Chapter 1 Function Transformations

Section 1.1 Horizontal and Vertical Translations

Section 1.1 Page 12 Question 1

a) For $y - 5 = f(x)$, $h = 0$ and $k = 5$.

b) For $y = f(x) - 4$, $h = 0$ and $k = -4$.

c) For $y = f(x + 1)$, $h = -1$ and $k = 0$.

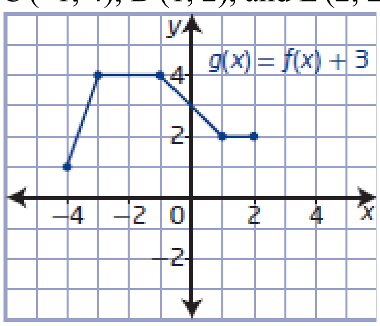
d) For $y + 3 = f(x - 7)$, $h = 7$ and $k = -3$.

e) For $y = f(x + 2) + 4$, $h = -2$ and $k = 4$.

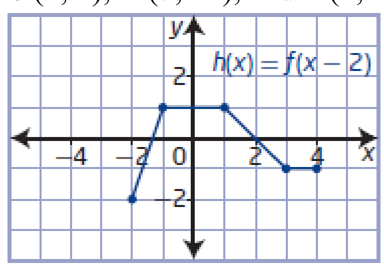
Section 1.1 Page 12 Question 2

A(–4, –2), B(–3, 1), C(–1, 1), D(1, –1), E(2, –1)

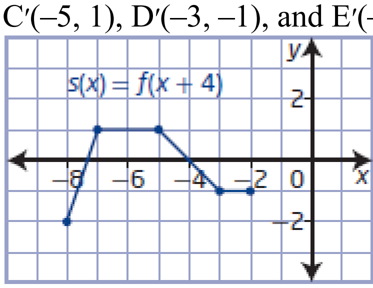
a) For $g(x) = f(x) + 3$, A′(–4, 1), B′(–3, 4), C′(–1, 4), D′(1, 2), and E′(2, 2).



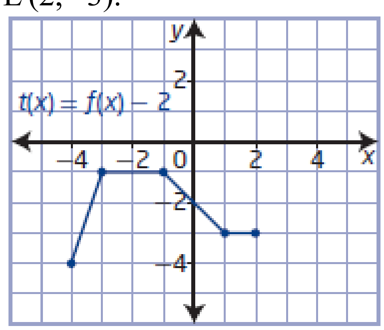
b) For $h(x) = f(x - 2)$, A′(–2, –2), B′(–1, 1), C′(1, 1), D′(3, –1), and E′(4, –1).



c) For $s(x) = f(x + 4)$, A′(–8, –2), B′(–7, 1), C′(–5, 1), D′(–3, –1), and E′(–2, –1).



d) For $t(x) = f(x) - 2$, A′(–4, –4), B′(–3, –1), C′(–1, –1), D′(1, –3), and E′(2, –3).



Section 1.1 Page 13 Question 3

For horizontal translation and vertical translation, every point (x, y) on the graph of $y = f(x)$ is transformed to $(x + h, y + k)$.

a) For $y = f(x + 10)$, $h = -10$ and $k = 0$: $(x, y) \rightarrow (x - 10, y)$.

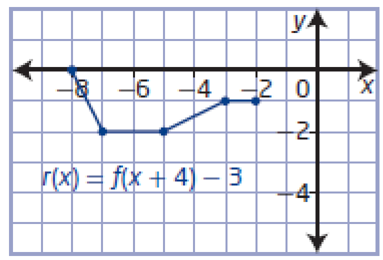
b) For $y + 6 = f(x)$, $h = 0$ and $k = -6$: $(x, y) \rightarrow (x, y - 6)$.

c) For $y = f(x - 7) + 4$, $h = 7$ and $k = 4$: $(x, y) \rightarrow (x + 7, y + 4)$.

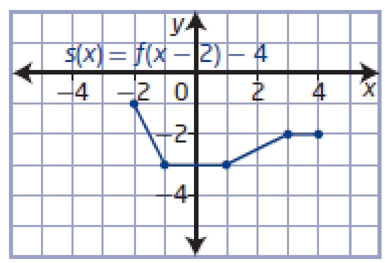
d) For $y - 3 = f(x - 1)$, $h = 1$ and $k = 3$: $(x, y) \rightarrow (x + 1, y + 3)$.

Section 1.1 Page 13 Question 4

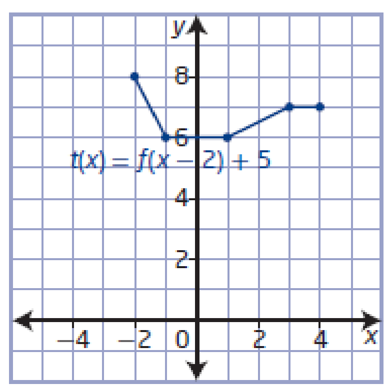
a) For $r(x) = f(x + 4) - 3$, $h = -4$ and $k = -3$. The graph of $r(x) = f(x + 4) - 3$ can be obtained from the graph of $y = f(x)$ by a vertical translation of 3 units down and a horizontal translation of 4 units to the left.
$(x, y) \rightarrow (x - 4, y - 3)$



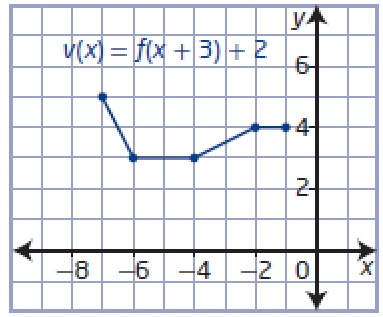
b) For $s(x) = f(x - 2) - 4$, $h = 2$ and $k = -4$. The graph of $s(x) = f(x - 2) - 4$ can be obtained from the graph of $y = f(x)$ by a vertical translation of 4 units down and a horizontal translation of 2 units to the right.
$(x, y) \rightarrow (x + 2, y - 4)$



c) For $t(x) = f(x - 2) + 5$, $h = 2$ and $k = 5$. The graph of $t(x) = f(x - 2) + 5$ can be obtained from the graph of $y = f(x)$ by a vertical translation of 5 units up and a horizontal translation of 2 units to the right.
$(x, y) \rightarrow (x + 2, y + 5)$



d) For $v(x) = f(x + 3) + 2$, $h = –3$ and $k = 2$. The graph of $v(x) = f(x + 3) + 2$ can be obtained from the graph of $y = f(x)$ by a vertical translation of 2 units up and a horizontal translation of 3 units to the left.
$(x, y) \rightarrow (x – 3, y + 2)$



Section 1.1 Page 13 Question 5

a) Translated 5 units to the left and 4 units up represents $h = –5$ and $k = 4$. The equation of the transformed function is $y – 4 = f(x + 5)$.

b) Translated 8 units to the right and 6 units up represents $h = 8$ and $k = 6$. The equation of the transformed function is $y – 6 = f(x – 8)$.

c) Translated 10 units to the right and 8 units down represents $h = 10$ and $k = –8$. The equation of the transformed function is $y + 8 = f(x – 10)$.

d) Translated 7 units to the left and 12 units down represents $h = –7$ and $k = –12$. The equation of the transformed function is $y + 12 = f(x + 7)$.

Section 1.1 Page 13 Question 6

The graph of $y = x^2$ passes through (4, 16). If the transformed graph passes through (4, 19), then a vertical translation of 3 units up has been applied.

Section 1.1 Page 13 Question 7

The graph of $y = x^2$ passes through (4, 16). If the transformed graph passes through (5, 16), then a horizontal translation of 1 unit to the right has been applied.

Section 1.1 Page 13 Question 8

Translation	Transformed Function	Transformation of Points
vertical	$y = f(x) + 5$	$(x, y) \rightarrow (x, y + 5)$
horizontal	$y = f(x + 7)$	$(x, y) \rightarrow (x - 7, y)$
horizontal	$y = f(x - 3)$	$(x, y) \rightarrow (x + 3, y)$
vertical	$y = f(x) - 6$	$(x, y) \rightarrow (x, y – 6)$
horizontal and vertical	$y + 9 = f(x + 4)$	$(x, y) \rightarrow (x – 4, y – 9)$
horizontal and vertical	$y + 6 = f(x – 4)$	$(x, y) \rightarrow (x + 4, y - 6)$
horizontal and vertical	$y – 3 = f(x + 2)$	$(x, y) \rightarrow (x - 2, y + 3)$
horizontal and vertical	$y = f(x - h) + k$	$(x, y) \rightarrow (x + h, y + k)$

Section 1.1 Page 13 Question 9

a) Translated 4 units to the left and 5 units up represents $h = –4$ and $k = 5$. The equation of the transformed function is $y = (x + 4)^2 + 5$.

b) For $y = (x + 4)^2 + 5$, the domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \geq 5, y \in R\}$.

c) To determine the image function's domain and range, add the horizontal and vertical translations to the domain and range, respectively, of the base function. Since the domain is the set of real numbers, nothing changes, but the range does change.

Section 1.1 Page 13 Question 10

Given $f(x) = |x|$ and $g(x) = f(x – 9) + 5$.
a) The equation of the transformed function is $g(x) = |x – 9| + 5$.

b) The graph of $g(x)$ is a vertical and horizontal translation of the graph of $f(x)$ by 5 units up and 9 units to the right.

c) Example:

Graph of $f(x)$	**Graph of $g(x)$**	
	Horizontal translation	Vertical translation
(–2, 2)	(7, 2)	(7, 7)
(0, 0)	(9, 0)	(9, 5)
(2, 2)	(11, 2)	(11, 7)

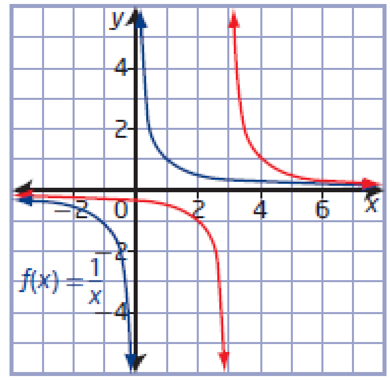
d) Example:

Graph of $f(x)$	**Graph of $g(x)$**	
	Vertical translation	Horizontal translation
(–2, 2)	(–2, 7)	(7, 7)
(0, 0)	(0, 5)	(9, 5)
(2, 2)	(2, 7)	(11, 7)

e) The coordinates of the image points from parts c) and d) are the same. The order in which the translations occur does not matter.

Section 1.1 Page 14 Question 11

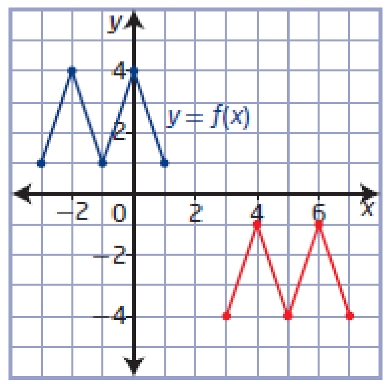
a) Choose key points on the graph of $f(x) = \dfrac{1}{x}$ and locate the corresponding image points on the graph of the translated function in red ($g(x)$).



***f*(*x*)**	***g*(*x*)**
(–2, –0.5)	→ (1, –0.5)
(–1, –1)	→ (2, –1)
(1, 1)	→ (4, 1)
(2, 0.5)	→ (5, 0.5)
(x, y)	→ (x + 3, y)

The equation of the translated function is $y = f(x - 3)$.

b) Choose key points on the graph of $y = f(x)$ and locate the corresponding image points on the graph of the translated function in red ($g(x)$).

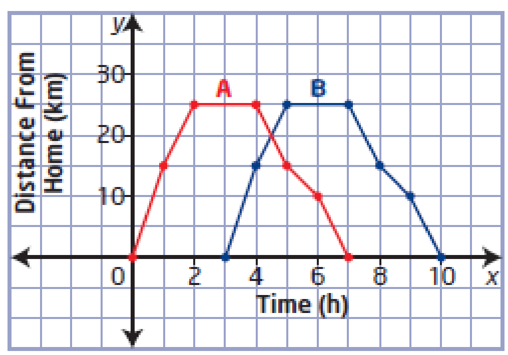


***f*(*x*)**	***g*(*x*)**
(–3, 1)	→ (3, –4)
(–2, 4)	→ (4, –1)
(–1, 1)	→ (5, –4)
(0, 4)	→ (6, –1)
(1, 1)	→ (7, –4)
(x, y)	→ (x + 6, y – 5)

The equation of the translated function is $y + 5 = f(x - 6)$.

Section 1.1 Page 14 Question 12

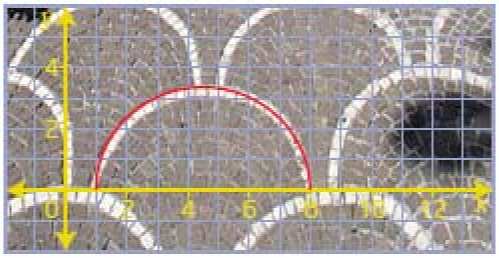
a) Example: Graph A shows that it took Janine 2 h to cycle to the lake, 25 km away. She rests at the lake for 2 h and then returns home in 3 h. If she left her home at 12 non, she returned at 7 p.m.



b) Example: Graph B shows the same trip as Graph A, but Janine does not leave her home until 3 p.m. and returns at 10 p.m.

c) The equation of graph B is $y = f(x - 3)$.

Section 1.1 Page 14 Question 13



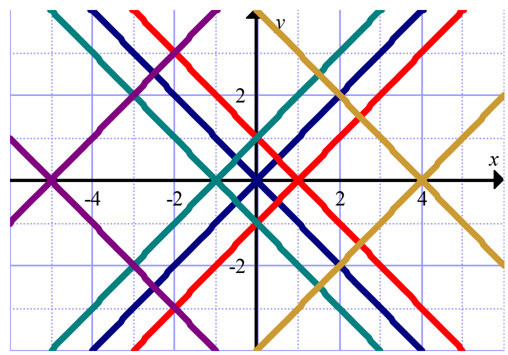
a) Example: The semicircle directly to the right is a translation of 8 units to the right of the base semicircle.

b) Example: The equation of the semicircle directly to the right is $y = f(x - 8)$. The equation of the semicircle to the right and up is $y = f(x - 4) + 3.5$. The equation of the semicircle to the left and up is $y = f(x + 4) + 3.5$.

Section 1.1 Page 14 Question 14

a) Example: A repeating X by using two linear functions $y = x$ and $y = -x$.

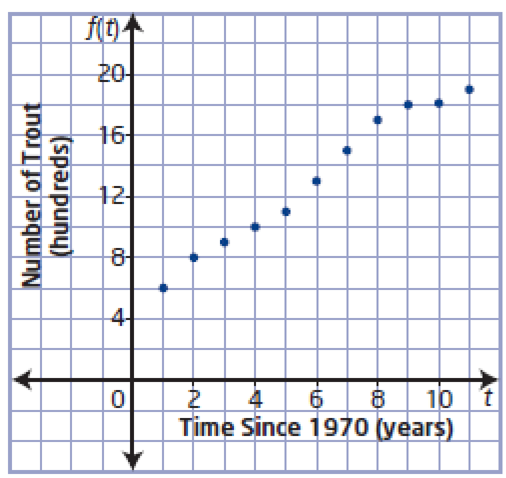
b) The original design is shown in blue.
Red: $y = f(x - 1)$
Green: $y = f(x + 1)$
Purple: $y = f(x + 5)$
Gold: $y = f(x - 4)$



Section 1.1 Page 15 Question 15

a) A vertical translation of 2 units up of the graph would result from having 200 more trout in 1970. $y = f(t) + 2$

b) A horizontal translation of 3 units to the right of the graph would result if recording the number of trout started in 1973. $y = f(t - 3)$



Section 1.1 Page 15 Question 16

If $n = f(A)$ gives the number of gallons, n, of paint needed to cover an area, A, in square metres, then $n = f(A) + 10$ represents the number of gallons Paul needs for a given area plus 10 more gallons. In this same context, $n = f(A + 10)$ represents how many gallons he needs to cover an area A less 10 units of area.

Section 1.1 Page 15 Question 17

a) The translated parabola will have an equation of the form $y = (x - h)^2 + k$. For an image parabola with zeros 7 and 1, the equation of the axis of symmetry is $x = 4$.
So, $h = 4$ and $y = (x - 4)^2 + k$. Use the coordinates of a given point to solve for k.
$y = (x - 4)^2 + k$
$0 = (1 - 4)^2 + k$
$0 = 9 + k$
$k = -9$
The equation of the image function is $y = (x - 4)^2 - 9$.

b) The graph of $y = x^2$ has been translated 4 units to the right and 9 units down.

c) Substitute $x = 0$.
$y = (x - 4)^2 - 9$
$y = (0 - 4)^2 - 9$
$y = 16 - 9$
$y = 7$
The y-intercept of the translated function is 7.

Section 1.1 Page 15 Question 18

a) The graph of $y - 4 = \frac{1}{x}$ is a vertical translation of 4 units up of the graph of $y = \frac{1}{x}$.

b) The graph of $y = \frac{1}{x+2}$ is a horizontal translation of 2 units to the left of the graph of $y = \frac{1}{x}$.

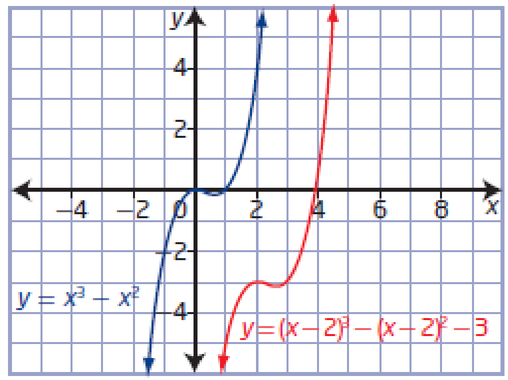
c) The graph of $y - 3 = \frac{1}{x-5}$ is a vertical translation of 3 units up and horizontal translation of 5 units to the right of the graph of $y = \frac{1}{x}$.

d) The graph of $y = \frac{1}{x+3} - 4$ is a vertical translation of 4 units down and horizontal translation of 3 units to the left of the graph of $y = \frac{1}{x}$.

Section 1.1 Page 15 Question 19

a) Prediction: The graph of $y + 3 = (x - 2)^3 - (x - 2)^2$ will be a vertical translation of 3 units down and horizontal translation of 2 units to the right of the graph of $y = x^3 - x^2$.

b) The prediction was correct.



Section 1.1 Page 15 Question C1

a) If the horizontal translation is performed before the vertical translation, the function $y = f(x)$ becomes $y = f(x - h)$ and then $y = f(x - h) + k$.
If the vertical translation is performed before the horizontal translation, the function $y = f(x)$ becomes $y = f(x) + k$ and then $y = f(x - h) + k$.
The final function equations are the same. So, translations can be applied in either order.

b) Potentially, the domain is shifted by h and the range is shifted by k.

Section 1.1 Page 15 Question C2

a) Completing the square reveals the location of the vertex and the translations of the graph of $y = x^2$.
$f(x) = x^2 + 2x + 1$
$f(x) = (x^2 + 2x + 1 - 1) + 1$
$f(x) = (x + 1)^2$
The graph of $y = x^2$ is translated 1 unit to the left to obtain the graph of $f(x) = x^2 + 2x + 1$.

b) Complete the square.
$g(x) = x^2 - 4x + 3$
$g(x) = (x^2 - 4x + 4 - 4) + 3$
$g(x) = (x - 2)^2 - 1$
The graph of $y = x^2$ is translated 2 units to the right and 1 unit down to obtain the graph of $g(x) = x^2 - 4x + 3$.

Section 1.1 Page 15 Question C3

Comparing the graphs of the corresponding functions for the equations $x^2 - x - 12 = 0$ and $(x - 5)^2 - (x - 5) - 12 = 0$ shows that the second is a horizontal translation of the first by 5 units to the right. Since the roots of the first equation are –3 and 4, the roots of the second equation will be –3 + 5 and 4 + 5, or 2 and 9.

Section 1.1 Page 15 Question C4

If the base function $f(x) = x$ is translated vertically, the function becomes $y = x + k$. So for 4 units up, the equation is $y = x + 4$.
If the base function $f(x) = x$ is translated horizontally, the function becomes $y = (x - h)$. So for 4 units to the left, the equation is $y = (x + 4)$ or $y = x + 4$.

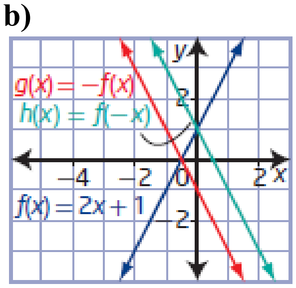
Section 1.2 Reflections and Stretches

Section 1.2 Page 28 Question 1

a)

x	$f(x) = 2x + 1$	$g(x) = -f(x)$	$h(x) = f(-x)$
−4	−7	7	9
−2	−3	3	5
0	1	−1	1
2	5	−5	−3
4	9	−9	−7

b)



c) The y-coordinates of the points on the graph of $g(x)$ have changed sign compared to the corresponding points on the graph of $f(x)$. The invariant point is (–0.5, 0).
The x-coordinates of the points on the graph of $h(x)$ have changed sign compared to the corresponding points on the graph of $f(x)$. The invariant point is (0, 1).

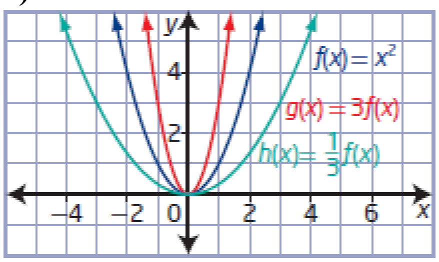
d) The graph of $g(x)$ is a reflection in the x-axis of the graph of $f(x)$. The graph of $h(x)$ is a reflection in the y-axis of the graph of $f(x)$.

Section 1.2 Page 28 Question 2

a)

x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
–6	36	108	12
–3	9	27	3
0	0	0	0
3	9	27	3
6	36	108	12

b)

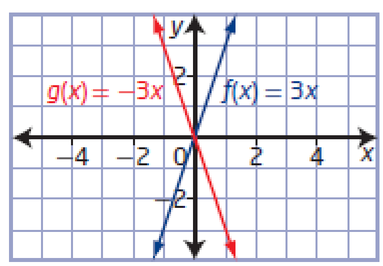


c) The y-coordinates of the points on the graph of $g(x)$ are three times far from the x-axis as the corresponding points on the graph of $f(x)$. The invariant point is (0, 0). The y-coordinates on the graph of $h(x)$ are one-third as far from the x-axis as the corresponding points on the graph of $f(x)$. The invariant point is (0, 0).

d) The graph of $g(x)$ is a vertical stretch by a factor of 3 of the graph of $f(x)$, while the graph of $h(x)$ is a vertical stretch by a factor of $\frac{1}{3}$ of the graph of $f(x)$.

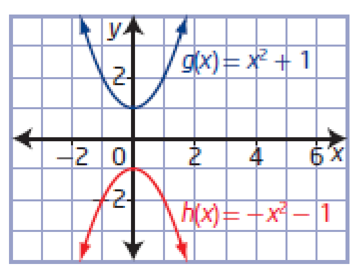
Section 1.2 Page 28 Question 3

a) A reflection in the x-axis of $f(x) = 3x$ results in the function $g(x) = –3x$.



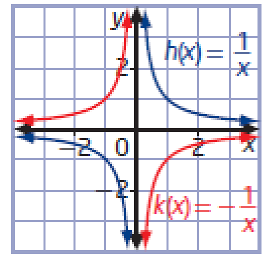
$f(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$
$g(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

b) A reflection in the x-axis of $g(x) = x^2 + 1$ results in the function $h(x) = –x^2 – 1$.



$g(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \geq 1, y \in R\}$
$h(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \leq -1, y \in R\}$

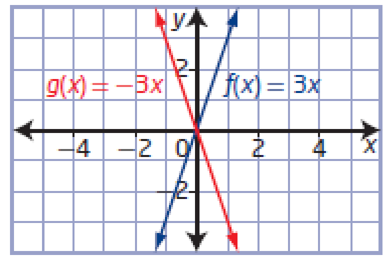
c) A reflection in the x-axis of $h(x) = \dfrac{1}{x}$ results in the function $k(x) = -\dfrac{1}{x}$.



$h(x)$: domain $\{x \mid x \neq 0, x \in \mathrm{R}\}$, range $\{y \mid y \neq 0, y \in \mathrm{R}\}$
$k(x)$: domain $\{x \mid x \neq 0, x \in \mathrm{R}\}$, range $\{y \mid y \neq 0, y \in \mathrm{R}\}$

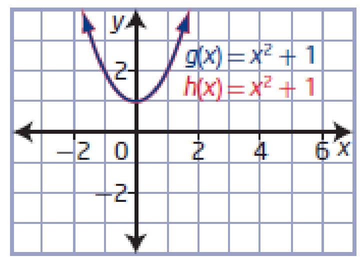
Section 1.2 Page 28 Question 4

a) A reflection in the y-axis of $f(x) = 3x$ results in the function $g(x) = -3x$.



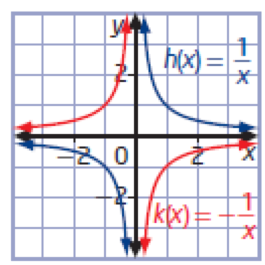
$f(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$
$g(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

b) A reflection in the y-axis of $g(x) = x^2 + 1$ results in the function $h(x) = x^2 + 1$.



$g(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq 1, y \in \mathrm{R}\}$
$h(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq 1, y \in \mathrm{R}\}$

c) A reflection in the y-axis of $h(x) = \dfrac{1}{x}$ results in the function $k(x) = -\dfrac{1}{x}$.



$h(x)$: domain $\{x \mid x \neq 0, x \in \mathrm{R}\}$, range $\{y \mid y \neq 0, y \in \mathrm{R}\}$
$k(x)$: domain $\{x \mid x \neq 0, x \in \mathrm{R}\}$, range $\{y \mid y \neq 0, y \in \mathrm{R}\}$

Section 1.2 Page 29 Question 5

a) Since $a = 4$, the graph of $y = 4f(x)$ is a vertical stretch of the graph of $y = f(x)$ about the x-axis by a factor of 4. The points on the graph of $y = 4f(x)$ relate to the points on the graph of $y = f(x)$ by the mapping $(x, y) \to (x, 4y)$.

b) Since $b = 3$, the graph of $y = f(3x)$ is a horizontal stretch of the graph of $y = f(x)$ about the y-axis by a factor of $\dfrac{1}{3}$. The points on the graph of $y = f(3x)$ relate to the points on the graph of $y = f(x)$ by the mapping $(x, y) \to \left(\dfrac{1}{3}x, y\right)$.

c) Since $a = -1$, the graph of $y = -f(x)$ is a reflection of the graph of $y = f(x)$ in the x-axis. The points on the graph of $y = -f(x)$ relate to the points on the graph of $y = f(x)$ by the mapping $(x, y) \rightarrow (x, -y)$.

d) Since $b = -1$, the graph of $y = f(-x)$ is a reflection of the graph of $y = f(x)$ in the y-axis. The points on the graph of $y = f(-x)$ relate to the points on the graph of $y = f(x)$ by the mapping $(x, y) \rightarrow (-x, y)$.

Section 1.2 Page 29 Question 6

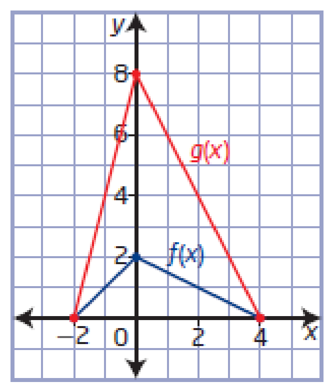
a) For the given graph of $y = f(x)$, the domain is $\{x \mid -6 \le x \le 6, x \in \mathrm{R}\}$ and the range is $\{y \mid -4 \le y \le 4, y \in \mathrm{R}\}$. If the graph of $y = f(x)$ is vertically stretched about the x-axis by a factor of 2, the domain remains the same but the range becomes $\{y \mid -8 \le y \le 8, y \in \mathrm{R}\}$.

b) Since all points on a graph that is vertically stretched relate to the points on the original graph by the mapping $(x, y) \rightarrow (x, ay)$, the domain remains the same but the range changes by a factor of a.

Section 1.2 Page 29 Question 7

a) Compare key points on the graphs of $y = f(x)$ and $y = g(x)$.

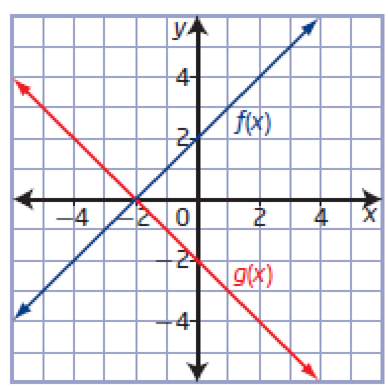
x	$y = f(x)$	$y = g(x)$
–2	0	0
–1	1	4
0	2	8
2	1	4
4	0	0



The transformation can be described by the mapping $(x, y) \rightarrow (x, 4y)$. This indicates a vertical stretch about the x-axis by a factor of 4. The equation of the transformed function is $g(x) = 4f(x)$.

b) Compare key points on the graphs of $y = f(x)$ and $y = g(x)$.

x	$y = f(x)$	$y = g(x)$
–4	–2	2
–2	0	0
0	2	–2
2	4	–4

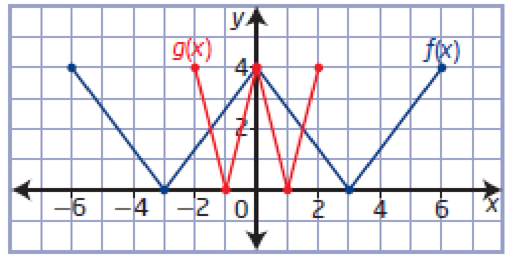


The transformation can be described by the mapping $(x, y) \rightarrow (x, -y)$. This indicates a reflection in the x-axis. The equation of the transformed function is $g(x) = -f(x)$.

c) Compare key points on the graphs of $y = f(x)$ and $y = g(x)$.

x	$y = f(x)$
–6	4
–3	0
0	4
3	0
6	4

x	$y = g(x)$
–2	0
1	4
0	8
1	4
2	0

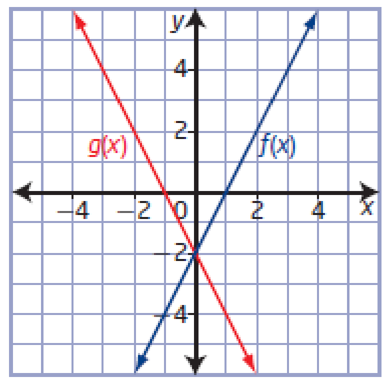


The transformation can be described by the mapping $(x, y) \rightarrow \left(\frac{1}{3}x, y\right)$. This indicates a horizontal stretch about the y-axis by a factor of $\frac{1}{3}$. The equation of the transformed function is $g(x) = f(3x)$.

d) Compare key points on the graphs of $y = f(x)$ and $y = g(x)$.

x	$y = f(x)$
–2	–6
–1	–4
0	–2
2	2

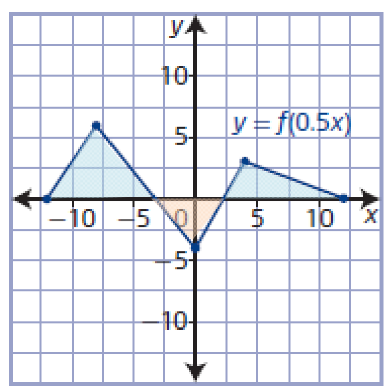
x	$y = g(x)$
2	–6
1	–4
0	–2
–2	–6



The transformation can be described by the mapping $(x, y) \rightarrow (-x, y)$. This indicates a reflection in the y-axis. The equation of the transformed function is $g(x) = f(-x)$.

Section 1.2 Page 29 Question 8

The transformation $y = f(0.5x)$ can be described by the mapping $(x, y) \rightarrow (2x, y)$.



Section 1.2 Page 30 Question 9

a) When x is replaced by $4x$, the equation becomes $y = f(4x)$ and the graph of $y = f(x)$ is stretched horizontally about the y-axis by a factor of $\frac{1}{4}$.

b) When x is replaced by $\frac{1}{4}x$, the equation becomes $y = f\left(\frac{1}{4}x\right)$ and the graph of $y = f(x)$ is stretched horizontally about the y-axis by a factor of 4.

c) When y is replaced by $2y$, the equation becomes $y = \frac{1}{2}f(x)$ and the graph of $y = f(x)$ is stretched vertically about the x-axis by a factor of $\frac{1}{2}$.

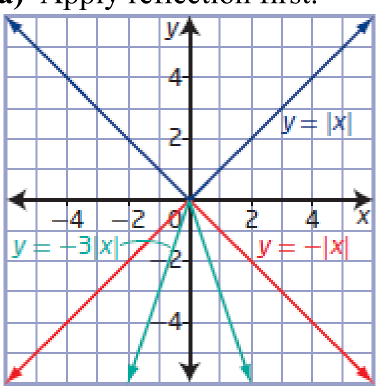
d) When y is replaced by $\frac{1}{4}y$, the equation becomes $y = 4f(x)$ and the graph of $y = f(x)$ is stretched vertically about the x-axis by a factor of 4.

e) When x is replaced by $–3x$, the equation becomes $y = f(–3x)$ and the graph of $y = f(x)$ is stretched horizontally about the y-axis by a factor of $\frac{1}{3}$ and reflected in the y-axis.

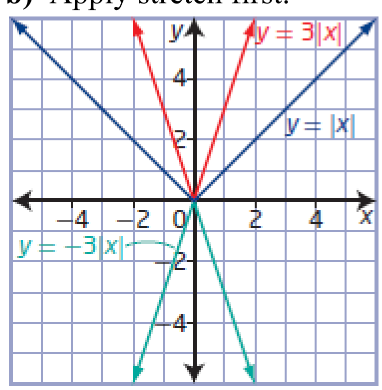
f) When y is replaced by $-\frac{1}{3}y$, the equation becomes $y = –3f(x)$ and the graph of $y = f(x)$ is stretched vertically about the x-axis by a factor of 3 and reflected in the x-axis.

Section 1.2 Page 30 Question 10

a) Apply reflection first.



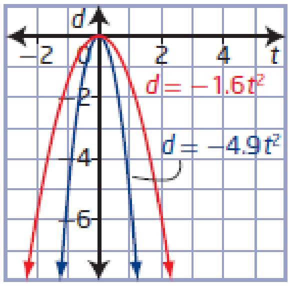
b) Apply stretch first.



c) Both Thomas and Sharyn are correct. Reflections and stretches can be applied in any order.

Section 1.2 Page 30 Question 11

a)



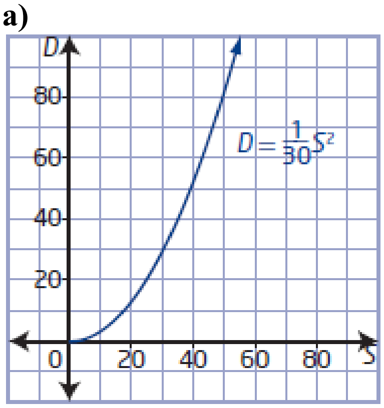
b) Both functions are reflections in the t-axis and vertical stretches of the base function $d = t^2$. The object falling on Earth ($d = -4.9t^2$) is stretched vertically more than the object falling on the moon.

Section 1.2 Page 30 Question 12

Example: When the graph of $y = f(x)$ is transformed to the graph of $y = f(bx)$, it undergoes a horizontal stretch about the y-axis by a factor of $\frac{1}{|b|}$ and only the x-coordinates are affected. When the graph of $y = f(x)$ is transformed to the graph of $y = af(x)$, it undergoes a vertical stretch about the x-axis by a factor of $|a|$ and only the y-coordinates are affected.

Section 1.2 Page 30 Question 13

a)

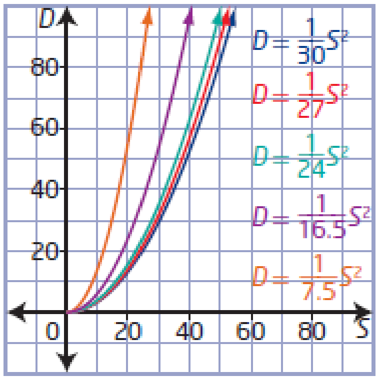


b) For asphalt, substitute $f = 0.9$ and $n = 1$ into $D = \frac{1}{30fn}S^2$: $D = \frac{1}{27}S^2$.

For gravel, substitute $f = 0.8$ and $n = 1$ into $D = \frac{1}{30fn}S^2$: $D = \frac{1}{24}S^2$.

For snow, substitute $f = 0.55$ and $n = 1$ into $D = \frac{1}{30fn}S^2$: $D = \frac{1}{16.5}S^2$.

For ice, substitute $f = 0.25$ and $n = 1$ into $D = \frac{1}{30fn}S^2$: $D = \frac{1}{7.5}S^2$.



As the drag factor decreases, the length of the skid mark increases for the same speed.

Section 1.2 Page 31 Question 14

The zeros of the function $f(x) = (x + 4)(x - 3)$ are -4 and 3.
a) Since the points on the graph $y = 4f(x)$ relate to the graph of $f(x)$ by $(x, y) \rightarrow (x, 4y)$, the zeros do not change.

b) Since the points on the graph $y = f(-x)$ relate to the graph of $f(x)$ by $(x, y) \rightarrow (-x, y)$, the zeros will occur at $x = 4$ and $x = -3$.

c) Since the points on the graph $y = f\left(\frac{1}{2}x\right)$ relate to the graph of $f(x)$ by $(x, y) \rightarrow (2x, y)$, the zeros will occur at $x = -8$ and $x = 6$.

d) Since the points on the graph $y = f(2x)$ relate to the graph of $f(x)$ by $(x, y) \rightarrow \left(\frac{1}{2}x, y\right)$, the zeros will occur at $x = -2$ and $x = 1.5$.

Section 1.2 Page 31 Question 15

The graph of a function $y = f(x)$ is contained completely in quadrant IV.
a) A transformation to $y = -f(x)$ represents a reflection in the x-axis: $(x, y) \rightarrow (x, -y)$.
If $y = f(x)$ is transformed to $y = -f(x)$, it will be in quadrant **I**.

b) A transformation to $y = f(-x)$ represents a reflection in the y-axis: $(x, y) \rightarrow (-x, y)$.
If $y = f(x)$ is transformed to $y = f(-x)$, it will be in quadrant **III**.

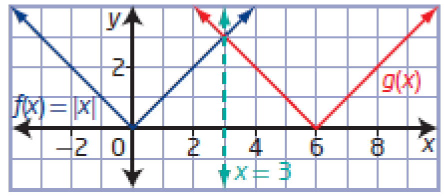
c) A transformation to $y = 4f(x)$ represents a vertical stretch about the x-axis: $(x, y) \rightarrow (x, 4y)$.
If $y = f(x)$ is transformed to $y = 4f(x)$, it will be in quadrant **IV**.

d) A transformation to $y = f\left(\frac{1}{4}x\right)$ represents a horizontal stretch about the y-axis:

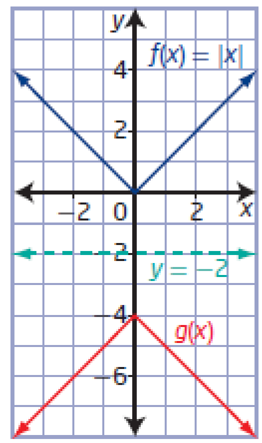
$(x, y) \rightarrow (4x, y)$.
If $y = f(x)$ is transformed to $y = 4f(x)$, it will be in quadrant **IV**.

Section 1.2 Page 31 Question 16

a) To reflect the graph of $f(x) = |x|$ in the line $x = 3$, each image point is the same distance from the line $x = 3$ as its corresponding key point.



b) To reflect the graph of $f(x) = |x|$ in the line $y = -2$, each image point is the same distance from the line $y = -2$ as its corresponding key point.



Section 1.2 Page 31 Question C1

Example: When the input values for $g(x)$ are b times the input values for $f(x)$, the scale factor must be $\frac{1}{b}$ (for $b > 0$) to maintain the same output values.

$g(x) = f\left(\frac{1}{b}(bx)\right) = f(x)$

Section 1.2 Page 31 Question C2

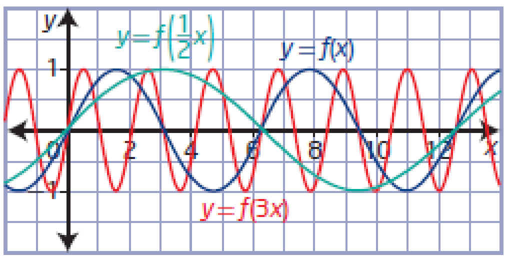
a) Transformations that result in the x-intercepts being invariant points are vertical stretches or a reflection in the x-axis.

b) Transformations that result in the y-intercepts being invariant points are horiontal stretches or a reflection in the y-axis.

Section 1.2 Page 31 Question C3

$f(x)$	$g(x)$	Transformation
(5, 6)	(5, –6)	reflection in the x-axis
(4, 8)	(–4, 8)	reflection in the y-axis
(2, 3)	(2, 12)	vertical stretch by a factor of 4
(4, –12)	(2, –6)	horizontal stretch by a factor of $\frac{1}{2}$ and vertical stretch by a factor of $\frac{1}{2}$

Section 1.2 Page 31 Question C4



Section 1.2 Page 31 Question C5

a) For the sequence –10, –6, –2, 2, 6, … $a = –10$ and $d = 4$. So, the general term is
$t_n = a + (n – 1)d$
$t_n = –10 + (n – 1)(4)$
$t_n = –14 + 4n$

b) For the sequence 10, 6, 2, –2, –6, … $a = 10$ and $d = –4$. So, the general term is
$t_n = a + (n – 1)d$
$t_n = 10 + (n – 1)(–4)$
$t_n = 14 – 4n$

c) Since the general term for part a) equals –1 times the general term in part b), the graphs will reflections of each other in the x-axis.

Section 1.3 Combining Transformations

Section 1.3 Page 38 Question 1

a) The graph of $y = x^2$ is stretched horizontally about the y-axis by a factor of 2 and then reflected in the x-axis: $b = 0.5$ and $a = –1$. Then, the transformed equation is $y = –f(0.5x)$ or

$$y = –(0.5x)^2$$
$$= –0.25x^2$$

b) The graph of $y = x^2$ is stretched horizontally about the y-axis by a factor of $\frac{1}{4}$, reflected in the y-axis, and then stretched vertically about the x-axis by a factor of $\frac{1}{4}$:
$b = -4$ and $a = \frac{1}{4}$. Then, the transformed equation is $y = \frac{1}{4}f(-4x)$. Then, $y = \frac{1}{4}(-4x)^2$, or $y = 4x^2$.

Section 1.3 Page 38 Question 2

a) Write $g(x) = -3f(4x - 16) - 10$ in the form $g(x) = af(b(x - h) + k$:
$g(x) = -3f(4(x - 4) - 10$
Then, $a = -3$, $b = 4$, $h = 4$, and $k = -10$.
The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the ***y*-axis** by a factor of $\mathbf{\frac{1}{4}}$. It is vertically stretched about the ***x*-axis** by a factor of **3**. It is reflected in the ***x*-axis**, and then translated **4** units to the right and **10** units down.

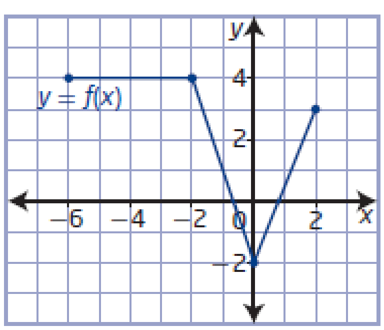
Section 1.3 Page 39 Question 3

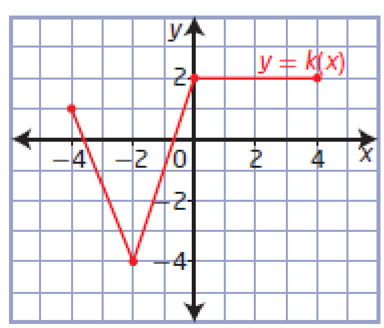
For $y - 4 = f(x - 5)$, $a = 1$, $b = 1$, $h = 5$, and $k = 4$.
For $y + 5 = 2f(3x)$, $a = 2$, $b = 3$, $h = 0$, and $k = -5$.
For $y = \frac{1}{2}f\left(\frac{1}{2}(x-4)\right)$, $a = \frac{1}{2}$, $b = \frac{1}{2}$, $h = 4$, and $k = 0$.
For $y + 2 = -3f(2(x + 2))$, $a = -3$, $b = 2$, $h = -2$, $k = -2$.

Function	Reflections	Vertical Stretch Factor	Horizontal Stretch Factor	Vertical Translation	Horizontal Translation
$y - 4 = f(x - 5)$	none	none	none	4	5
$y + 5 = 2f(3x)$	none	2	$\frac{1}{3}$	−5	none
$y = \frac{1}{2}f\left(\frac{1}{2}(x-4)\right)$	none	$\frac{1}{2}$	2	none	4
$y + 2 = -3f(2(x + 2))$	x-axis	3	$\frac{1}{2}$	−2	−2

Section 1.3 Page 39 Question 4

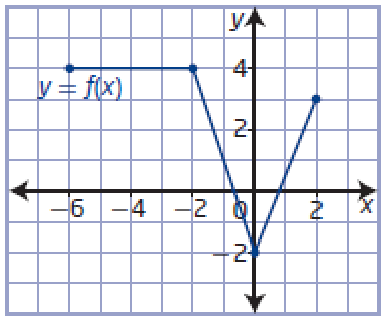
a) Locate key points on the graph of $f(x)$ and their image points on the graph of $k(x)$.
$(-6, 4) \to (4, 2)$
$(-2, 4) \to (0, 2)$
$(0, -2) \to (-2, -4)$
$(2, 3) \to (-4, 1)$

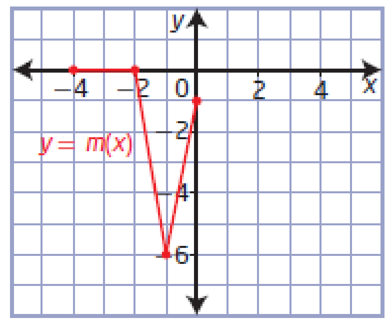




Since the x-coordinates have all changed sign, the graph has been reflected in the y-axis. The overall width and height have not changed, so the graph has not been vertically or horizontally stretched. Since the y-intercept has changed, the graph has been translated 2 units to the left and 2 units down. So, $a = 1$, $b = –1$, $h = –2$, $k = –2$, and the equation of the transformed graph is $y = f(–(x + 2)) – 2$.

b) Locate key points on the graph of $f(x)$ and their image points on the graph of $m(x)$.
$(–6, 4) \rightarrow (–4, 0)$
$(–2, 4) \rightarrow (–2, 0)$
$(0, –2) \rightarrow (–1, –6)$
$(2, 3) \rightarrow (0, –1)$





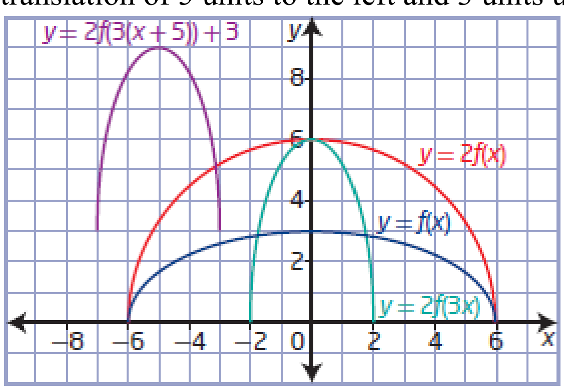
Since the orientation is unchanged, the graph has not been reflected in either axis. The overall width has changed, so the graph has been horizontally stretched by a factor of 0.5. Since the y-intercept has changed, the graph has been translated 1 unit to the left and 4 units down. So, $a = 1$, $b = 2$, $h = –1$, $k = –4$, and the equation of the transformed graph is $y = f(2(x + 1)) – 4$.

Section 1.3 Page 39 Question 5

a) vertical stretch about the x-axis by a factor of 2: $a = 2$, $y = 2f(x)$

horizontal stretch about the y-axis by a factor of $\frac{1}{3}$: $b = 3$, $y = 2f(3x)$

translation of 5 units to the left and 3 units up: $h = –5$ and $k = 3$, $y = 2f(3(x + 5)) + 3$

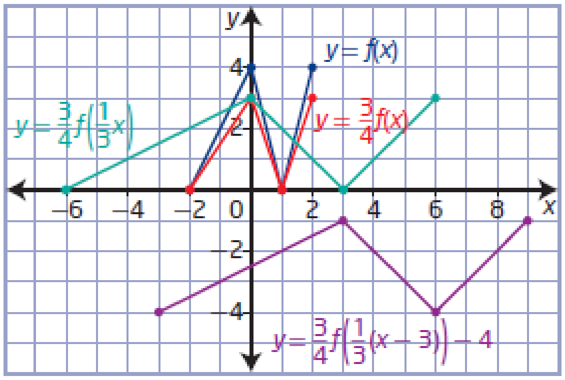


b) vertical stretch about the x-axis by a factor of $\frac{3}{4}$: $a = \frac{3}{4}$, $y = \frac{3}{4}f(x)$

horizontal stretch about the y-axis by a factor of 3: $b = \frac{1}{3}$, $y = \frac{3}{4}f\left(\frac{1}{3}x\right)$

translation of 3 units to the right and 4 units down: $h = 3$ and $k = –4$,

$y = \frac{3}{4}f\left(\frac{1}{3}(x-3)\right)-4$



Section 1.3 Page 39 Question 6

a) The graph of $y + 6 = f(x - 4)$ is related to the graph of $y = f(x)$ by the mapping $(x, y) \to (x + 4, y - 6)$. So, the key point $(-12, 18)$ becomes the image point $(-8, 12)$.

b) The graph of $y = 4f(3x)$ is related to the graph of $y = f(x)$ by the mapping $(x, y) \to \left(\frac{1}{3}x, 4y\right)$. So, the key point $(-12, 18)$ becomes the image point $(-4, 72)$.

c) The graph of $y = -2f(x - 6) + 4$ is related to the graph of $y = f(x)$ by the mapping $(x, y) \to (x + 6, -2y + 4)$. So, the key point $(-12, 18)$ becomes the image point $(-6, -32)$.

d) The graph of $y = -2f\left(-\frac{2}{3}x - 6\right) + 4$ or $y = -2f\left(-\frac{2}{3}(x + 9)\right) + 4$ is related to the graph of $y = f(x)$ by the mapping $(x, y) \to \left(-\frac{3}{2}x - 9, -2y + 4\right)$. So, the key point $(-12, 18)$ becomes the image point $(9, -32)$.

e) The graph of $y + 3 = -\frac{1}{3}f\left(2(x + 6)\right)$ is related to the graph of $y = f(x)$ by the mapping $(x, y) \to \left(\frac{1}{2}x - 6, -\frac{1}{3}y - 3\right)$. So, the key point $(-12, 18)$ becomes the image point $(-12, -9)$.

Section 1.3 Page 40 Question 7

a) For $y = 2f(x - 3) + 4$, $a = 2$, $b = 1$, $h = 3$, and $k = 4$. The graph of $y = f(x)$ is vertically stretched about the x-axis by a factor of 2 and then translated 3 units to the right and 4 units up.
$(x, y) \to (x + 3, 2y + 4)$

b) For $y = -f(3x) - 2$, $a = -1$, $b = 3$, $h = 0$, and $k = -2$. The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{3}$, reflected in the x-axis, and then translated 2 units down.

$(x, y) \rightarrow \left(\frac{1}{3}x, -y - 2\right)$

c) For $y = -\frac{1}{4}f(-(x + 2))$, $a = -\frac{1}{4}$, $b = -1$, $h = -2$, and $k = 0$. The graph of $y = f(x)$ is reflected in the y-axis, vertically stretched about the x-axis by a factor of $\frac{1}{4}$, reflected in the x-axis, and then translated 2 units to the left.

$(x, y) \rightarrow \left(-x - 2, -\frac{1}{4}y\right)$

d) For $y - 3 = -f(4(x - 2))$, $a = -1$, $b = 4$, $h = 2$, and $k = 3$. The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{4}$, reflected in the x-axis, and then translated 2 units to the right and 3 units up.

$(x, y) \rightarrow \left(\frac{1}{4}x + 2, -y + 3\right)$

e) For $y = -\frac{2}{3}f\left(-\frac{3}{4}x\right)$, $a = -\frac{2}{3}$, $b = -\frac{3}{4}$, $h = 0$, and $k = 0$. The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{4}{3}$, reflected in the y-axis, vertically stretched about the x-axis by a factor of $\frac{2}{3}$, and then reflected in the x-axis.

$(x, y) \rightarrow \left(-\frac{4}{3}x, -\frac{2}{3}y\right)$

f) Write $3y - 6 = f(-2x + 12)$ in the form $y - k = af(b(x - h)$:

$3y - 6 = f(-2x + 12)$
$3(y - 2) = f(-2(x - 6))$
$y - 2 = \frac{1}{3}f(-2(x - 6))$

Then, $a = \frac{1}{3}$, $b = -2$, $h = 6$, and $k = 2$. The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$, reflected in the y-axis, vertically stretched about the x-axis by a factor of $\frac{1}{3}$, and then translated 6 units to the right and 2 units up.

$(x, y) \rightarrow \left(-\frac{1}{2}x + 6, \frac{1}{3}y + 2\right)$

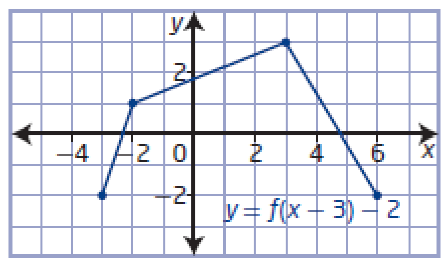
Section 1.3 Page 40 Question 8

a) For a vertical stretch about the *x*-axis by a factor of 3, a reflection in the *x*-axis, a horizontal translation of 4 units to the left, and a vertical translation of 5 units down, $a = -3$, $b = 1$, $h = -4$, and $k = -5$. So, the equation of the transformed function is $y + 5 = -3f(x + 4)$.

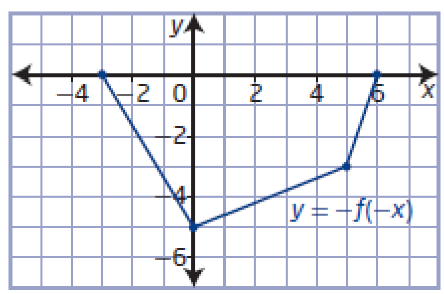
b) For a horizontal stretch about the *y*-axis by a factor of $\frac{1}{3}$, a vertical stretch about the *x*-axis by a factor of $\frac{3}{4}$, a reflection in both the *x*-axis and the *y*-axis, and a translation of 6 units to the right and 2 units up, $a = -\frac{3}{4}$, $b = -3$, $h = 6$, and $k = 2$. So, the equation of the transformed function is $y - 2 = -\frac{3}{4}f(-3(x - 6))$.

Section 1.3 Page 40 Question 9

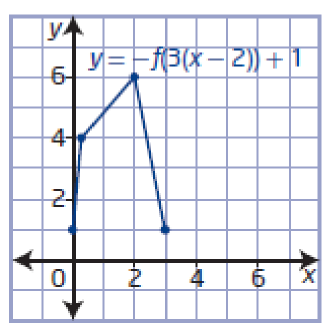
a) To obtain the graph of $y + 2 = f(x - 3)$, the graph of $y = f(x)$ is translated 3 units to the left and 2 units down.



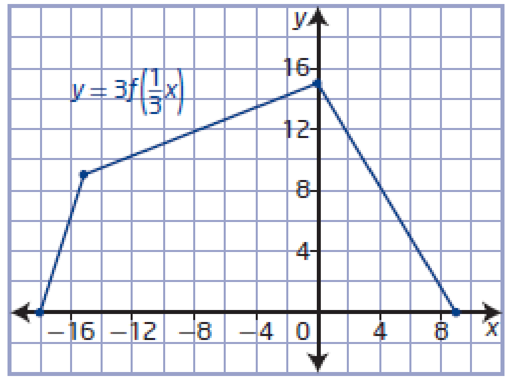
b) To obtain the graph of $y = -f(-x)$, the graph of $y = f(x)$ is reflected in the *x*-axis and the *y*-axis.



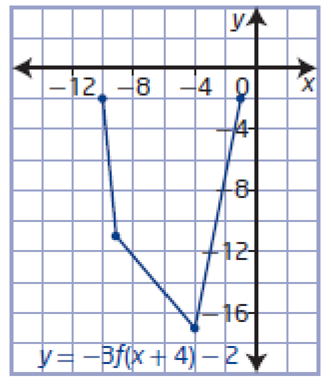
c) To obtain the graph of $y = f(3(x - 2)) + 1$, the graph of $y = f(x)$ is stretched horizontally about the *y*-axis by a factor of $\frac{1}{3}$ and translated 2 units to the right and 1 unit up.



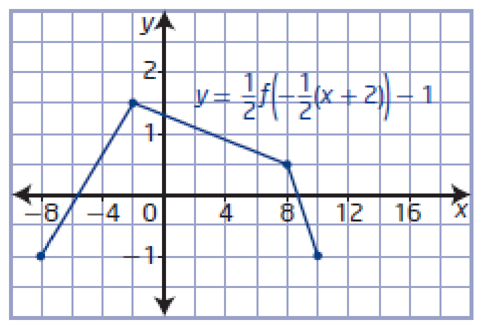
d) To obtain the graph of $y = 3f\left(\frac{1}{3}x\right)$, the graph of $y = f(x)$ is stretched horizontally about the y-axis by a factor of 3 and stretched vertically about the x-axis by a factor of 3.



e) To obtain the graph of $y + 2 = -3f(x + 4)$, the graph of $y = f(x)$ is stretched vertically about the x-axis by a factor of 3, reflected in the x-axis, and translated 4 units to the left and 2 units down.

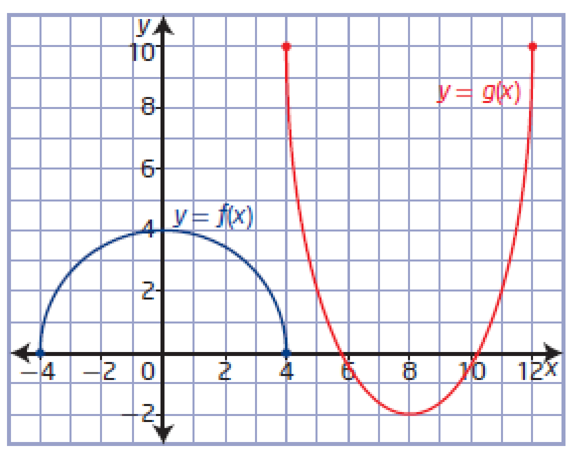


f) To obtain the graph of $y = \frac{1}{2}f\left(-\frac{1}{2}(x+2)\right)-1$, the graph of $y = f(x)$ is stretched horizontally about the y-axis by a factor of 2, reflected in the y-axis, stretched vertically about the x-axis by a factor of $\frac{1}{2}$, and translated 2 units to the left and 1 unit down.

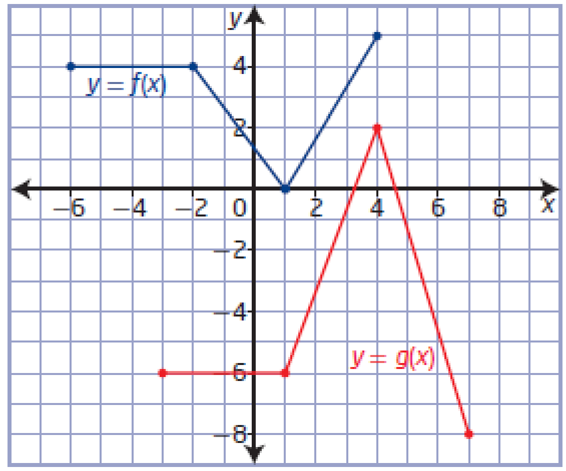


Section 1.3 Page 40 Question 10

a) Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.
(–4, 0) → (4, 10)
(0, 4) → (8, –2)
(4, 0) → (12, 10)
The orientation is changed, the graph has been reflected in the x-axis. The overall height has changed, so the graph has been vertically stretched by a factor of 3. Since the y-intercept has changed, the graph has been translated 8 units to the right and 10 units up. So, $a = -3$, $b = 1$, $h = 8$, $k = 10$, and the equation of the transformed graph is $y = -3f(x - 8) + 10$.



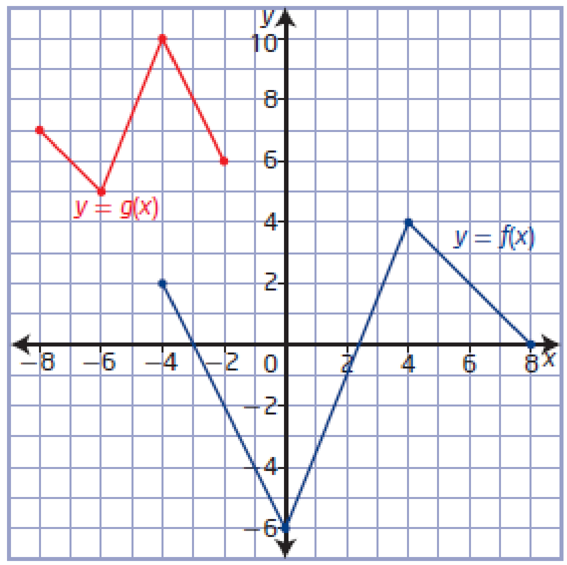
b) Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.



$(-6, 4) \rightarrow (-3, -6)$
$(-2, 4) \rightarrow (1, -6)$
$(1, 0) \rightarrow (4, 2)$
$(4, 5) \rightarrow (7, -8)$

The orientation is changed, the graph has been reflected in the x-axis. The overall height has changed, so the graph has been vertically stretched by a factor of 2. Since the x-intercept has changed, the graph has been translated 3 units to the right and 2 units up. So, $a = -2$, $b = 1$, $h = 3$, $k = 2$, and the equation of the transformed graph is $y = -2f(x - 3) + 2$.

c) Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.



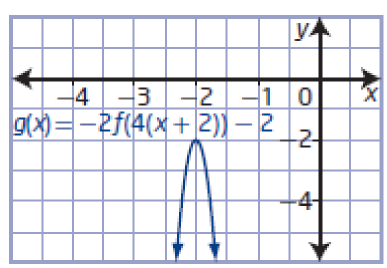
$(8, 0) \rightarrow (-8, 7)$
$(4, 4) \rightarrow (-6, 5)$
$(0, -6) \rightarrow (-4, 10)$
$(-4, 2) \rightarrow (-2, 6)$

The orientation is changed, the graph has been reflected in the x-axis and in the y-axis. The overall width and height have changed, so the graph has been vertically stretched by a factor of $\frac{1}{2}$ and horizontally stretched by a factor of $\frac{1}{2}$.

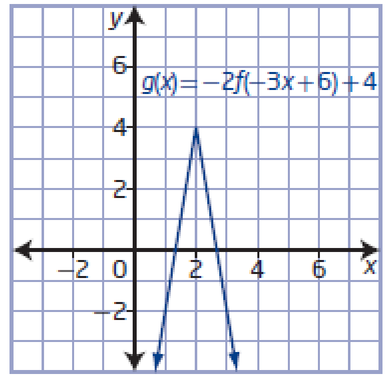
The graph has been translated 4 units to the left and 7 units up. So, $a = -\frac{1}{2}$, $b = -2$, $h = -4$, and $k = 7$, and the equation of the transformed graph is $y = -\frac{1}{2}f\left(-2(x+4)\right)+7$.

Section 1.3 Page 40 Question 11

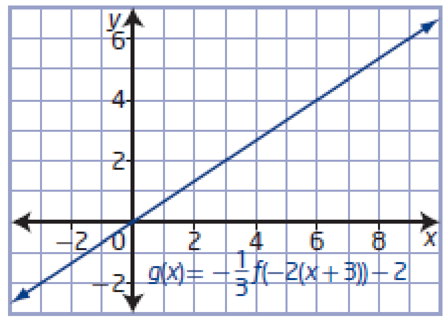
a) For $g(x) = -2f(4(x + 2)) - 2$, $a = -2$, $b = 4$, $h = -2$, and $k = -2$. The graph of $f(x) = x^2$ is stretched horizontally about the y-axis by a factor of 0.25, stretched vertically by a factor of 2, reflected in the x-axis, and translated 2 units to the left and 2 units down.



b) Write $g(x) = -2f(-3x + 6) + 4$ in the form $y = af(b(x - h) + k$: $g(x) = -2f(-3(x - 2)) + 4$
For $g(x) = -2f(-3(x - 2)) + 4$, $a = -2$, $b = -3$, $h = 2$, and $k = 4$. The graph of $f(x) = |x|$ is stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, reflected in the y-axis, stretched vertically by a factor of 2, reflected in the x-axis, and translated 2 units to the right and 4 units up.



c) For $g(x) = -\frac{1}{3}f(-2(x + 3)) - 2$, $a = -\frac{1}{3}$, $b = -2$, $h = -3$, and $k = -2$. The graph of $f(x) = x$ is stretched horizontally about the y-axis by a factor of 0.5, reflected in the y-axis, stretched vertically by a factor of $\frac{1}{3}$, reflected in the x-axis, and translated 3 units to the left and 2 units down.



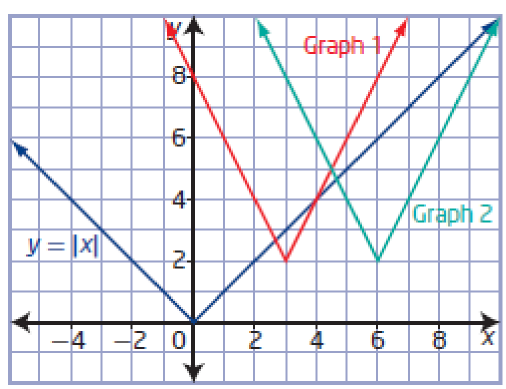
Section 1.3 Page 41 Question 12

a) To transform the original design by a horizontal stretch about the y-axis by a factor of 2, a reflection in the x-axis, and a translation of 4 units up and 3 units to the left, use the mapping $(x, y) \rightarrow (2x - 3, -y + 4)$.
A(–4, 6) → A′(–11, –2)
B(–2, –2) → B′(–7, 6)
C(0, 0) → C′(–3, 4)
D(1, –1) → D′(–1, 5)
E(3, 6) → E′(3, –2)
b) The equation of the design resulting from the transformations is
$y = -f\left(\frac{1}{2}(x+3)\right)+4$.

Section 1.3 Page 41 Question 13

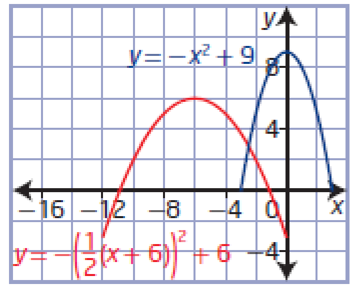
a) The graphs are in two locations because the transformations Gil performed to obtain Graph 2 do not match those in $y = |2x - 6| + 2$. Gil forgot to factor out the coefficient of the x-term, 2, from –6. The horizontal translation should have been 3 units to the right, not 6 units.

b) Gil should have rewritten the function equation $y = |2x - 6| + 2$ as $y = |2(x - 3)| + 2$.



Section 1.3 Page 41 Question 14

a) The first arch is modelled by the function $y = -x^2 + 9$, with a range of $0 \le y \le 9$. For the second arch that spans twice the distance and is translated 6 units to the left and 3 units down, use the mapping $(x, y) \to (2x - 6, y - 3)$.



b) For the second arch, $a = 1$, $b = \frac{1}{2}$, $h = -6$, and $k = -3$.

So, the equation of the second arch is $y = f\left(\frac{1}{2}(x+6)\right) - 3$

or $y = -\left(\frac{1}{2}(x+6)\right)^2 + 6$.

Section 1.3 Page 41 Question 15

a) For $y = -f(-x)$, use the mapping $(x, y) \to (-x, -y)$.
$(a, 0) \to (-a, 0)$
$(0, b) \to (0, -b)$

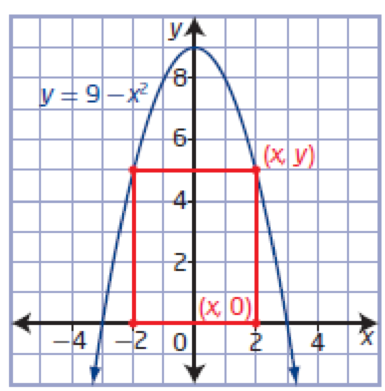
b) For $y = 2f\left(\frac{1}{2}x\right)$, use the mapping $(x, y) \to (2x, 2y)$.

$(a, 0) \to (2a, 0)$
$(0, b) \to (0, 2b)$

c) and **d)** There is not enough information to determine the locations of the new intercepts. When a transformation involves translations, the locations of the new intercepts will vary with different base functions.

Section 1.3 Page 41 Question 16

a) The area of the rectangle can be modelled by the function $A = 2x(9 - x^2)$ or $A = -2x^3 + 18x$.



b) For a horizontal stretch by a factor of 4, the equation of the parabola becomes $y = 9 - \frac{1}{16}x^2$. So, the area of the rectangle can be modeled by the function

$A = 2x\left(9 - \frac{1}{16}x^2\right)$ or $A = -\frac{1}{8}x^3 + 18x$.

c) For (2, 5), the area of the rectangle in part a) is 2(2)(5), or 20 square units.

Verify the function in part a):
$A = -2x^3 + 18x$
$A = -2(2)^3 + 18(2)$
$A = -16 + 36$
$A = 20$
For a horizontal stretch by a factor of 4, the point (2, 5) becomes the point (8, 5). Then, the area of the rectangle in part b) is 2(8)(5), or 80 square units.
Verify the function in part b):

$A = -\frac{1}{8}x^3 + 18x$

$A = -\frac{1}{8}(8)^3 + 18(8)$

$A = -64 + 144$
$A = 80$

Section 1.3 Page 42 Question 17

If the function is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down, then $a = 2$, $b = 3$, $h = 2$, $k = -4$, and $y = 2f(3(x - 2)) - 4$. Given the function $y = 2x^2 + x + 1$, the equation of the transformed function is
$y = 2[2(3(x - 2))^2 + 3(x - 2) + 1] - 4$
$y = 2[18(x - 2)^2 + 3(x - 2) + 1] - 4$
$y = 36(x - 2)^2 + 6(x - 2) + 2 - 4$
$y = 36(x - 2)^2 + 6(x - 2) - 2$
or $y = 36x^2 - 138x + 130$

Section 1.3 Page 42 Question 18

Example: Transformations in which the order does not matter include vertical stretches and horizontal stretches followed by reflections.
For example:

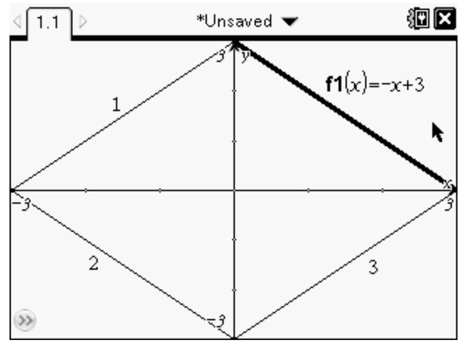
Graph of $f(x)$	**Graph of $g(x)$**	
	Horizontal stretch	Vertical stretch
(x, y)	$\left(\frac{x}{b}, y\right)$	$\left(\frac{x}{b}, ay\right)$

Graph of $f(x)$	**Graph of $g(x)$**	
	Vertical stretch	Horizontal stretch
(x, y)	(x, ay)	$\left(\frac{x}{b}, ay\right)$

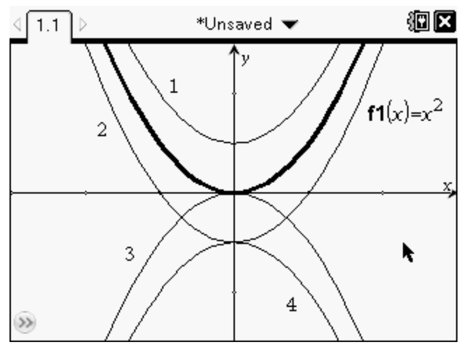
The end result is the same.

Section 1.3 Page 42 Question C1

Step 1 Graph 1: a reflection in the y-axis; $y = x + 3$
Graph 2: a reflection in the y-axis and in the x-axis: $y = -x - 3$
Graph 3: a reflection in the x-axis; $y = x - 3$



Step 2 Graph 1: a translation of 1 unit up; $y = x^2 + 1$
Graph 2: a translation of 1 unit down: $y = x^2 - 1$
Graph 3: a reflection in the x-axis; $y = -x^2$
Graph 4: a reflection in the x-axis and a translation of 1 unit down; $y = -x^2 - 1$



Section 1.3 Page 42 Question C2

They make b bracelets per week at a cost of $f(b)$.
a) Then, $f(b + 12)$ represents the cost of making $b + 12$ bracelets. This relates to transformations as a horizontal translation.

b) Then, $f(b) + 12$ represents the cost of making b bracelets plus 12 more dollars. This relates to transformations as a vertical translation.

c) Then, $3f(b)$ represents triple the cost of making b bracelets. This relates to transformations as a vertical stretch.

d) Then, $f(2b)$ represents the cost of making $0.5b$ bracelets. This relates to transformations as a horizontal stretch.

Section 1.3 Page 42 Question C3

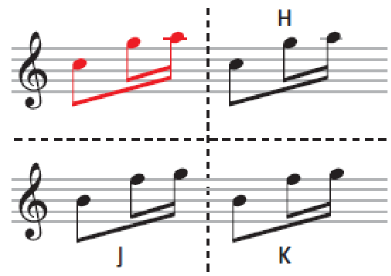
Complete the square.
$y = 2x^2 - 12x + 19$
$y = 2(x^2 - 6x) + 19$
$y = 2(x^2 - 6x + 9 - 9) + 19$
$y = 2(x^2 - 6x + 9) - 18 + 19$
$y = 2(x - 3)^2 + 1$

This form of the equation shows that the function is a vertical stretch by a factor of 2 and then a translation of 3 units to the right and 1 unit up of the graph of $y = x^2$.

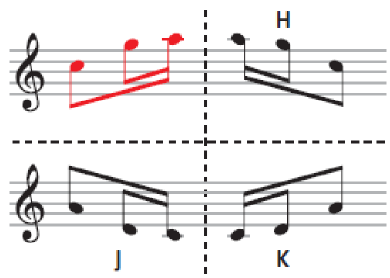
Section 1.3 Page 43 Question C4

Musical notes can be repeated (translated horizontally), transposed (translated vertically), inverted (horizontal mirror), in retrograde (vertical mirror), or in retrograde inversion (180° rotation).

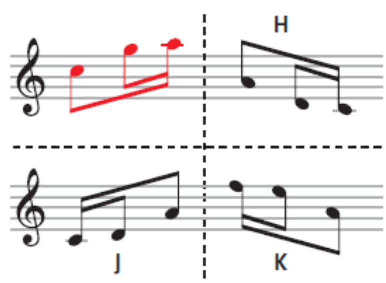
a) H is repeated; J is transposed; K is repeated and transposed



b) H is in retrograde; J is inverted; K is in retrograde and inverted



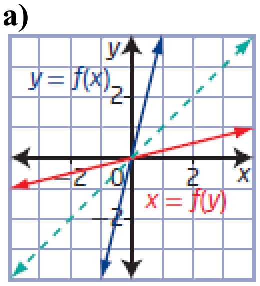
c) H is inverted, repeated, and transposed; J is in retrograde, inverted, and repeated; K is in retrograde and transposed



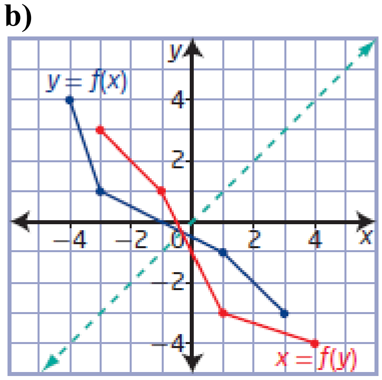
Section 1.4 Inverse of a Relation

Section 1.4 Page 51 Question 1

a)

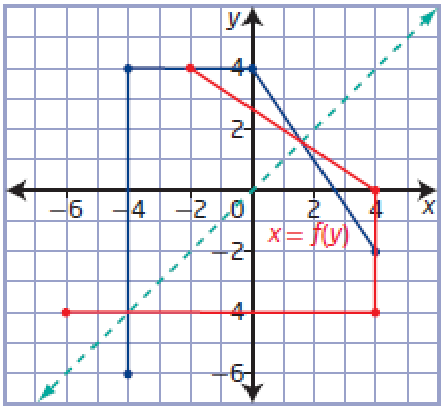


b)

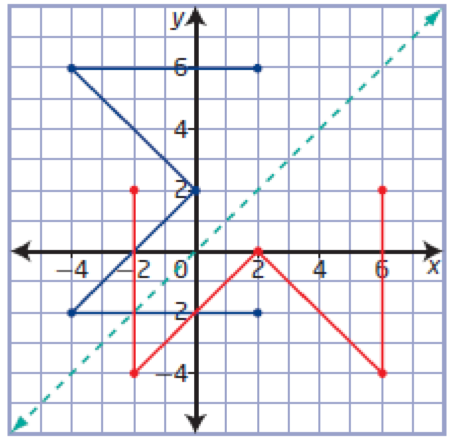


Section 1.4 Page 51 Question 2

a)

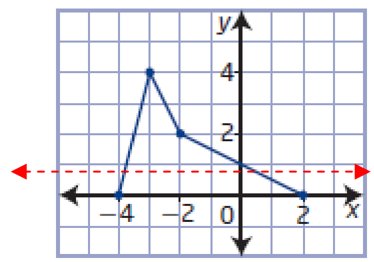


b)

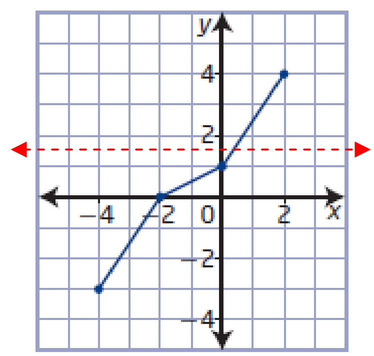


Section 1.4 Page 52 Question 3

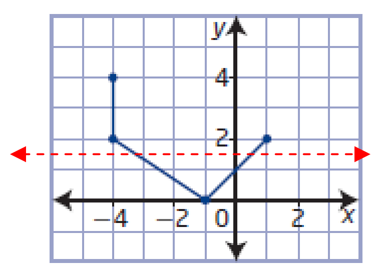
a) The graph is a function; it passes the vertical line test. The inverse will be a relation since the graph does not pass the horizontal line test.



b) The graph is a function; it passes the vertical line test. The inverse will be a function since the graph passes the horizontal line test.



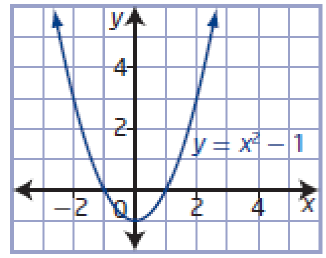
c) The graph is not a function; it does not pass the vertical line test. The inverse will be a relation since the graph does not pass the horizontal line test.



Section 1.4 Page 52 Question 4

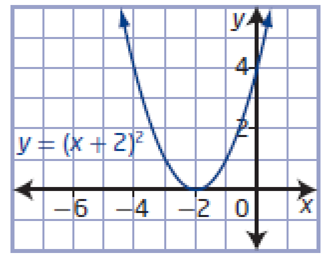
Examples:

a) A restricted domain for which the function has an inverse that is also a function is the left or right half of the parabola:



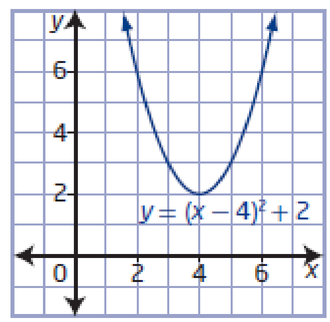
$\{x \mid x \leq 0, x \in \mathrm{R}\}$ or $\{x \mid x \geq 0, x \in \mathrm{R}\}$.

b) A restricted domain for which the function has an inverse that is also a function is the left or right half of the parabola:



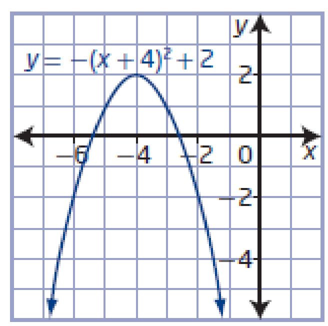
$\{x \mid x \leq -2, x \in \mathrm{R}\}$ or $\{x \mid x \geq -2, x \in \mathrm{R}\}$.

c) A restricted domain for which the function has an inverse that is also a function is the left or right half of the parabola:



$\{x \mid x \leq 4, x \in \mathrm{R}\}$ or $\{x \mid x \geq 4, x \in \mathrm{R}\}$.

d) A restricted domain for which the function has an inverse that is also a function is the left or right half of the parabola:



$\{x \mid x \leq -4, x \in \mathrm{R}\}$ or $\{x \mid x \geq -4, x \in \mathrm{R}\}$.

Section 1.4 Page 52 Question 5

Let $y = f(x)$. To find the equation of the inverse, interchange x and y, and then solve for y.

a) $f(x) = 7x$

$y = 7x$

$x = 7y$

$y = \frac{1}{7}x$

$f^{-1}(x) = \frac{1}{7}x$

b) $f(x) = -3x + 4$

$y = -3x + 4$

$x = -3y + 4$

$x - 4 = -3y$

$y = -\frac{1}{3}(x - 4)$

$f^{-1}(x) = -\frac{1}{3}(x - 4)$

c) $f(x) = \frac{x+4}{3}$

$y = \frac{x+4}{3}$

$x = \frac{y+4}{3}$

$3x = y + 4$

$y = 3x - 4$

$f^{-1}(x) = 3x - 4$

d) $f(x) = \frac{x}{3} - 5$

$y = \frac{x}{3} - 5$

$x = \frac{y}{3} - 5$

$x + 5 = \frac{y}{3}$

$y = 3(x + 5)$

$f^{-1}(x) = 3(x + 5)$

e) $f(x) = 5 - 2x$

$y = 5 - 2x$

$x = 5 - 2y$

$x - 5 = -2y$

$y = -\frac{1}{2}(x - 5)$

$f^{-1}(x) = -\frac{1}{2}(x - 5)$

f) $f(x) = \frac{1}{2}(x+6)$

$y = \frac{1}{2}(x+6)$

$x = \frac{1}{2}(y+6)$

$2x = y + 6$

$y = 2x - 6$

$f^{-1}(x) = 2x - 6$

Section 1.4 Page 52 Question 6

a) The inverse of $y = 2x + 5$ is $y = \frac{1}{2}(x - 5)$: choice **E**.

b) The inverse of $y = \frac{1}{2}x - 4$ is $y = 2(x + 4)$: choice **C**.

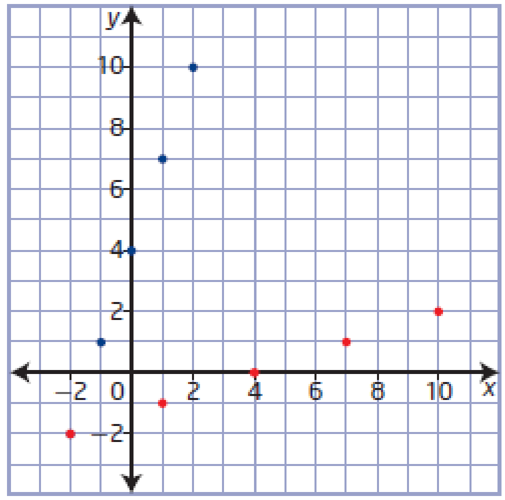
c) The inverse of $y = 6 - 3x$ is $y = -\frac{1}{3}(x - 6)$: choice **B**.

d) The inverse of $y = x^2 - 12$, $x \geq 0$ is $y = \sqrt{x+12}$: choice **A**.

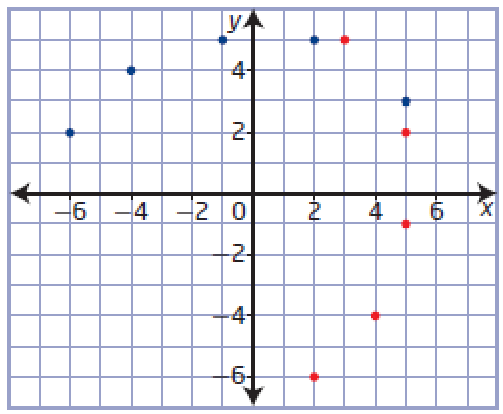
e) The inverse of $y = \frac{1}{2}(x + 1)^2$, $x \leq -1$ is $y = -\sqrt{2x} - 1$: choice **D**.

Section 1.4 Page 53 Question 7

a) Function (blue): domain {–2, –1, 0, 1, 2}, range {–2, 1, 4, 7, 10}
Inverse (red): domain {–2, 1, 4, 7, 10}, range {–2, –1, 0, 1, 2}

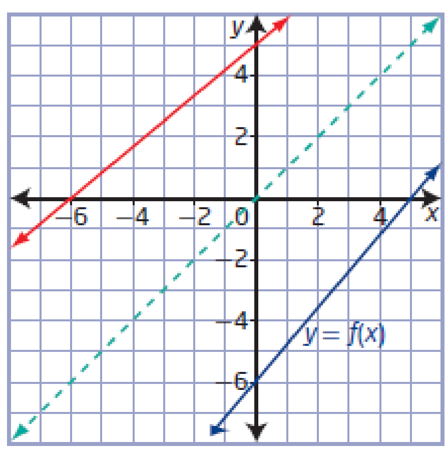


b) a) Function (blue):
domain {–6, –4, –1, 2, 5}, range {2, 4, 5, 3}
Inverse (red):
domain {2, 4, 5, 3},
range {–6, –4, –1, 2, 5}

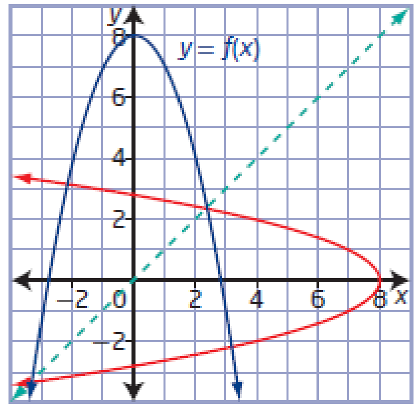


Section 1.4 Page 53 Question 8

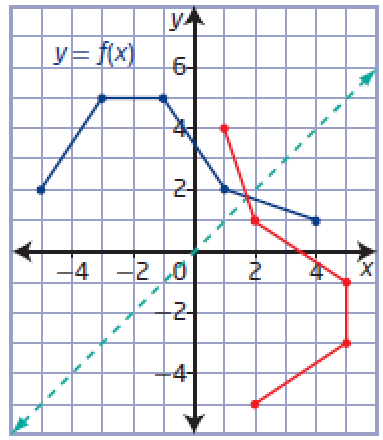
a) The inverse is a function; it passes the vertical line test.



b) The inverse is not a function; it does not pass the vertical line test.



c) The inverse is not a function; it does not pass the vertical line test.



Section 1.4 Page 53 Question 9

a) $f(x) = 3x + 2$

$y = 3x + 2$

$x = 3y + 2$

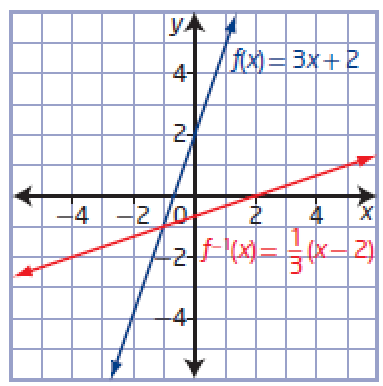
$x - 2 = 3y$

$y = \frac{1}{3}(x - 2)$

$f^{-1}(x) = \frac{1}{3}(x - 2)$

$f(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

$f^{-1}(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$



b) $f(x) = 4 - 2x$

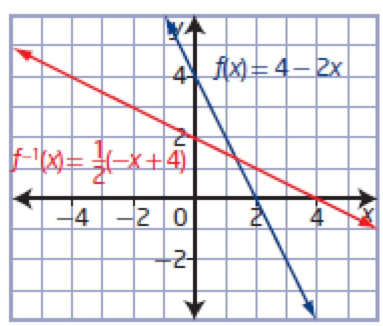
$y = 4 - 2x$

$x = 4 - 2y$

$x - 4 = -2y$

$y = -\frac{1}{2}(x - 4)$

$f^{-1}(x) = -\frac{1}{2}(x - 4)$



$f(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$
$f^{-1}(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

c) $f(x) = \frac{1}{2}x - 6$

$y = \frac{1}{2}x - 6$

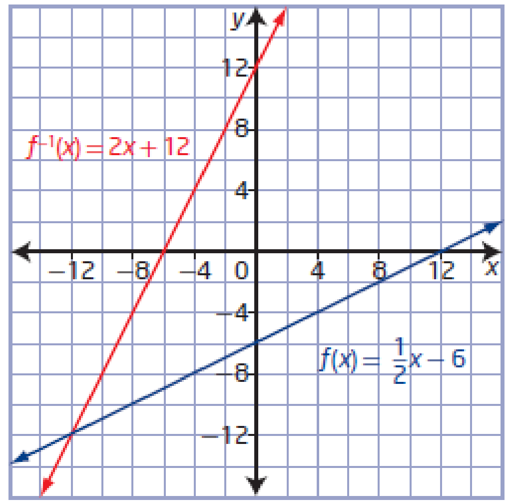
$x = \frac{1}{2}y - 6$

$x + 6 = \frac{1}{2}y$

$y = 2(x + 6)$

$f^{-1}(x) = 2(x + 6)$

$f(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$
$f^{-1}(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$



d) $f(x) = x^2 + 2, x \le 0$

$y = x^2 + 2$

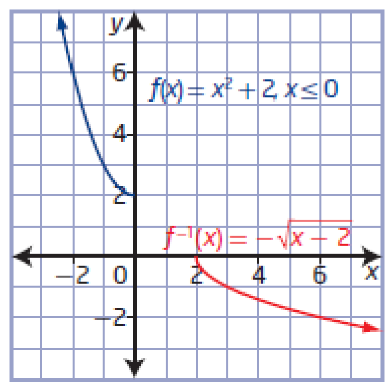
$x = y^2 + 2$

$x - 2 = y^2$

$y = -\sqrt{x - 2}$

$f^{-1}(x) = -\sqrt{x - 2}$

$f(x)$: domain $\{x \mid x \le 0, x \in \mathrm{R}\}$, range $\{y \mid y \ge 2, y \in \mathrm{R}\}$
$f^{-1}(x)$: domain $\{x \mid x \ge 2, x \in \mathrm{R}\}$, range $\{y \mid y \le 0, y \in \mathrm{R}\}$



e) $f(x) = 2 - x^2, x \ge 0$

$y = 2 - x^2$

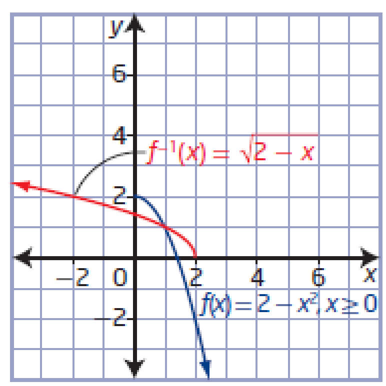
$x = 2 - y^2$

$-x - 2 = -y^2$

$y = \sqrt{-x - 2}$

$f^{-1}(x) = \sqrt{-x - 2}$

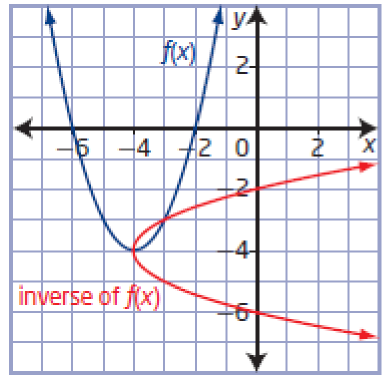
$f(x)$: domain $\{x \mid x \ge 0, x \in \mathrm{R}\}$, range $\{y \mid y \le 2, y \in \mathrm{R}\}$
$f^{-1}(x)$: domain $\{x \mid x \le 2, x \in \mathrm{R}\}$, range $\{y \mid y \ge 0, y \in \mathrm{R}\}$



Section 1.4 Page 53 Question 10

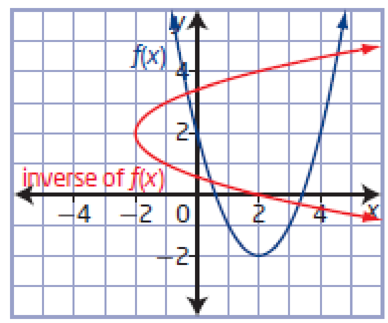
a) i) Complete the square.
$f(x) = x^2 + 8x + 12$
$f(x) = (x^2 + 8x + 16 - 16) + 12$
$f(x) = (x + 4)^2 - 4$
Determine the equation of the inverse of $f(x)$.
$y = (x + 4)^2 - 4$
$x = (y + 4)^2 - 4$
$x + 4 = (y + 4)^2$
$y = \pm\sqrt{x+4} - 4$

ii)



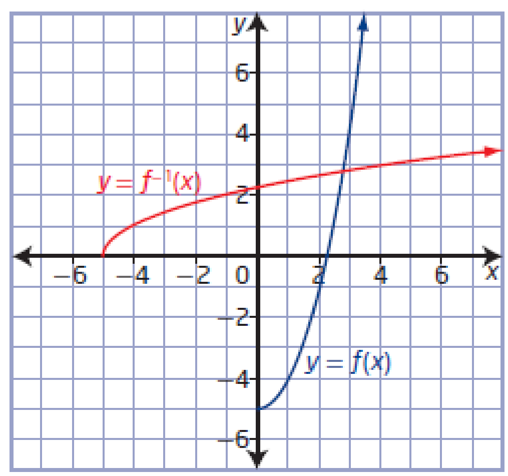
b) i) Complete the square.
$f(x) = x^2 - 4x + 2$
$f(x) = (x^2 - 4x + 4 - 4) + 2$
$f(x) = (x - 2)^2 - 2$
Determine the equation of the inverse of $f(x)$.
$y = (x - 2)^2 - 2$
$x = (y - 2)^2 - 2$
$x + 2 = (y - 2)^2$
$y = \pm\sqrt{x+2} + 2$

ii)



Section 1.4 Page 53 Question 11

Since the graphs are reflections of each other in the line $y = x$, the functions are inverses of each other.



Section 1.4 Page 54 Question 12

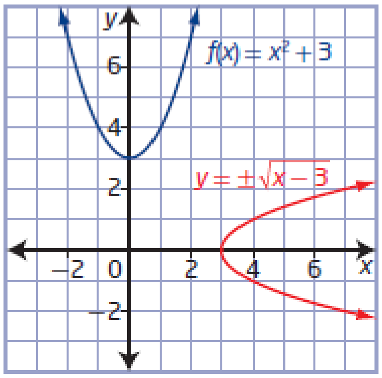
a) $f(x) = x^2 + 3$

$y = x^2 + 3$

$x = y^2 + 3$

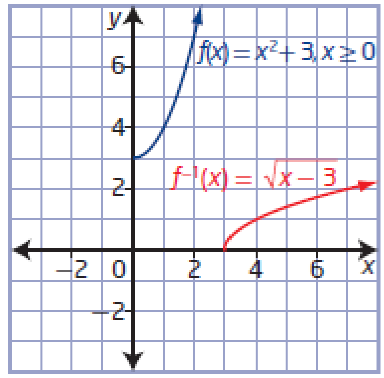
$x - 3 = y^2$

$y = \pm\sqrt{x-3}$



Example: A restricted domain for which the function has an inverse that is also a function is right half of the parabola: $\{x \mid x \geq 0, x \in \mathrm{R}\}$.

$f(x) = x^2 + 3, x \geq 0$ and $f^{-1}(x) = \sqrt{x-3}$



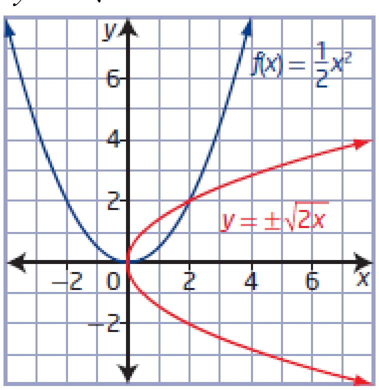
b) $f(x) = \frac{1}{2}x^2$

$y = \frac{1}{2}x^2$

$x = \frac{1}{2}y^2$

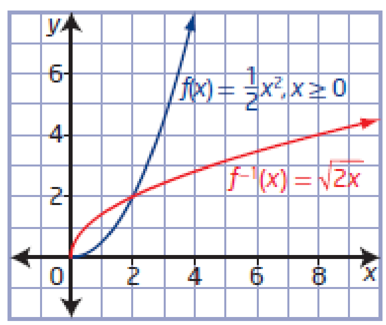
$2x = y^2$

$y = \pm\sqrt{2x}$



Example: A restricted domain for which the function has an inverse that is also a function is right half of the parabola: $\{x \mid x \geq 0, x \in \mathrm{R}\}$.

$f(x) = \frac{1}{2}x^2, x \geq 0$ and $f^{-1}(x) = \sqrt{2x}$



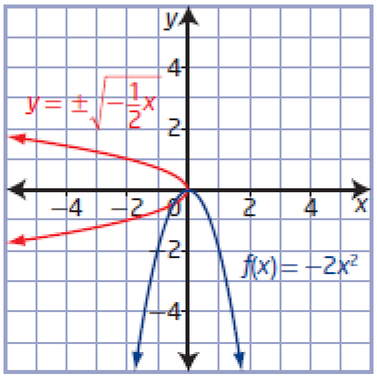
c) $f(x) = -2x^2$

$y = -2x^2$

$x = -2y^2$

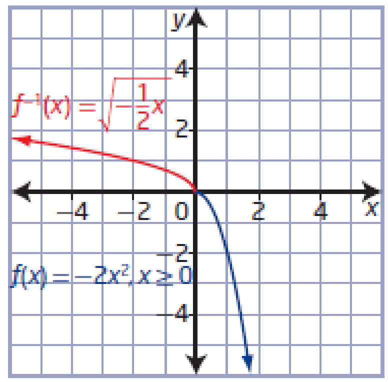
$-\frac{1}{2}x = y^2$

$y = \pm\sqrt{-\frac{1}{2}x}$



Example: A restricted domain for which the function has an inverse that is also a function is right half of the parabola: $\{x \mid x \geq 0, x \in \mathrm{R}\}$.

$f(x) = -2x^2, x \geq 0$ and $f^{-1}(x) = \sqrt{-\frac{1}{2}x}$



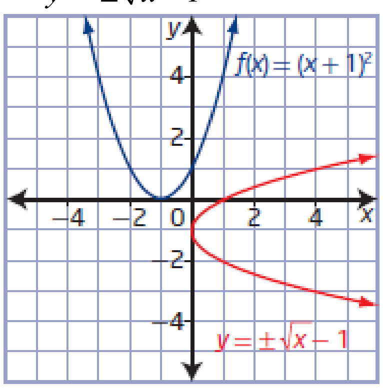
d) $f(x) = (x + 1)^2$

$y = (x + 1)^2$

$x = (y + 1)^2$

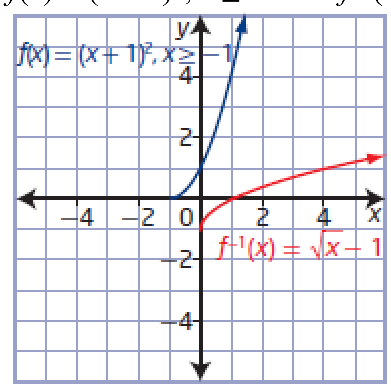
$\pm\sqrt{x} = y + 1$

$y = \pm\sqrt{x} - 1$



Example: A restricted domain for which the function has an inverse that is also a function is right half of the parabola: $\{x \mid x \geq -1, x \in \mathrm{R}\}$.

$f(x) = (x + 1)^2, x \geq -1$ and $f^{-1}(x) = \sqrt{x} - 1$



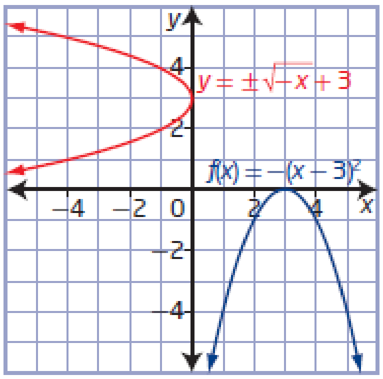
e) $f(x) = -(x-3)^2$

$y = -(x-3)^2$

$x = -(y-3)^2$

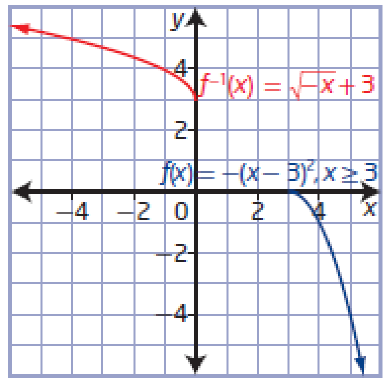
$\pm\sqrt{-x} = y - 3$

$y = \pm\sqrt{-x} + 3$



Example: A restricted domain for which the function has an inverse that is also a function is right half of the parabola: $\{x \mid x \geq 3, x \in \mathrm{R}\}$.

$f(x) = -(x-3)^2, x \geq 3$ and $f^{-1}(x) = \sqrt{-x} + 3$



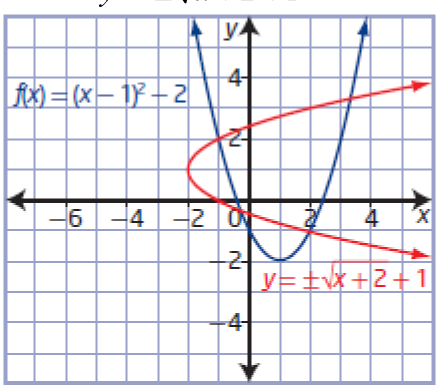
f) $f(x) = (x-1)^2 - 2$

$y = (x-1)^2 - 2$

$x = (y-1)^2 - 2$

$\pm\sqrt{x+2} = y - 1$

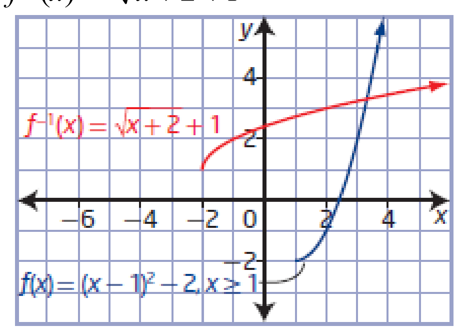
$y = \pm\sqrt{x+2} + 1$



Example: A restricted domain for which the function has an inverse that is also a function is right half of the parabola: $\{x \mid x \geq 1, x \in \mathrm{R}\}$.

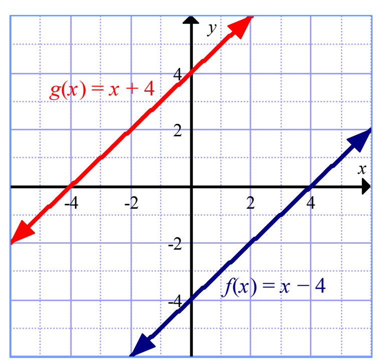
$f(x) = (x-1)^2 - 2, x \geq 1$ and

$f^{-1}(x) = \sqrt{x+2} + 1$

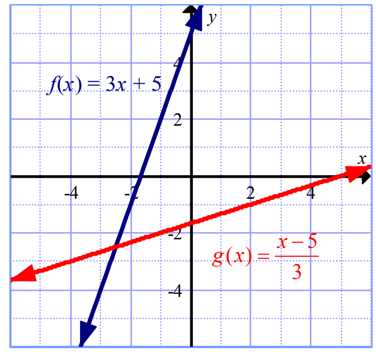


Section 1.4 Page 54 Question 13

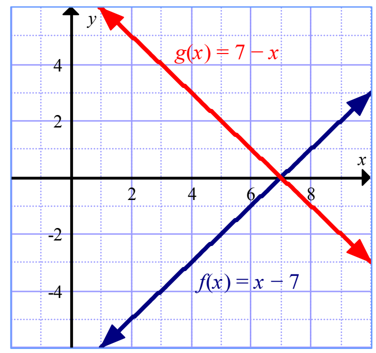
a) Since the graphs are reflections of each other in the line $y = x$, the functions are inverses of each other.



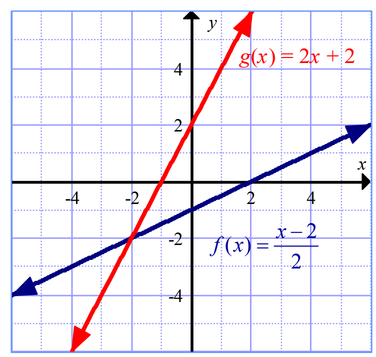
b) Since the graphs are reflections of each other in the line $y = x$, the functions are inverses of each other.



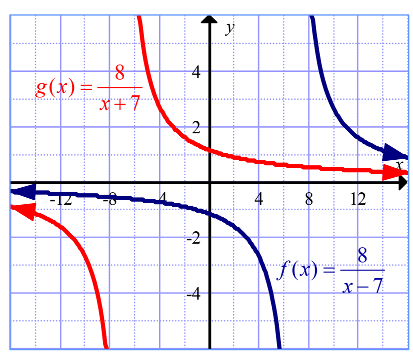
c) The graphs are not reflections of each other in the line $y = x$, so the functions are not inverses of each other.



d) Since the graphs are reflections of each other in the line $y = x$, the functions are inverses of each other.



e) The graphs are not reflections of each other in the line $y = x$, so the functions are not inverses of each other.



Section 1.4 Page 54 Question 14

Examples: A restricted domain for which these functions have an inverse that is also a function is the left or right half of the parabola.

a) For $f(x) = x^2 + 4$, the vertex is located at (0, 4). So, two ways to restrict the domain are $\{x \mid x \leq 0, x \in \mathrm{R}\}$ or $\{x \mid x \geq 0, x \in \mathrm{R}\}$.

b) For $f(x) = 2 - x^2$, the vertex is located at (0, 2). So, two ways to restrict the domain are $\{x \mid x \leq 0, x \in \mathrm{R}\}$ or $\{x \mid x \geq 0, x \in \mathrm{R}\}$.

c) For $f(x) = (x - 3)^2$, the vertex is located at (3, 0). So, two ways to restrict the domain are $\{x \mid x \leq 3, x \in \mathrm{R}\}$ or $\{x \mid x \geq 3, x \in \mathrm{R}\}$.

d) For $f(x) = (x + 2)^2 - 4$, the vertex is located at (–2, –4). So, two ways to restrict the domain are $\{x \mid x \leq -2, x \in \mathrm{R}\}$ or $\{x \mid x \geq -2, x \in \mathrm{R}\}$.

Section 1.4 Page 54 Question 15

$f(x) = 4x - 2$
$y = 4x - 2$
$x = 4y - 2$
$x + 2 = 4y$
$y = \frac{1}{4}(x + 2)$
$f^{-1}(x) = \frac{1}{4}(x + 2)$

a) Substitute $x = 4$.

$f^{-1}(x) = \frac{1}{4}(x + 2)$

$f^{-1}(4) = \frac{1}{4}(4 + 2)$

$f^{-1}(4) = \frac{3}{2}$

b) Substitute $x = -2$.

$f^{-1}(x) = \frac{1}{4}(x + 2)$

$f^{-1}(-2) = \frac{1}{4}(-2 + 2)$

$f^{-1}(-2) = 0$

c) Substitute $x = 8$.

$f^{-1}(x) = \frac{1}{4}(x + 2)$

$f^{-1}(8) = \frac{1}{4}(8 + 2)$

$f^{-1}(8) = \frac{5}{2}$

d) Substitute $x = 0$.

$f^{-1}(x) = \frac{1}{4}(x + 2)$

$f^{-1}(0) = \frac{1}{4}(0 + 2)$

$f^{-1}(0) = \frac{1}{2}$

Section 1.4 Page 54 Question 16

a) Substitute $x = 90$ into $y = \frac{5}{9}(x - 32)$.

$y = \frac{5}{9}(90 - 32)$

$y = \frac{5}{9}(58)$

$y \approx 32.22$

The equivalent temperature in degrees Celsius for 90 °F is approximately 32.22 °C.

b) For the inverse of the function, let *x* represent the temperature, in degrees Celsius, and *y* represent the temperature, in degrees Fahrenheit.

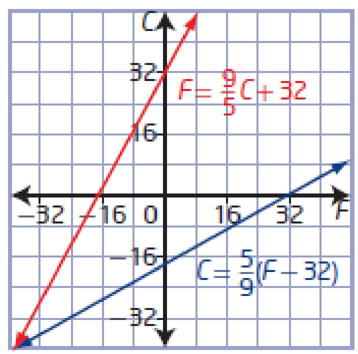
$$y = \frac{5}{9}(x - 32)$$
$$x = \frac{5}{9}(y - 32)$$
$$\frac{9}{5}x = y - 32$$
$$y = \frac{9}{5}x + 32$$

c) Substitute $x = 32$ into $y = \frac{9}{5}x + 32$.

$$y = \frac{9}{5}(32) + 32$$
$$y = 89.6$$

The equivalent temperature in degrees Fahrenheit for 32 °C is 89.6 °F.

d) The invariant point represents when the temperature is the same in both scales (–40 °C = –40 °F).



Section 1.4 Page 54 Question 17

a) Substitute $x = 45.47$ into each function.

For a male: $y = 2.32x + 65.53$
$y = 2.32(45.47) + 65.53$
$y = 171.0204$

For a female: $y = 2.47x + 54.13$
$y = 2.47(45.47) + 54.13$
$y = 166.4409$

The height of a male and of a female with a femur length of 45.47 cm is 171.02 cm and 166.44 cm, respectively.

b) For the inverse functions, let *y* represent the length of the femur and let *x* represent the height of the person, both in centimetres.

For a male: $y = 2.32x + 65.53$
$x = 2.32y + 65.53$
$x - 65.53 = 2.32y$
$$y = \frac{x - 65.53}{2.32}$$

For a female: $y = 2.47x + 54.13$
$x = 2.47y + 54.13$
$x - 54.13 = 2.47y$
$$y = \frac{x - 54.13}{2.47}$$

i) Substitute $x = 187.9$.

$$y = \frac{187.9 - 65.53}{2.32}$$

$y = 52.745\ldots$

The femur length of a male whose height is 187.9 cm is 52.75 cm.

ii) Substitute $x = 175.26$.

$$y = \frac{175.26 - 54.13}{2.47}$$

$y = 49.040\ldots$

The femur length of a female whose height is 175.26 cm is 49.04 cm.

Section 1.4 Page 54 Question 18

a) Substitute $x = 49.3$.

$$y = \frac{x - 36.5}{2.55}$$

$$y = \frac{49.3 - 36.5}{2.55}$$

$y \approx 5$

The whole-number ring size that corresponds to a finger circumference of 49.3 mm is 5.

b) For the inverse functions, let x represent the numerical ring size and let y represent finger circumference, in millimetres.

$$y = \frac{x - 36.5}{2.55}$$

$$x = \frac{y - 36.5}{2.55}$$

$2.55x = y - 36.5$

$y = 2.55x + 36.5$

c) Substitute ring sizes into $y = 2.55x + 36.5$.

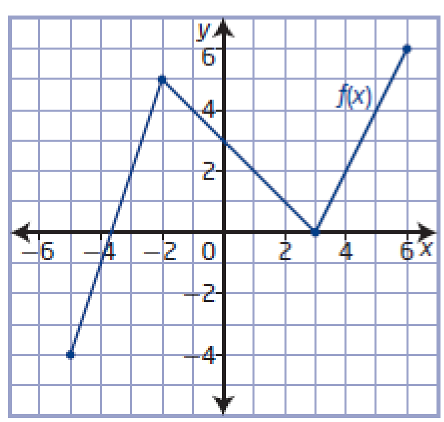
Substitute $x = 6$. $y = 2.55(6) + 36.5$ $y = 51.8$	Substitute $x = 7$. $y = 2.55(7) + 36.5$ $y = 54.35$	Substitute $x = 9$. $y = 2.55(9) + 36.5$ $y = 59.45$

The finger circumferences that correspond to ring sizes of 6, 7, and 9 are 51.8 mm, 54.35 mm, and 59.45 mm.

Section 1.4 Page 55 Question 19

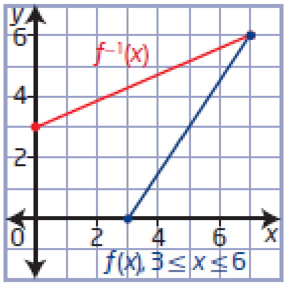
a) The function is increasing over the intervals $-5 \le x \le -2$ and $3 \le x \le 6$.

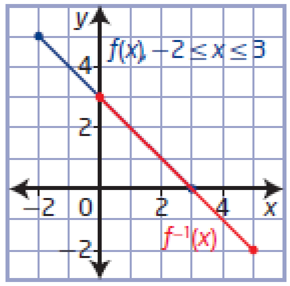
The function is decreasing over the interval $-2 \le x \le 3$.



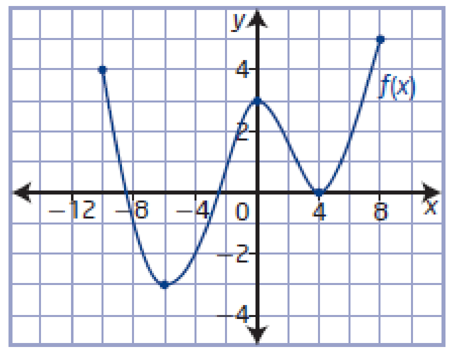
Example:

i) For $3 \le x \le 6$ **ii)** For $-2 \le x \le 3$



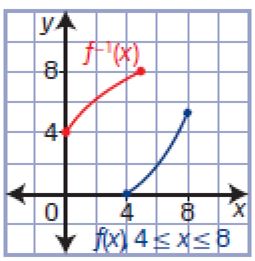


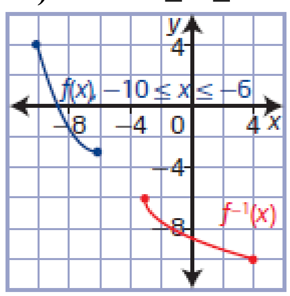
b) The function is increasing over the intervals $-6 \le x \le 0$ and $4 \le x \le 8$.
The function is decreasing over the intervals $-10 \le x \le -6$ and $0 \le x \le 4$.



Example:

i) For $4 \le x \le 8$ **ii)** For $-10 \le x \le -6$





Section 1.4 Page 55 Question 20

The domain and range of $f(x)$ become the range and domain, respectively, of $f^{-1}(x)$.

a) If $f(17) = 5$, then $f^{-1}(5) = 17$.

b) If $f^{-1}\left(\sqrt{3}\right) = -2$, then $f(-2) = \sqrt{3}$.

c) Substitute $x = 1$ into $f(x) = 2x^2 + 5x + 3, x \ge -1.25$.
$f(1) = 2(1)^2 + 5(1) + 3$
$f(1) = 10$
If $f(1) = 10$, then $f^{-1}(10) = 1$ and $a = 10$.

Section 1.4 Page 55 Question 21

Given the point (10, 8) is on the graph of $y = f(x)$. Then, the point (8, 10) is on the graph of $y = f^{-1}(x)$.

a) The graph of $y = f^{-1}(x + 2)$ is a translation of 2 units to the left of the graph of $y = f^{-1}(x)$. The point (8, 10) on the graph of $y = f^{-1}(x)$ becomes (6, 10).

b) The graph of $y = 2f^{-1}(x) + 3$ is a vertical stretch by a factor of 2 and a translation of 3 units up of the graph of $y = f^{-1}(x)$. The point (8, 10) on the graph of $y = f^{-1}(x)$ becomes (8, 23).

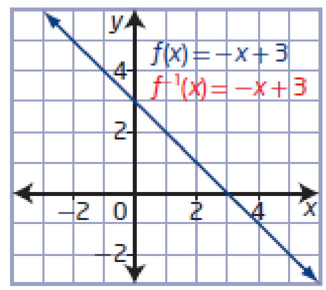
c) The graph of $y = -f^{-1}(-x) + 1$ is a reflection in the x-axis, a reflection in the y-axis, and a translation of 1 unit up of the graph of $y = f^{-1}(x)$. The point (8, 10) on the graph of $y = f^{-1}(x)$ becomes (–8, –9).

Section 1.4 Page 55 Question C1

a) The inverse sequence of operations for $f(x) = 6x + 12$ are subtract 12 and divide by 6.
b) The inverse sequence of operations for $f(x) = (x + 3)^2 - 1$ are add 1, take the positive and negative square root, and subtract 3.

Section 1.4 Page 55 Question C2

a) $f(x) = -x + 3$
$y = -x + 3$
$x = -y + 3$
$x - 3 = -y$
$y = -x + 3$
$f^{-1}(x) = -x + 3$



b) Example: The graph of the original linear function is perpendicular to $y = x$. So, after a reflection in the line $y = x$, the graph of the inverse is the same.

c) If a function and its inverse are the same, their graphs are perpendicular to the line $y = x$.

Section 1.4 Page 55 Question C3

Example: If the original function passes the vertical line test, then it is a function. If the original function passes the horizontal line test, then the inverse is a function.

Section 1.4 Page 55 Question C4

Step 1

x	$f(x) = \frac{x+5}{3}$
1	2
4	3
–8	–1
a	$\frac{a+5}{3}$

x	$g(x) = 3x - 5$
2	1
3	4
–1	–8
$\frac{a+5}{3}$	a

The output values of $g(x)$ are the same as the input values for $f(x)$. This occurs because the functions are inverses of each other.
Example: Use the output values of one function as the input values for second function. If the output values for that second function are the same as the input values of the first function, then the two functions are inverses of each other.

Step 2

x	**g(x) = 3x – 5**
1	–2
4	7
–8	–29
a	3*a* – 5

x	$f(x) = \frac{x+5}{3}$
–2	1
7	4
–29	–8
3*a* – 5	*a*

If two functions are inverses, then the order in which you apply them does not change the final result.

Step 3 Example:

x	**f(x) = x + 2**
1	3
4	6
–8	–6
a	*a* + 2

x	**g(x) = x – 2**
3	1
6	4
–6	–8
a + 2	*a*

x	**g(x) = x – 2**
1	–1
4	2
–8	–10
a	*a* – 2

x	**f(x) = x + 2**
–1	1
2	4
–10	–8
a – 2	*a*

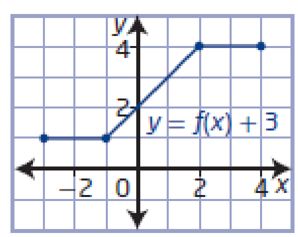
The hypothesis and conclusion from steps 1 and 2 hold.

Step 4 The statement is saying that if you have a function that when given *a* outputs *b* and another that when given *b* outputs *a*, then the functions are inverses of each other.

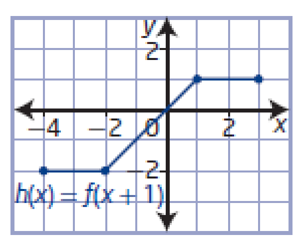
Chapter 1 Review

Chapter 1 Review Page 56 Question 1

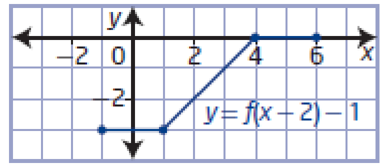
a) For the graph of $y - 3 = f(x)$, translate the graph of $y = f(x)$ up 3 units.



b) For the graph of $h(x) = f(x + 1)$, translate the graph of $y = f(x)$ to the left 1 unit.



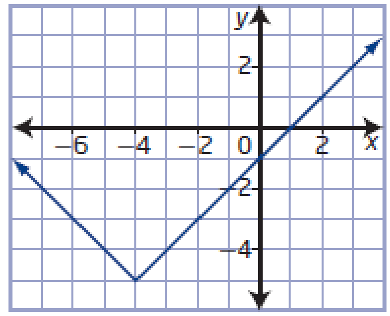
c) For the graph of $y + 1 = f(x - 2)$, translate the graph of $y = f(x)$ to the right 2 units and down 1 unit.



Chapter 1 Review Page 56 Question 2

To obtain the graph of the function shown, translate the graph of $y = |x|$ to the left 4 units and down 5 units.

The equation of the transformed function is $y + 5 = |x + 4|$.



Chapter 1 Review Page 56 Question 3

For $y = f(x - 2) + 4$, $h = 2$ and $k = 4$, representing a translation of 2 units to the right and 4 units up. If the range of the function $y = f(x)$ is $\{y \mid -2 \le y \le 5, y \in \mathrm{R}\}$, then the range of $y = f(x - 2) + 4$ will be $\{y \mid 2 \le y \le 9, y \in \mathrm{R}\}$.

Chapter 1 Review Page 56 Question 4

James is incorrect. If the point (a, b) is on the graph of $y = f(x)$, then the point $(a + 5, b - 4)$ is the image point on the graph of $y + 4 = f(x - 5)$, not $(a - 5, b + 4)$. He should have compared $y + 4 = f(x - 5)$ to the form $y - k = f(x - h)$.
$y - (-4) = f(x - (+5))$

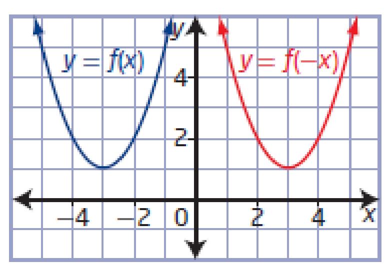
Chapter 1 Review Page 56 Question 5

a) When the graph of $y = f(x)$ is transformed to $y = -f(x)$ it is reflected in the x-axis. The key point (3, 5) becomes the image point (3, –5).

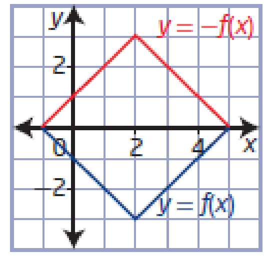
b) When the graph of $y = f(x)$ is transformed to $y = f(-x)$ it is reflected in the y-axis. The key point (3, 5) becomes the image point (–3, 5).

Chapter 1 Review Page 56 Question 6

a) $y = f(-x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \ge 1, y \in \mathrm{R}\}$
The invariant point is (0, 10).

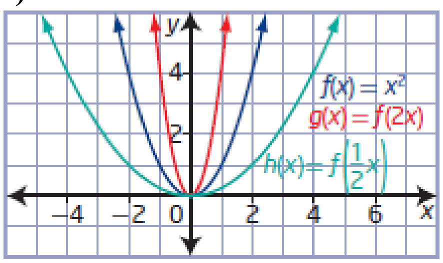


b) $y = -f(x)$: domain $\{x \mid -1 \le x \le 5, x \in \mathrm{R}\}$, range $\{y \mid 0 \le y \le 3, y \in \mathrm{R}\}$
The invariant points are (–1, 0) and (5, 0).



Chapter 1 Review Page 56 Question 7

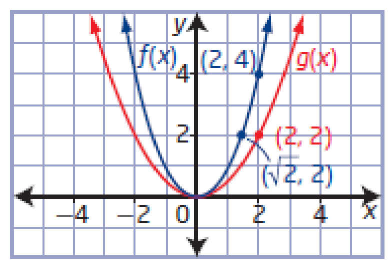
a)



b) If the coefficient of x is greater than 1, then the graph of the transformed function moves closer to the y-axis. If the coefficient of x is between 0 and 1, then the graph of the transformed function moves farther from the y-axis.

Chapter 1 Review Page 56 Question 8

a) Compare a key point on the graph of $f(x)$ to the image point on the graph of $g(x)$.
Case 1: Pattern in the y-coordinates
$f(x)$ $g(x)$
$(2, 4) \to (2, 2)$
$(x, y) \to \left(x, \frac{1}{2}y\right)$



This indicates a vertical stretch about the x-axis by a factor of $\frac{1}{2}$.

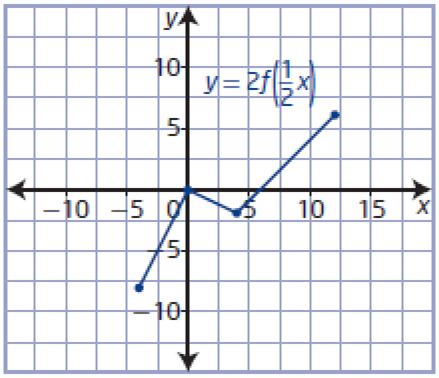
Case 2: Pattern in the x-coordinates
$f(x)$ $g(x)$
$\left(\sqrt{2}, 2\right) \to (2, 2)$
$(x, y) \to \left(\sqrt{2}x, y\right)$

This indicates a horizontal stretch about the y-axis by a factor of $\sqrt{2}$.
The graph of $g(x)$ could be a horizontal or a vertical stretch of the graph of $f(x)$.

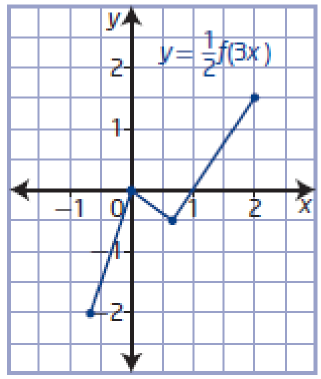
b) Example: $g(x) = \frac{1}{2}f(x)$

Chapter 1 Review Page 57 Question 9

a) The graph of $y = f(x)$ is stretched horizontally about the y-axis by a factor of 2 and stretched vertically about the x-axis by a factor of 2.



b) The graph of $y = f(x)$ is stretched horizontally about the y-axis by a factor of $\frac{1}{3}$ and stretched vertically about the x-axis by a factor of $\frac{1}{2}$.



Chapter 1 Review Page 57 Question 10

Rewrite $y = f(4x + 1)$ as $y = f(4(x + 0.25))$.
Compare $y = f(4(x + 1))$ and $y = f(4(x + 0.25))$.

Both transformations represent a horizontal stretch about the y-axis by a factor of $\frac{1}{4}$.

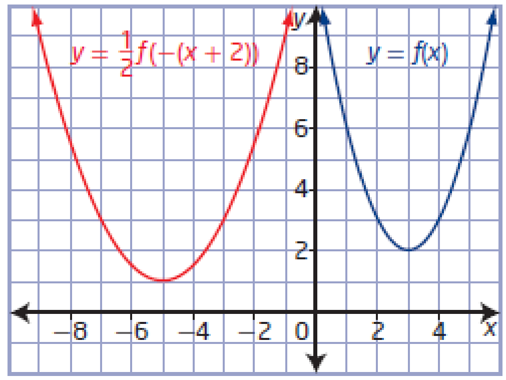
Both have also been translated to the left, but by different amounts; the first is 1 unit left and the second is 0.25 units left.

Chapter 1 Review Page 57 Question 11

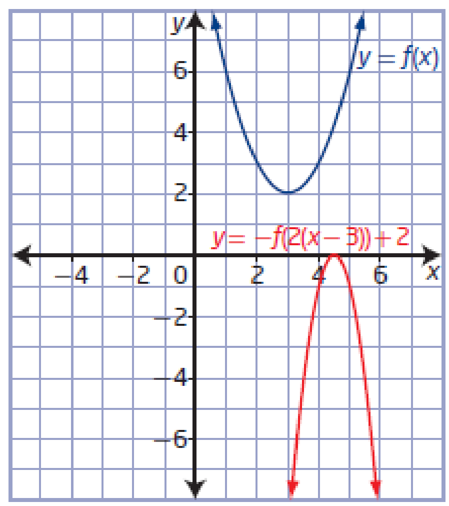
Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.
$(-2, 2) \rightarrow (4, 0)$
$(0, 0) \rightarrow (5, -2)$
$(2, 2) \rightarrow (6, 0)$
The orientation is unchanged, the graph has not been reflected.
The overall width has changed, so the graph has been horizontally stretched by a factor of 0.5. Since the point (0, 0) is not affected by stretches, the graph has been translated 5 units to the right and 2 units down.
So, $a = 1$, $b = 2$, $h = 5$, $k = -2$, and the equation of the transformed graph is $g(x) = f(2(x - 5)) - 2$.

Chapter 1 Review Page 57 Question 12

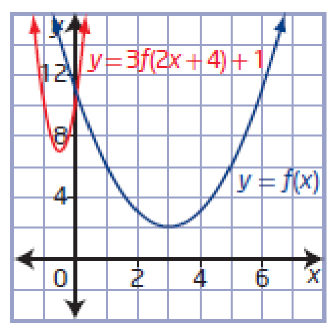
a) For $y = \frac{1}{2}f(-(x + 2))$, $a = \frac{1}{2}$, $b = -1$, $h = -2$, and $k = 0$. This represents a reflection in the y-axis, a vertical stretch about the x-axis by a factor of $\frac{1}{2}$, and a translation of 2 units to the left of the graph of $y = f(x)$.



b) For $y - 2 = -f(2(x - 3))$, $a = -1$, $b = 2$, $h = 3$, and $k = 2$. This represents a reflection in the x-axis, a horizontal stretch about the y-axis by a factor of $\frac{1}{2}$, and a translation of 3 units to the right and 2 units up of the graph of $y = f(x)$.

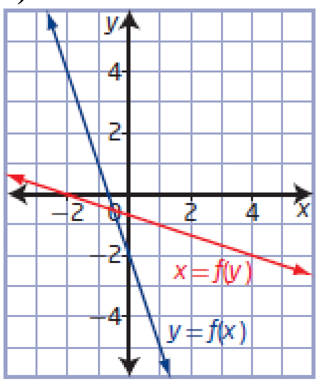


c) For $y - 1 = 3f(2x + 4)$ or $y - 1 = 3f(2(x + 2))$, $a = 3$, $b = 2$, $h = -2$, and $k = 1$. This represents a horizontal stretch about the y-axis by a factor of $\frac{1}{2}$, a vertical stretch about the x-axis by a factor of 3, and a translation of 2 units to the left and 1 unit up of the graph of $y = f(x)$.



Chapter 1 Review   Page 57   Question 13

a)



b) To obtain the graph of $x = f(y)$, reflect the graph of $y = f(x)$ in the line $y = x$. The invariant point is on the line of reflection, (–0.5, –0.5).

c) $f(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$
$f(y)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

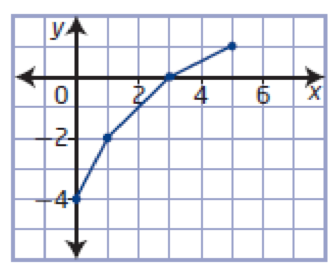
Chapter 1 Review   Page 57   Question 14

Interchange the x- and y-coordinates.

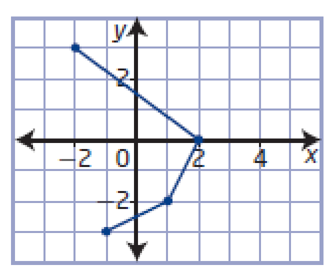
$y = f(x)$		$y = f^{-1}(x)$	
x	y	x	y
−3	7	7	−3
2	4	4	2
10	−12	−12	10

Chapter 1 Review   Page 57   Question 15

a) Reflect the given graph in the line $y = x$. Both the relation and its inverse are functions, since the relation passes both the vertical line test and the horizontal line test.



b) Reflect the given graph in the line $y = x$. The relation is a function but its inverse is not, since the relation passes the vertical line test but not the horizontal line test.



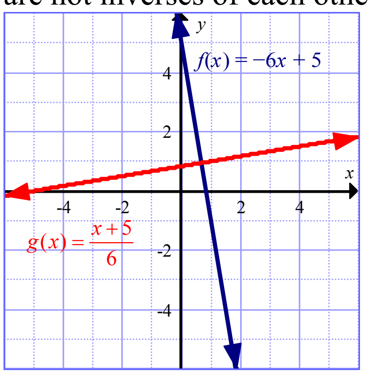
Chapter 1 Review Page 57 Question 16

$$y = (x - 3)^2 + 1$$
$$x = (y - 3)^2 + 1$$
$$\pm\sqrt{x-1} = y - 3$$
$$y = \pm\sqrt{x-1} + 3$$

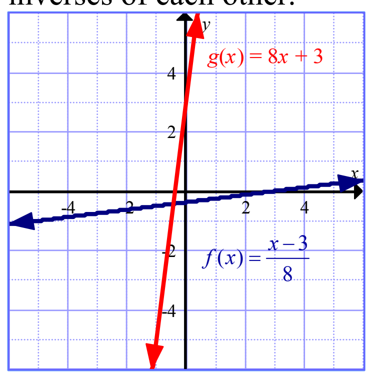
Example: A restricted domain for which the function has an inverse that is also a function is right half of the parabola.
For $y = (x - 3)^2 + 1$, the vertex is located at (3, 1). So, restrict the domain to $\{x \mid x \geq 3, x \in \mathrm{R}\}$.
$y = (x - 3)^2 + 1, x \geq 3$ and $y = \sqrt{x-1} + 3$

Chapter 1 Review Page 57 Question 17

a) Since the graphs are not reflections of each other in the line $y = x$, the functions are not inverses of each other.



b) Since the graphs are reflections of each other in the line $y = x$, the functions are inverses of each other.



Chapter 1 Practice Test

Chapter 1 Practice Test Page 58 Question 1

The equation $y = (x + 1)^2$ represents a translation of 1 unit to the left of the graph $y = x^2$. The graph is the same shape but translated to the left: choice **D**.

Chapter 1 Practice Test Page 58 Question 2

Since the point (0, 0) is not affected by stretches, the graph of $y = |x|$ has been translated 4 units to the left and 6 units down.
So, $h = -4$, $k = -6$, and the equation of the transformed graph is $y + 6 = |x + 4|$: choice **D**.

Chapter 1 Practice Test Page 58 Question 3

The graph of $y = f(x + 2)$ is a translation of 2 units to the left of the graph of $y = f(x)$. The point (a, b) becomes $(a - 2, b)$: choice **B**.

Chapter 1 Practice Test Page 58 Question 4

A reflection in the y-axis is represented by $y = f(-x)$. So, the image of $y = x^2 + 2$ is $y = x^2 + 2$: choice **B**.

Chapter 1 Practice Test Page 58 Question 5

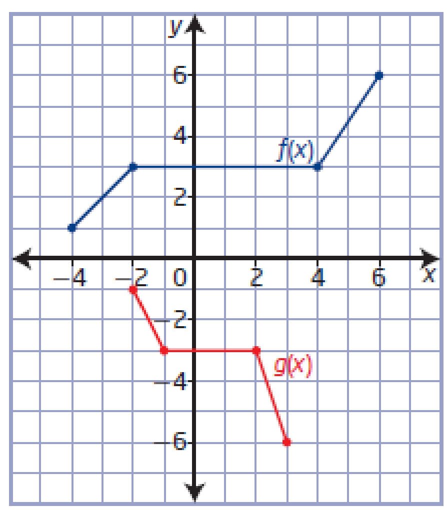
The graph of $y = \frac{1}{4}f(3x)$ is a vertical stretch by a factor of $\frac{1}{4}$ and a horizontal stretch by a factor of $\frac{1}{3}$: choice **B**.

Chapter 1 Practice Test Page 58 Question 6

The location of the y-intercept will not be affected by a horizontal stretch. The graph of $f(-9x)$ will have the same y-intercept as the graph of $f(x)$: choice **C**.

Chapter 1 Practice Test Page 58 Question 7

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.
$(-4, 1) \rightarrow (-2, -1)$
$(-2, 3) \rightarrow (-1, -3)$
$(4, 3) \rightarrow (2, -3)$
$(6, 6) \rightarrow (3, -6)$
The orientation is changed, the graph has been reflected in the x-axis. The overall width has changed, so the graph has been horizontally stretched by a factor of $\frac{1}{2}$. So, $a = -1$, $b = 2$, and the equation of the transformed graph is $g(x) = -f(2x)$: choice **C**.

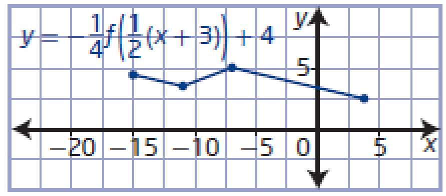


Chapter 1 Practice Test Page 59 Question 8

For $y = f(x + 2) - 1$, $h = -2$ and $k = -1$, representing a translation of 2 units to the left and 1 unit down. If the domain of the function $y = f(x)$ is $\{x \mid -3 \le x \le 4, x \in \mathrm{R}\}$, then the domain of $y = f(x + 2) - 1$ will be $\{x \mid -5 \le x \le 2, x \in \mathrm{R}\}$.

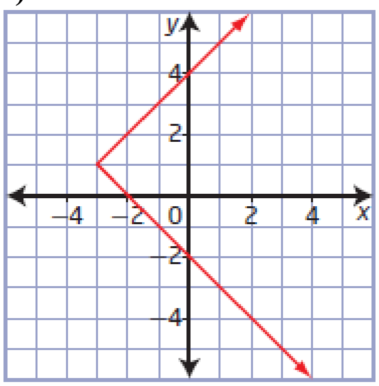
Chapter 1 Practice Test Page 59 Question 9

The graph of $y = f(x)$ is stretched horizontally about the y-axis by a factor of 2, stretched vertically by a factor of $\frac{1}{4}$, reflected in the x-axis, and translated 3 units to the left and 4 units up.



Chapter 1 Practice Test Page 59 Question 10

a)



b) The image points of an inverse are determined by interchanging the x- and y-coordinates of the key points.

c) Any invariant points lie on the line of reflection: (–1, –1).

Chapter 1 Practice Test Page 59 Question 11

$y = 5x + 2$

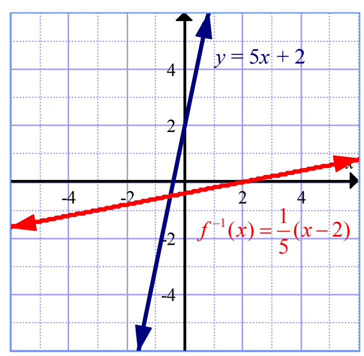
$x = 5y + 2$

$x - 2 = 5y$

$y = \frac{1}{5}(x - 2)$

$f^{-1}(x) = \frac{1}{5}(x - 2)$

Since the graphs are reflections of each other in the line $y = x$, the functions are inverses of each other.



Chapter 1 Practice Test Page 59 Question 12

For a horizontal stretch about the y-axis by a factor of 2, a horizontal reflection in the y-axis, a vertical stretch about the x-axis by a factor of 3, and a horizontal translation of 2 units to the right, $a = 3$, $b = -\frac{1}{2}$, $h = 2$, and $k = 0$. The equation of the transformed function is $y = 3f\left(-\frac{1}{2}(x-2)\right)$.

Chapter 1 Practice Test Page 59 Question 13

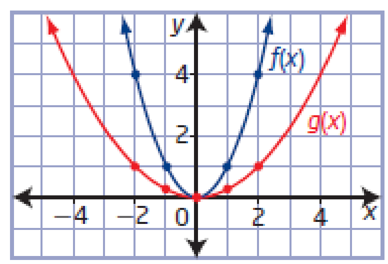
a) For $g(x) = f(x + 2) - 7$, $h = -2$ and $k = -7$. The graph of $f(x) = |x|$ will be translated 2 units to the left and 7 units down.

b) The equation of the transformed function is $f(x) = |x + 2| - 7$.

c) The minimum value of $g(x)$ occurs at the vertex of the V-shaped graph, (–2, –7).

d) Invariant points are points that are not affected by a transformation. In this case, the points on the graph of $f(x)$ are related to the corresponding points on the graph of $g(x)$ by the mapping $(x, y) \rightarrow (x - 2, y - 7)$. So, while the domains are the same, each point is affected by the transformation, so there are no invariant points.

Chapter 1 Practice Test Page 59 Question 14



a) The equation of the base function is $f(x) = x^2$.

b) For $g(x) = af(x)$, determine the pattern in the y-coordinates of corresponding points: $a = \frac{1}{4}$. So, the equation of the transformed function is $g(x) = \frac{1}{4}f(x)$, which is a vertical stretch about the x-axis by a factor of $\frac{1}{4}$.

c) For $g(x) = f(bx)$, determine the pattern in the x-coordinates of corresponding points: $b = 2$. So, the equation of the transformed function is $g(x) = f\left(\frac{1}{2}x\right)$, which is a horizontal stretch about the y-axis by a factor of 2.

d) $g(x) = \frac{1}{4}f(x)$

$g(x) = \frac{1}{4}x^2$

$g(x) = f\left(\frac{1}{2}x\right)$

$g(x) = \left(\frac{1}{2}x\right)^2$

$g(x) = \frac{1}{4}x^2$

The two forms are equivalent.

Chapter 1 Practice Test Page 59 Question 15

a) If the graph of $h(x)$ passes the horizontal line test, then the inverse of $h(x)$ will be a function. A horizontal line would pass through the quadratic function $h(x)$ twice, so its inverse is not a function.

b) $h(x) = -(x + 3)^2 - 5$

$$y = -(x + 3)^2 - 5$$
$$x = -(y + 3)^2 - 5$$
$$\pm\sqrt{-x-5} = y + 3$$
$$y = \pm\sqrt{-x-5} - 3$$

c) Example: A restricted domain for which the function has an inverse that is also a function is right half of the parabola.
For $h(x) = -(x + 3)^2 - 5$, the vertex is located at $(-3, -5)$. So, restrict the domain to $\{x \mid x \geq -3, x \in \mathrm{R}\}$.