)^2} = \sqrt{\frac{1}{36}}$$

$$x - \frac{7}{6} = \pm\sqrt{\frac{1}{36}}$$

$$x = \frac{7}{6} \pm \frac{1}{6}$$

$$x = \frac{7}{6} + \frac{1}{6} \quad \text{or} \quad x = \frac{7}{6} - \frac{1}{6}$$

$$x = \frac{8}{6} \qquad\qquad x = \frac{6}{6}$$

$$x = \frac{4}{3} \qquad\qquad x = 1$$

Example 2 (cont.)

d) Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(6)(8)}}{2(6)}$$

$$x = \frac{14 \pm \sqrt{196 - 193}}{12}$$

$$x = \frac{14 \pm \sqrt{4}}{12}$$

$$x = \frac{14 \pm 2}{12}$$

$$x = \frac{14 + 2}{12} \quad \text{or} \quad x = \frac{14 - 2}{12}$$

$$x = \frac{16}{12} \qquad x = \frac{12}{12}$$

$$x = \frac{4}{3} \qquad x = 1$$

Try: Solve the equation $0.57x^2 - 3.7x - 2.5 = 0$ using a method of your choice.

Topic 3 — The Discriminant

The discriminant lets you determine the nature of the roots for a quadratic equations of the form $ax^2 + bx + c = 0$. It is the expression $b^2 - 4ac$ which is under the radical sign in the quadratic formula.

$b^2 - 4ac > 0$ 2 distinct real roots

$b^2 - 4ac = 0$ 2 equal real roots (one distinct real root)

$b^2 - 4ac < 0$ NO real roots

Example 1

Use the Discriminant to Determine the nature of the Roots

Example: Determine the nature of the roots of $-2x^2 + 3x + 8 = 0$. Verify your answer with your calculator.

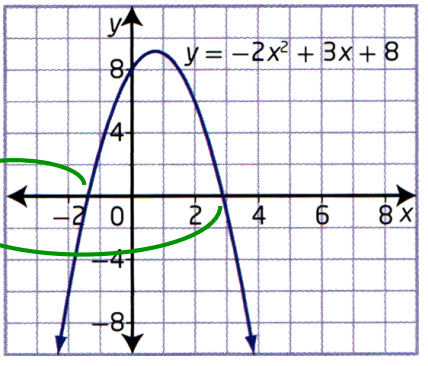
$b^2 - 4ac$

$(3)^2 - 4(-2)(8)$

$9 + 64$

73 ← Greater than 0, so there are 2 distinct real roots.

Graphing confirms 2 real roots.



Try: Determine the nature of the roots for each equation and verify your solution.

a) $2x^2 - 8x = -9$

b) $3x^2 - 4x + \frac{4}{3} = 0$