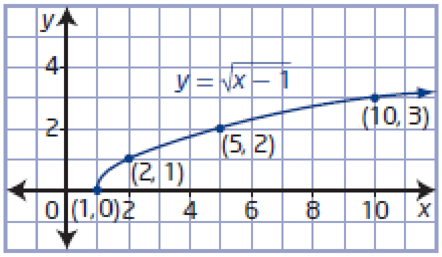
Chapter 2 Radical Functions

Section 2.1 Radical Functions and Transformations

Section 2.1 Page 72 Question 1

a)

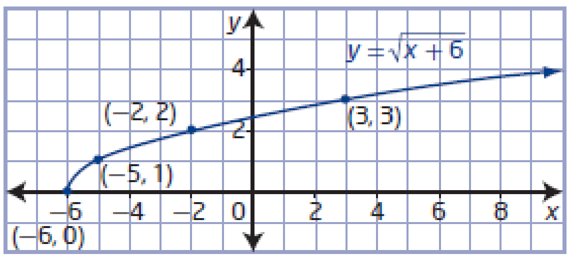
x	$y = \sqrt{x-1}$
1	0
2	1
5	2
10	3



domain $\{x \mid x \geq 1, x \in \mathrm{R}\}$
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

b)

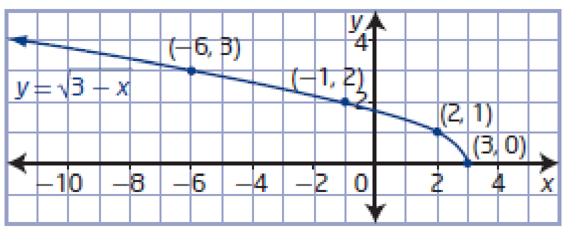
x	$y = \sqrt{x+6}$
–6	0
–5	1
–2	2
3	3



domain
$\{x \mid x \geq -6, x \in \mathrm{R}\}$
range
$\{y \mid y \geq 0, y \in \mathrm{R}\}$

c)

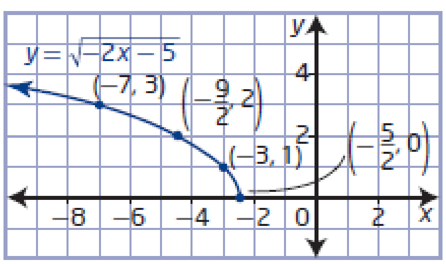
x	$y = \sqrt{3-x}$
3	0
2	1
–1	2
–6	3



domain
$\{x \mid x \leq 3, x \in \mathrm{R}\}$
range
$\{y \mid y \geq 0, y \in \mathrm{R}\}$

d)

x	$y = \sqrt{-2x-5}$
3	0
2	1
–1	2
–6	3



domain
$\left\{x \mid x \leq -\frac{5}{2}, x \in \mathrm{R}\right\}$
range
$\{y \mid y \geq 0, y \in \mathrm{R}\}$

Section 2.1 Page 72 Question 2

a) For $y = 7\sqrt{x-9}$, $a = 7$, $b = 1$, $h = 9$, and $k = 0$. The graph of $y = \sqrt{x}$ is vertically stretched by a factor of 7 and translated 9 units to the right.
domain $\{x \mid x \geq 9, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

b) For $y = \sqrt{-x} + 8$, $a = 1$, $b = –1$, $h = 0$, and $k = 8$. The graph of $y = \sqrt{x}$ is reflected in the y-axis and translated 8 units up.
domain $\{x \mid x \le 0, x \in R\}$, range $\{y \mid y \ge 8, y \in R\}$

c) For $y = -\sqrt{0.2x}$, $a = –1$, $b = 0.2$, $h = 0$, and $k = 0$. The graph of $y = \sqrt{x}$ is reflected in the x-axis and stretched horizontally by a factor of 5.
domain $\{x \mid x \ge 0, x \in R\}$, range $\{y \mid y \le 0, y \in R\}$

d) For $4 + y = \frac{1}{3}\sqrt{x+6}$, $a = \frac{1}{3}$, $b = 1$, $h = –6$, and $k = –4$. The graph of $y = \sqrt{x}$ is vertically stretched by a factor of $\frac{1}{3}$ and translated 6 units to the left and 4 units down.
domain $\{x \mid x \ge –6, x \in R\}$, range $\{y \mid y \ge 4, y \in R\}$

Section 2.1 Page 72 Question 3

a) For $y = \sqrt{x} - 2$, $a = 1$, $b = 1$, $h = 0$, and $k = –2$. The graph of $y = \sqrt{x}$ is translated 2 units down: graph **B**.

b) For $y = \sqrt{-x} + 2$, $a = 1$, $b = –1$, $h = 0$, and $k = 2$. The graph of $y = \sqrt{x}$ is reflected in the y-axis and translated 2 units up: graph **A**.

c) For $y = -\sqrt{x+2}$, $a = –1$, $b = 1$, $h = –2$, and $k = 0$. The graph of $y = \sqrt{x}$ is reflected in the x-axis and translated 2 units to the left: graph **D**.

d) For $y = -\sqrt{-(x-2)}$, $a = –1$, $b = –1$, $h = 2$, and $k = 0$. The graph of $y = \sqrt{x}$ is reflected in the x-axis and in the y-axis and translated 2 units to the right: graph **C**.

Section 2.1 Page 73 Question 4

a) For a vertical stretch by a factor of 4 and a horizontal translation of 6 units left, $a = 4$, $h = –6$, and the equation of the transformed function is $y = 4\sqrt{x+6}$.

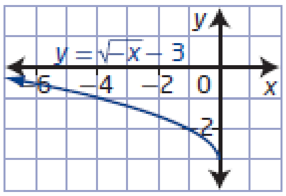
b) For a horizontal stretch by a factor of $\frac{1}{8}$ and a vertical translation of 5 units down, $b = 8$, $k = –5$, and the equation of the transformed function is $y = \sqrt{8x} - 5$.

c) For a horizontal reflection in the y-axis, a horizontal translation of 4 units right, and a vertical translation of 11 units up, $b = –1$, $h = 4$, $k = 11$, and the equation of the transformed function is $y = \sqrt{-(x-4)} + 11$.

d) For a vertical stretch by a factor of 0.25, a vertical reflection in the x-axis, and a horizontal stretch by a factor of 10, $a = -0.25$, $b = \frac{1}{10}$, and the equation of the transformed function is $y = -0.25\sqrt{\frac{1}{10}x}$.

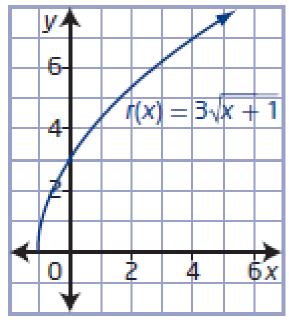
Section 2.1 Page 73 Question 5

a)



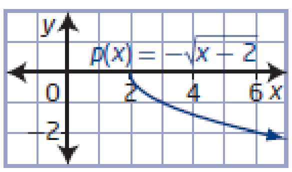
domain $\{x \mid x \le 0, x \in \mathrm{R}\}$,
range $\{y \mid y \ge -3, y \in \mathrm{R}\}$

b)



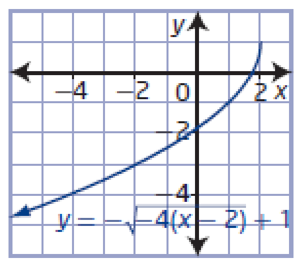
domain $\{x \mid x \ge -1, x \in \mathrm{R}\}$,
range $\{y \mid y \ge 0, y \in \mathrm{R}\}$

c)



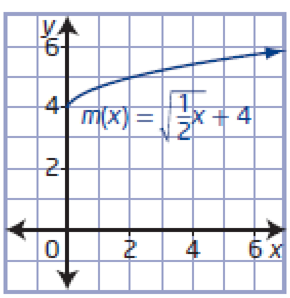
domain $\{x \mid x \ge 2, x \in \mathrm{R}\}$,
range $\{y \mid y \le 0, y \in \mathrm{R}\}$

d)



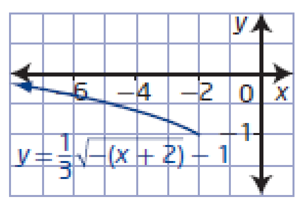
domain $\{x \mid x \le 2, x \in \mathrm{R}\}$,
range $\{y \mid y \le 1, y \in \mathrm{R}\}$

e)



domain $\{x \mid x \ge 0, x \in \mathrm{R}\}$,
range $\{y \mid y \ge 4, y \in \mathrm{R}\}$

f)



domain $\{x \mid x \le -2, x \in \mathrm{R}\}$,
range $\{y \mid y \ge -1, y \in \mathrm{R}\}$

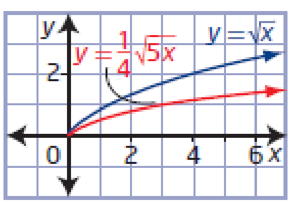
Section 2.1 Page 73 Question 6

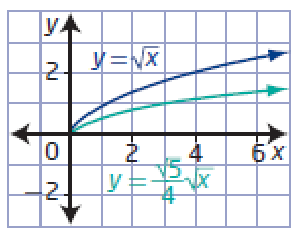
a) For $f(x) = \frac{1}{4}\sqrt{5x}$, $a = \frac{1}{4}$ and $b = 5$. The graph of $y = \sqrt{x}$ is vertically stretched by a factor of $\frac{1}{4}$ and horizontally stretched by a factor of $\frac{1}{5}$.

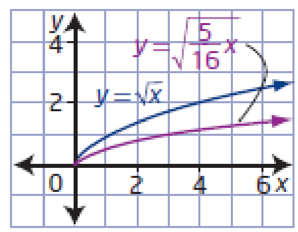
b) Two functions equivalent to $f(x)$: $y = \frac{\sqrt{5}}{4}\sqrt{x}$ and $y = \sqrt{\frac{5}{16}x}$.

c) The function $y = \frac{\sqrt{5}}{4}\sqrt{x}$ represents a vertical stretch by a factor of $\frac{\sqrt{5}}{4}$, and the function $y = \sqrt{\frac{5}{16}x}$ represents a horizontal stretch by a factor of $\frac{16}{5}$.

d) The graphs are the same.







Section 2.1 Page 73 Question 7

a) The radius, r, of a circle as a function of area, A:

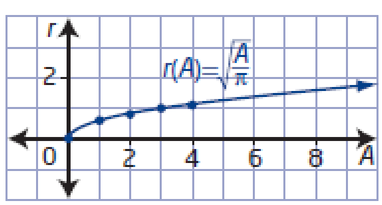
$$A = \pi r^2$$

$$\frac{A}{\pi} = r^2$$

$$r = \sqrt{\frac{A}{\pi}}$$

b)

A	r
0	0
1	0.6
2	0.8
3	1.0
4	1.1



Section 2.1 Page 73 Question 8

a) For $d = \sqrt{1.50h}$, $b = 1.50$. The graph of $d = \sqrt{h}$ is horizontally stretched by a factor of $\frac{1}{1.50}$ or $\frac{2}{3}$.

b) An approximate equivalent function of the form $d = a\sqrt{h}$ is $d \approx 1.22\sqrt{h}$.
Example: I prefer the original function because the values are exact.

c) Substitute $h = 20$ into $d = \sqrt{1.50h}$.
$d = \sqrt{1.50(20)}$
$d = 5.477\ldots$
The lifeguard can see approximately 5.5 miles.

Section 2.1 Page 73 Question 9

a) The function $y = -\sqrt{3x} + 4$ has domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$ and range $\{y \mid y \leq 4, y \in \mathrm{R}\}$.
After it is translated 9 units up and reflected in the x-axis, the domain remains as $\{x \mid x \geq 0, x \in \mathrm{R}\}$ but the range becomes $\{y \mid y \geq -13, y \in \mathrm{R}\}$.

b) Compared to the base function y = $\sqrt{x}$, the transformed function has not been translated horizontally but has been translated vertically 13 units down.

Section 2.1 Page 74 Question 10

a) Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph.
$(0, 0) \to (-3, 4)$
$(1, 1) \to (-2, 3)$
$(4, 2) \to (1, 2)$
$(9, 3) \to (6, 1)$
The overall width and height have not changed, so the graph has not been vertically or horizontally stretched. From the general shape, the graph has been reflected in the x-axis. From the endpoint, the graph has been translated 3 units to the left and 4 units up. So, $a = -1$, $b = 1$, $h = -3$, $k = 4$, and the equation of the transformed graph is $y = -\sqrt{x+3} + 4$.

b) Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph.
$(0, 0) \to (-5, -3)$
$(1, 1) \to (-1, -2)$
$(4, 2) \to (11, -1)$

The overall width has changed: the graph has been horizontally stretched by a factor of 4. From the general shape, the graph has not been reflected. From the endpoint, the graph has been translated 5 units to the left and 3 units down. So, $a = 1$, $b = \frac{1}{4}$, $h = –5$, $k = –3$, and the equation of the transformed graph is $y = \sqrt{\frac{1}{4}(x+5)} - 3$ or alternatively $y = \frac{1}{2}\sqrt{x+5} - 3$.

c) Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph.
(0, 0) → (5, –1)
(1, 1) → (4, 1)
(4, 2) → (1, 3)
(3, 9) → (–4, 5)
The overall height has changed: the graph has been vertically stretched by a factor of 2. From the general shape, the graph has been reflected in the y-axis. From the endpoint, the graph has been translated 5 units to the right and 1 unit down. So, $a = 2$, $b = –1$, $h = 5$, $k = –1$, and the equation of the transformed graph is $y = 2\sqrt{-(x-5)} - 1$ or alternatively $y = \sqrt{-4(x-5)} - 1$.

d) Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph.
(0, 0) → (4, 5)
(1, 1) → (3, 1)
(4, 2) → (0, –3)
(3, 9) → (–5, –7)
The overall height has changed: the graph has been vertically stretched by a factor of 4. From the general shape, the graph has been reflected in both the x-axis and the y-axis. From the endpoint, the graph has been translated 4 units to the right and 5 units up. So, $a = –4$, $b = –1$, $h = 4$, $k = 5$, and the equation of the transformed graph is $y = -4\sqrt{-(x-4)} + 5$ or alternatively $y = -\sqrt{-16(x-5)} - 1$.

Section 2.1 Page 74 Question 11

Compare each given domain and range to the those of the base function $y = \sqrt{x}$: domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
Examples:
a) For domain $\{x \mid x \geq 6, x \in \mathrm{R}\}$, the function has been translated 6 units to the right: $h = 6$. For range $\{y \mid y \geq 1, y \in \mathrm{R}\}$, the function has been translated 1 unit up: $k = 1$. Then, the equation of the radical function is $y = \sqrt{x-6} + 1$.

b) For domain $\{x \mid x \geq -7, x \in \mathrm{R}\}$, the function has been translated 7 units to the left: $h = -7$. For range $\{y \mid y \leq -9, y \in \mathrm{R}\}$, the function has been translated 9 units down and reflected in the x-axis: $k = -9$ and $a = -1$. Then, the equation of the radical function is $y = -\sqrt{x+7} - 9$.

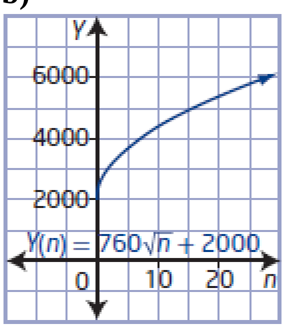
c) For domain $\{x \mid x \leq 4, x \in \mathrm{R}\}$, the function has been translated 4 units to the right and reflected in the y-axis: $h = 4$ and $b = -1$. For range $\{y \mid y \geq -3, y \in \mathrm{R}\}$, the function has been translated 3 units down: $k = -3$. Then, the equation of the radical function is $y = \sqrt{-(x-4)} - 3$.

d) For domain $\{x \mid x \leq -5, x \in \mathrm{R}\}$, the function has been translated 5 units to the left and reflected in the y-axis: $h = -5$ and $b = -1$. For range $\{y \mid y \leq 8, y \in \mathrm{R}\}$, the function has been translated 8 units up and reflected in the x-axis: $k = 8$ and $a = -1$. Then, the equation of the radical function is $y = -\sqrt{-(x+5)} + 8$.

Section 2.1 Page 74 Question 12

a) For $Y(n) = 760\sqrt{n} + 2000$, $a = 760$, $b = 1$, $h = 0$, and $k = 2000$. The graph will be vertically stretched by a factor of 760 and translated 2000 units up.

b)



c) domain $\{n \mid n \geq 0, n \in \mathrm{R}\}$, range $\{Y \mid Y \geq 2000, Y \in \mathrm{R}\}$

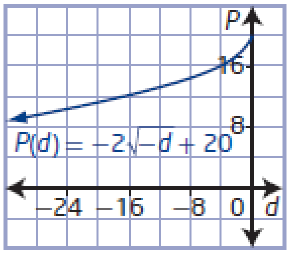
d) Example: The minimum yield is 2000 kg/hectare. The domain and range imply that the more nitrogen added, the greater the yield without end. This is not realistic.

Section 2.1 Page 75 Question 13

a) For $P(d) = -2\sqrt{-d} + 20$, the domain is $\{d \mid -100 \leq d \leq 0, d \in \mathrm{R}\}$ and the range is $\{P \mid 0 \leq P \leq 20, n \in \mathrm{R}\}$. The domain is negative indicating days remaining before the phone's release, and the maximum value of P, the number of pre-orders, is 20 million.

b) For $P(d) = -2\sqrt{-d} + 20$, $a = -2$, $b = -1$, $h = 0$, and $k = 20$. The graph will be vertically stretched by a factor of 2, reflected in the d-axis, reflected in the P-axis, and translated 20 units up.

c)



Since d represents the number of days remaining before release, d is negative. The function has a maximum of 20 million pre-orders.

d) Substitute $d = –30$.

$P(d) = -2\sqrt{-(-30)} + 20$

$P(d) = 9.045\ldots$

The manufacturer can expect to have approximately 9.05 million pre-orders.

Section 2.1 Page 75 Question 14

a) The graph shows that the polling error decreases as the election approaches.

b) Since the graph starts on the origin, there are no translations: $h = 0$ an $k = 0$. Since the graph is reflected in the y-axis, $b = –1$. So, the equation is of the form $y = a\sqrt{-x}$. Use the coordinates of a point, say (–150, 6) to determine the value of a.

$6 = a\sqrt{-(-150)}$

$a = \dfrac{6}{\sqrt{150}}$

$a \approx 0.49$

c) The function has been stretched vertically by a factor of approximately 0.49 and reflected in the y-axis.

Section 2.1 Page 75 Question 15

Since the graph is reflected in the y-axis, $b = –1$. So, the equation is of the form $y = a\sqrt{-x}$. Use the coordinates of a point, say (–5.4, 4.8) to determine the value of a.

$4.8 = a\sqrt{-(-5.4)}$

$a = \dfrac{4.8}{\sqrt{5.4}}$

$a \approx 2.07$

An equation to represent the shape of the greenhouse roof is $y = 2.07\sqrt{-x}$.

Section 2.1 Page 75 Question 16

Examples:

a) Since the endpoint is (2, 5), $h = 2$ and $k = 5$.

Try an equation of the form $y = a\sqrt{x-2}+5$. Use the coordinates of the point (6, 1) to determine the value of a.

$$1 = a\sqrt{6-2}+5$$
$$-4 = 2a$$
$$a = -2$$

The equation of the function is $y = -2\sqrt{x-2}+5$.

b) Since the endpoint is (3, –2), $h = 3$ and $k = –2$.

Try an equation of the form $y = a\sqrt{x-3}-2$. Use the coordinates of the point (–6, 0) to determine the value of a.

$$0 = a\sqrt{-6-3}-2$$
$$2 = a\sqrt{-9}$$

This is not possible as you cannot have a negative radicand.

Try an equation of the form $y = \sqrt{b(x-3)}-2$. Use the coordinates of the point (–6, 0) to determine the value of b.

$$0 = \sqrt{b(-6-3)}-2$$
$$2 = \sqrt{-9b}$$
$$4 = -9b$$
$$b = -\frac{4}{9}$$

The equation of the function is $y = \sqrt{-\frac{4}{9}(x-3)}-2$ or $y = \frac{2}{3}\sqrt{(3-x)}-2$.

Section 2.1 Page 76 Question 17

a) China, India, and USA might feel that the "one nation, one vote" system is unfair. The larger the country, the more unfair the this system becomes. Tuvalu, Nauru, Vatican City might feel that the "one person, one vote" system is unfair. The smaller the nation, the more unfair this system becomes.

b) For each country, calculate $\frac{\text{Population}}{\text{World Population}} \times 100\%$.

Nation	Percentage
China	18.6%
India	17.1%
US	4.5%
Canada	0.48%
Tuvalu	0.000 151%
Nauru	0.000 137%
Vatican City	0.000 014%

c) The Penrose system can be represented by the function $V(x) = \frac{1}{1000}\sqrt{x}$.

d) For each country, calculate $\frac{V(x)}{765} \times 100\%$.

Nation	Percentage
China	4.82%
India	4.62%
US	2.36%
Canada	0.77%
Tuvalu	0.014%
Nauru	0.013%
Vatican City	0.004%

e) Example: The Penrose system gives larger nations votes based on population but also provides an opportunity for smaller nations to provide influence.

Section 2.1 Page 76 Question 18

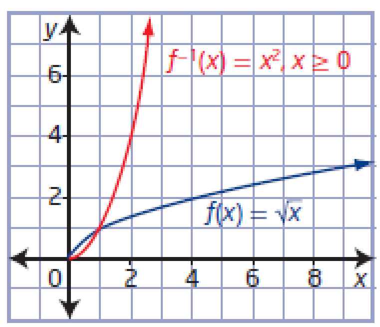
Answers may vary.
Step 2 and Step 3 In general, the periods of the pendulums should decrease as the length of the string decreases.

Step 4 In general, the graph should resemble the graph of $y = a\sqrt{x}$.

Step 5 The graph should be a vertical stretch by a factor of approximately 0.2 of the graph of $y = \sqrt{x}$. A function that approximates the graph is $T = 0.2\sqrt{L}$.

Section 2.1 Page 77 Question 19

a) The domain and range of a function become the range and domain, respectively, of the inverse of the function. The positive domain of the inverse is the same as the range of the original function.



b) i) $g(x) = -\sqrt{x-5}$, $x \geq 5$

$$y = -\sqrt{x-5}$$
$$x = -\sqrt{y-5}$$
$$x^2 = y - 5$$
$$y = x^2 + 5$$
$$g^{-1}(x) = x^2 + 5, x \leq 0$$

ii) $h(x) = \sqrt{-x} + 3$, $x \leq 0$

$$y = \sqrt{-x} + 3$$
$$x = \sqrt{-y} + 3$$
$$x - 3 = \sqrt{-y}$$
$$(x - 3)^2 = -y$$
$$y = -(x - 3)^2$$
$$h^{-1}(x) = -(x - 3)^2, x \geq 3$$

iii) $j(x) = \sqrt{2x-7} - 6$, $x \geq \frac{7}{2}$

$$y = \sqrt{2x-7} - 6$$
$$x = \sqrt{2y-7} - 6$$
$$x + 6 = \sqrt{2y-7}$$
$$(x + 6)^2 = 2y - 7$$
$$y = \frac{1}{2}(x + 6)^2 + \frac{7}{2}$$
$$j^{-1}(x) = \frac{1}{2}(x + 6)^2 + \frac{7}{2}, x \geq -6$$

Section 2.1 Page 77 Question 20

For $f(x) = \frac{5}{8}\sqrt{-\frac{7}{12}x}$, the key points on the graph of $y = \sqrt{x}$ are changed by the mapping $(x, y) \to \left(-\frac{12}{7}x, \frac{5}{8}y\right)$.

For $g(x) = -\frac{2}{5}\sqrt{6(x+3)} - 4$, the key points on the graph of $y = \sqrt{x}$ are changed by the mapping $(x, y) \to \left(\frac{1}{6}x - 3, -\frac{2}{5}y - 4\right)$.

To transform the graph of $f(x)$ to create the graph of $g(x)$, determine the mapping needed to transform points $\left(-\frac{12}{7}x, \frac{5}{8}y\right)$ to $\left(\frac{1}{6}x - 3, -\frac{2}{5}y - 4\right)$.

$$-\frac{7}{72}\left(-\frac{12}{7}x\right) - 3$$
$$= \frac{1}{6}x - 3$$

Apply a horizontal stretch by a factor of $\frac{7}{72}$, a reflection in the y-axis, and a translation of 3 units to the left.

$$-\frac{16}{25}\left(\frac{5}{8}y\right) - 4$$
$$= -\frac{2}{5}y - 4$$

Apply a vertical stretch by a factor of $\frac{16}{25}$, a reflection in the x-axis, and a translation of 4 units down.

Section 2.1 Page 77 Question C1

For a radical function $y = a\sqrt{b(x-h)} + k$, the parameters *b* and *h* affect the domain. The radicand must be greater than or equal to zero.

Case 1: $b > 0$	Case 2: $b < 0$
$b(x-h) \geq 0$	$b(x-h) \geq 0$
$x - h \geq 0$	$x - h \leq 0$
$x \geq h$	$x \leq h$

For example, $y = \sqrt{x}$ has domain $x \geq 0$ but $y = \sqrt{2(x-3)}$ has domain $x \geq 3$ and $y = \sqrt{-2(x-3)}$ has domain $x \leq 3$.

For a radical function $y = a\sqrt{b(x-h)} + k$, the parameters *a* and *k* affect the range. The graph of a radical function starts with an endpoint whose value of *x* makes $b(x-h) = 0$. So, the range restriction value is determined by the value of *k* and the direction of the inequality symbol is determined by the value of *a*.
For example, $y = \sqrt{x}$ has range $y \geq 0$ but $y = 2\sqrt{x} - 4$ has range $y \geq -4$ and $y = -2\sqrt{x} - 4$ has range $y \leq -4$.

Section 2.1 Page 77 Question C2

Yes, any given radical function can be simplified so that there is no value of *b*, only a value of *a*.

$y = a\sqrt{bx}$

$y = \left(a\sqrt{b}\right)\sqrt{x}$

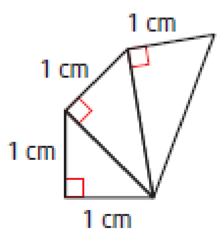
For example, $y = 2\sqrt{9x}$ can be simplified to $y = 6\sqrt{x}$.

Section 2.1 Page 77 Question C3

The processes are similar because the parameters *a*, *b*, *h*, and *k* have the same effect on radical functions and quadratic functions. The processes are different because the base functions are different: one is the shape of a parabola and the other is the shape of half of a parabola.

Section 2.1 Page 77 Question C4

Steps 1–4



Triangle Number, *n*	Length of Hypotenuse, *L*
First	$\sqrt{2} = 1.414...$
Second	$\sqrt{3} = 1.732...$
Third	$\sqrt{4} = 2$

Step 5 The function that represents the hypotenuse length as it relates to its triangle number is $L = \sqrt{n+1}$. This represents the translation of 1 unit to the left of the base square root function.

Section 2.2 Square Root of a Function

Section 2.2 Page 86 Question 1

$f(x)$	$\sqrt{f(x)}$
36	6
0.09	0.3
1	1
–9	undefined
2.56	1.6
0	0

Section 2.2 Page 86 Question 2

a) For (4, 12), the corresponding point on the graph of $y = \sqrt{f(x)}$ is $\left(4, \sqrt{12}\right)$ or approximately (4, 3.46).

b) For (–2, 0.4), the corresponding point on the graph of $y = \sqrt{f(x)}$ is $\left(-2, \sqrt{0.4}\right)$ or approximately (–2, 0.63).

c) For (10, –2), there is no corresponding point on the graph of $y = \sqrt{f(x)}$ because $\sqrt{-2}$ does not exist.

d) For (0.09, 1), the corresponding point on the graph of $y = \sqrt{f(x)}$ is $\left(0.09, \sqrt{1}\right)$ or (0.09, 1).

e) For (–5, 0), the corresponding point on the graph of $y = \sqrt{f(x)}$ is $\left(-5, \sqrt{0}\right)$ or (–5, 0).

f) For (m, n), the corresponding point on the graph of $y = \sqrt{f(x)}$ is $\left(m, \sqrt{n}\right)$ for $n \geq 0$.

Section 2.2 Page 86 Question 3

a) Since the graph of $y = f(x)$ is on or above the x-axis for $x \leq -2$ or $x \geq 2$, the graph of $y = \sqrt{f(x)}$ exists only for these values: graph **C**.

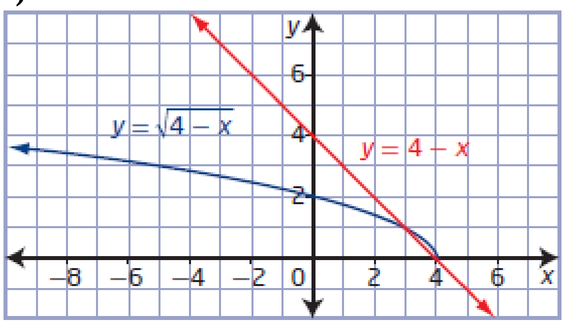
b) Since the graph of $y = f(x)$ is on or above the x-axis for $-2 \le x \le 2$, the graph of $y = \sqrt{f(x)}$ exists only for these values: graph **D**.

c) Since the graph of $y = f(x)$ is above the x-axis for all values of x, the graph of $y = \sqrt{f(x)}$ exists only for these values: graph **A**.

d) Since the graph of $y = f(x)$ is on or above the x-axis for $-4 \le x \le 4$, the graph of $y = \sqrt{f(x)}$ exists only for these values: graph **B**.

Section 2.2 Page 87 Question 4

a)

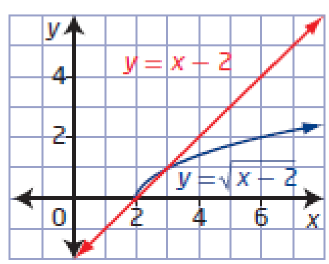


b) The graph of $y = \sqrt{4-x}$ exists for values where the graph of $y = 4 - x$ is on or above the x-axis, $x \le 4$. The graph of $y = \sqrt{4-x}$ is above the graph of $y = 4 - x$ for $3 < x < 4$, and the graph of $y = \sqrt{4-x}$ is below the graph of $y = 4 - x$ for $x > 3$.

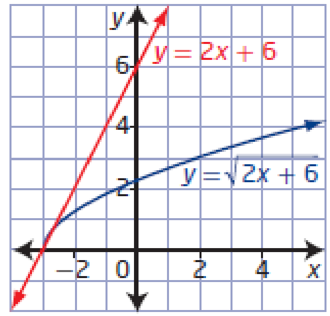
c) For $y = 4 - x$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{4-x}$, the domain is $\{x \mid x \le 4, x \in \mathrm{R}\}$ and the range is $\{y \mid y \ge 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{4-x}$ is undefined for $x > 4$.
The ranges differ because $y = \sqrt{4-x}$ is undefined for $y < 0$.

Section 2.2 Page 87 Question 5

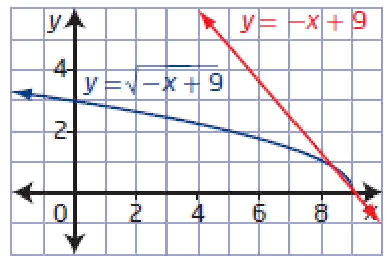
a) For $y = x - 2$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{x-2}$, the domain is $\{x \mid x \ge 2, x \in \mathrm{R}\}$ and the range is $\{y \mid y \ge 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{x-2}$ is undefined for $x < 2$.
The ranges differ because $y = \sqrt{x-2}$ is undefined for $y < 0$.



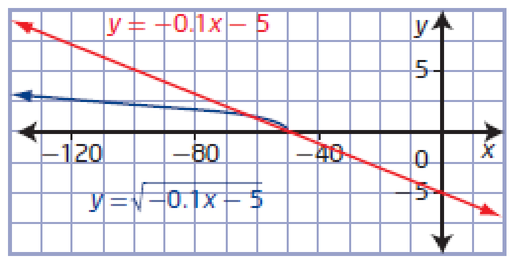
b) For $y = 2x + 6$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{2x+6}$, the domain is $\{x \mid x \geq -3, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{2x+6}$ is undefined for $x < -3$. The ranges differ because $y = \sqrt{2x+6}$ is undefined for $y < 0$.



c) For $y = -x + 9$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{-x+9}$, the domain is $\{x \mid x \leq 9, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{-x+9}$ is undefined for $x > 9$. The ranges differ because $y = \sqrt{-x+9}$ is undefined for $y < 0$.



d) For $y = -0.1x - 5$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{-0.1x-5}$, the domain is $\{x \mid x \leq 50, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{-0.1x-5}$ is undefined for $x > 50$. The ranges differ because $y = \sqrt{-0.1x-5}$ is undefined for $y < 0$.



Section 2.2 Page 87 Question 6

a) For $y = x^2 - 9$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -9, y \in \mathrm{R}\}$.
For $y = \sqrt{x^2 - 9}$, the domain is $\{x \mid x \leq -3 \text{ and } x \geq 3, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{x^2 - 9}$ is undefined for $-3 < x < 3$. The ranges differ because $y = \sqrt{x^2 - 9}$ is undefined for $y < 0$.

b) For $y = 2 - x^2$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \leq 2, y \in \mathrm{R}\}$.
For $y = \sqrt{2 - x^2}$, the domain is $\{x \mid -\sqrt{2} \leq x \leq \sqrt{2}, x \in \mathrm{R}\}$ and the range is $\{y \mid 0 \leq y \leq \sqrt{2}, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{2 - x^2}$ is undefined for $x < -\sqrt{2}$ and $x > \sqrt{2}$.
The ranges differ because $y = \sqrt{2 - x^2}$ is undefined for $y < 0$ and has a maximum value of $\sqrt{2}$.

c) For $y = x^2 + 6$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 6, y \in \mathrm{R}\}$.
For $y = \sqrt{x^2 + 6}$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq \sqrt{6}, y \in \mathrm{R}\}$.
The domains are the same because the entire graph of $y = x^2 + 6$ is above the x-axis.
The ranges differ because $y = \sqrt{x^2 + 6}$ has a minimum value of $\sqrt{6}$.

d) For $y = 0.5x^2 + 3$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 3, y \in \mathrm{R}\}$.
For $y = \sqrt{0.5x^2 + 3}$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq \sqrt{3}, y \in \mathrm{R}\}$.
The domains are the same because the entire graph of $y = 0.5x^2 + 3$ is above the x-axis.
The ranges differ because $y = \sqrt{0.5x^2 + 3}$ has a minimum value of $\sqrt{3}$.

Section 2.2 Page 87 Question 7

a) For $y = x^2 - 25$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -25, y \in \mathrm{R}\}$.
For $y = \sqrt{x^2 - 25}$, the domain is $\{x \mid x \leq -5 \text{ and } x \geq 5, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{x^2 - 25}$ is undefined for $-5 < x < 5$. The ranges differ because $y = \sqrt{x^2 - 9}$ is undefined for $y < 0$.

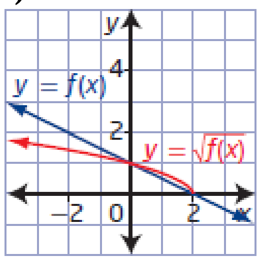
b) For $y = x^2 + 3$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 3, y \in \mathrm{R}\}$.
For $y = \sqrt{x^2 + 3}$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq \sqrt{3}, y \in \mathrm{R}\}$.
The domains are the same because the entire graph of $y = x^2 + 3$ is above the x-axis.
The ranges differ because $y = \sqrt{x^2 + 3}$ has a minimum value of $\sqrt{3}$.

c) For $y = 32 - 2x^2$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \leq 32, y \in \mathrm{R}\}$.
For $y = \sqrt{32 - 2x^2}$, the domain is $\{x \mid -4 \leq x \leq 4, x \in \mathrm{R}\}$ and the range is $\{y \mid 0 \leq y \leq \sqrt{32}, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{2 - x^2}$ is undefined for $x < -\sqrt{2}$ and $x > \sqrt{2}$.
The ranges differ because $y = \sqrt{2 - x^2}$ is undefined for $y < 0$ and has a maximum value of $\sqrt{2}$.

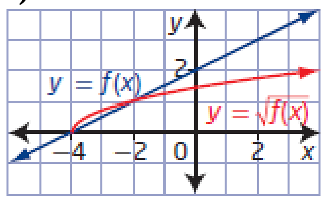
d) For $y = 5x^2 + 50$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 50, y \in \mathrm{R}\}$.
For $y = \sqrt{5x^2 + 50}$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq \sqrt{50}, y \in \mathrm{R}\}$.
The domains are the same because the entire graph of $y = 5x^2 + 50$ is above the x-axis.
The ranges differ because $y = \sqrt{5x^2 + 50}$ has a minimum value of $\sqrt{50}$.

Section 2.2 Page 87 Question 8

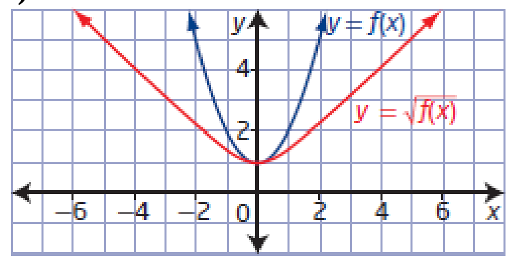
a)



b)



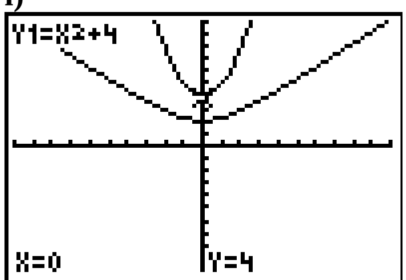
c)



Section 2.2 Page 87 Question 9

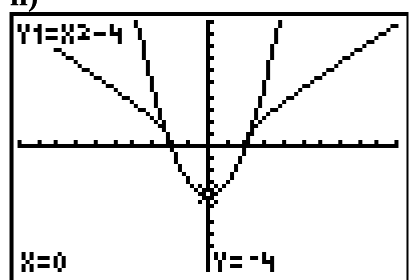
a) and **b)**

i)



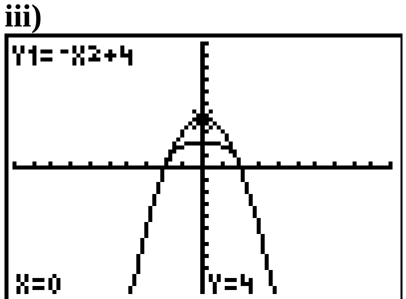
For $f(x) = x^2 + 4$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 4, y \in \mathrm{R}\}$.

ii)



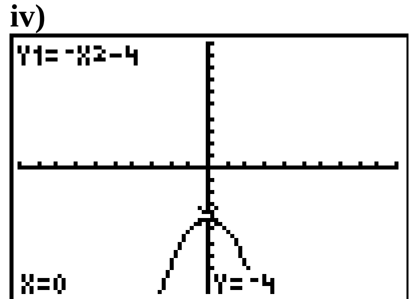
For $g(x) = x^2 - 4$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -4, y \in \mathrm{R}\}$.

iii)



For $h(x) = -x^2 + 4$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \leq 4, y \in \mathrm{R}\}$.

iv)



For $j(x) = -x^2 - 4$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \leq -4, y \in \mathrm{R}\}$.

c) The graph of $y = \sqrt{j(x)}$ does not exist. This is because the entire graph of $y = j(x)$ lies below the x-axis. For $j(x) = -x^2 - 4$, there are no values of x for which $\sqrt{-x^2 - 4}$ is defined.

$$-x^2 - 4 \geq 0$$
$$-x^2 \geq 4$$
$$x^2 \leq -4$$

This is never true.

d) The domains of the square root of a function are the same as the domains of the function when the value of the function is greater than or equal to zero. The domains of the square root of a function do not exist when the value of the function is less than zero. The ranges of the square root of a function are the square root of the range of the original function, except when the value of the function is less than zero then the range is undefined.

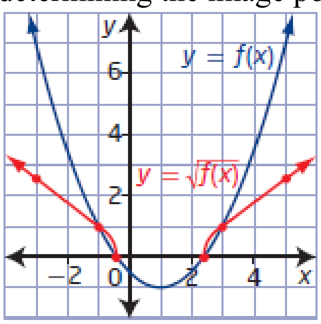
Section 2.2 Page 87 Question 10

a) For $y = x^2 - 4$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -4, y \in \mathrm{R}\}$.
For $y = \sqrt{x^2 - 4}$, the domain is $\{x \mid x \leq -2 \text{ and } x \geq 2, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

b) The function $y = \sqrt{x^2 - 4}$ is undefined for $-2 < x < 2$ because $y = x^2 - 4$ is less than zero. So, the domain does not contain this interval.

Section 2.2 Page 87 Question 11

a) I sketched the graph by locating key points, including invariant points, and determining the image points on the graph of the square root of the function.



b) For $y = f(x)$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -1, y \in \mathrm{R}\}$.
Determine the equation of the parabola: $y = \frac{1}{2}(x - 1)^2 - 1$ or $y = \frac{1}{2}x^2 - x - \frac{1}{2}$.
Use the quadratic formula or graphing technology to determine the zeros: $x = 1 \pm \sqrt{2}$.
For $y = \sqrt{f(x)}$, the domain is $\{x \mid x \leq 1 - \sqrt{2} \text{ and } x \geq 1 + \sqrt{2}, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{f(x)}$ is undefined for $1 - \sqrt{2} < x < 1 + \sqrt{2}$. The ranges differ because $y = \sqrt{f(x)}$ is undefined for $y < 0$.

Section 2.2 Page 88 Question 12

a) $d^2 = (h + 6378)^2 - 6378^2$
$d^2 = h^2 + 12\ 756h + 6378^2 - 6378^2$
$d^2 = h^2 + 12\ 756h$

$d = \sqrt{h^2 + 12\,756h}$

The equation for the distance, in kilometres, to the horizon is $d = \sqrt{h^2 + 12\,756h}$.

b) The domain is $\{h \mid h \geq 0, h \in \mathrm{R}\}$ and the range is $\{d \mid d \geq 0, d \in \mathrm{R}\}$.

c) You can use a graph to find the distance to the horizon for a satellite that is 800 km above Earth's surface by finding the *d*-coordinate of the point where the *h*-coordinate is 800. In this case, *d* is approximately 3293. The distance to the horizon for this satellite is approximately 3293 km.

d) If this function is not in context then the domain is $\{h \mid h \leq -12\,756 \text{ or } h \geq 0, h \in \mathrm{R}\}$ and the range remains as $\{d \mid d \geq 0, d \in \mathrm{R}\}$.

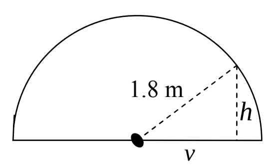
Section 2.2 Page 88 Question 13

a) No, I do not agree with Chris. The function $y = \sqrt{f(x)}$ is undefined for $f(x) < 0$, where $f(x)$ represents the range of $y = f(x)$, not the domain.

b) To determine whether a graph shows the function or the square root of the function, check for points below the *x*-axis. If there are such points, the graph is of the function. If there are not, then it could be a graph of the square root of the function.

Section 2.2 Page 88 Question 14

a) The radius of the iglu is 1.8 m. Any point on the iglu will be at the end of the hypotenuse of a right triangle, in which $r^2 = h^2 + v^2$. So, $v^2 = 1.8^2 - h^2$. A function that gives the vertical height in terms of the horizontal distance from the centre is $v = \sqrt{3.24 - h^2}$.



b) The domain is $\{h \mid 0 \leq h \leq 1.8, h \in \mathrm{R}\}$ and the range is $\{v \mid 0 \leq v \leq 1.8, v \in \mathrm{R}\}$. Both variables represent distances and must be non-negative.

c) 1 m in from the bottom edge of the wall: $h = 0.8$

$v = \sqrt{3.24 - 0.8^2}$

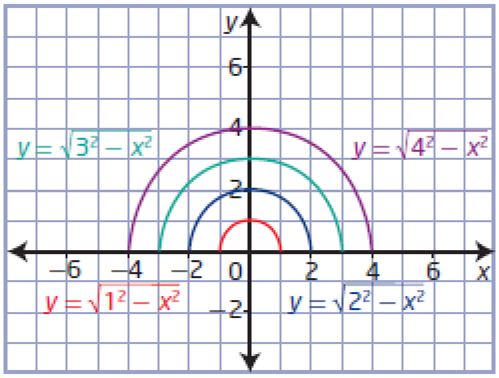
$v = \sqrt{2.6}$

$v = 1.612\ldots$

The height of the iglu at a point 1 m from the bottom edge is approximately 1.61 m.

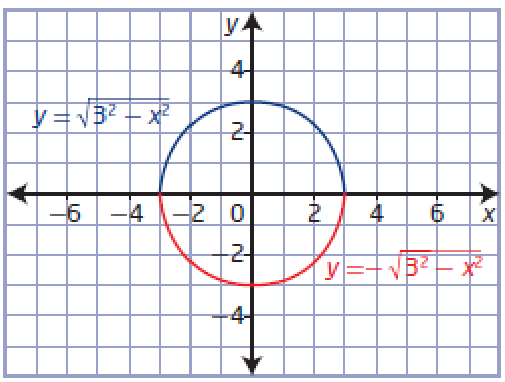
Section 2.2 Page 88 Question 15

Step 1 Example:



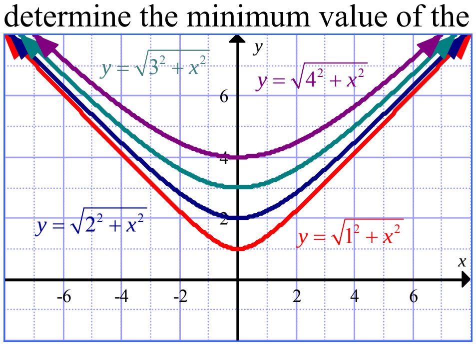
Step 2 The value of a determines the minimum value and the maximum value of the domain: $\{x \mid -a \le x \le a, x \in \mathrm{R}\}$. It also determines the maximum value of the range: $\{y \mid 0 \le y \le a, y \in \mathrm{R}\}$.

Step 3 Example: Choose $a = 3$. The equation for the function reflected in the x-axis is $y = -\sqrt{3^2 - x^2}$.

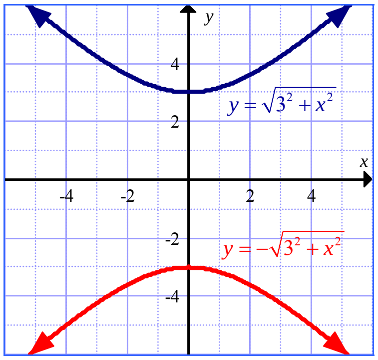


The graph forms a circle.

Step 4 Example: The value of a has no effect on the domain: $\{x \mid x \in \mathrm{R}\}$. It does determine the minimum value of the range: $\{y \mid y \ge a, y \in \mathrm{R}\}$.



Choose $a = 3$. The equation for the function reflected in the x-axis is $y = -\sqrt{3^2 + x^2}$.



Section 2.2 Page 89 Question 16

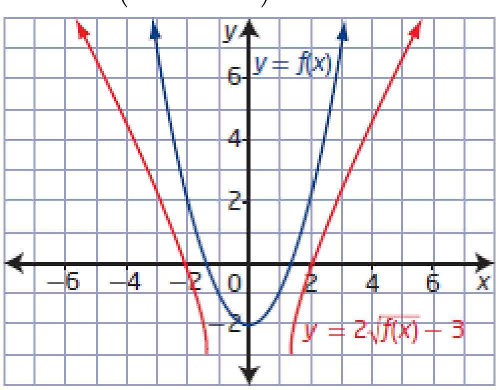
a) For $y = 4f(x + 3)$, $a = 4$, $b = 1$, $h = –3$, and $k = 0$. For $y = \sqrt{4f(x+3)}$, use the mapping $(x, y) \to \left(x-3, \sqrt{4y}\right)$. So, the point (–24, 12) becomes (–27, $\sqrt{48}$) or (–27, $4\sqrt{3}$).

b) For $y = f(4x)$, $a = 1$, $b = 4$, $h = 0$, and $k = 0$. For $y = -\sqrt{f(4x)} + 12$, use the mapping $(x, y) \to \left(\frac{1}{4}x, -\sqrt{y} + 12\right)$. So, the point (–24, 12) becomes (–6, $-\sqrt{12} + 12$) or (–27, $-2\sqrt{3} + 12$).

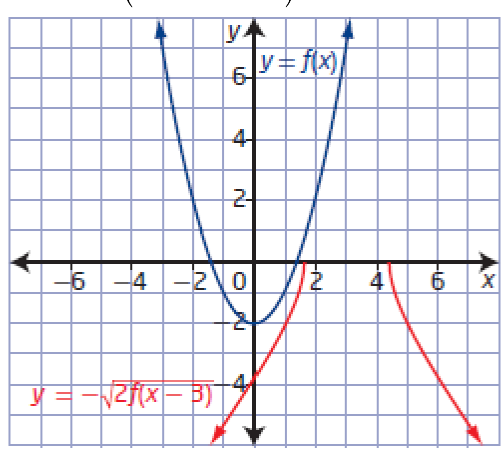
c) For $y = f(–(x – 2)) – 4$, $a = 1$, $b = –1$, $h = 2$, and $k = –4$. For $y = -2\sqrt{f(-(x-2)) - 4} + 6$, use the mapping $(x, y) \to \left(-x+2, -2\sqrt{y} + 6\right)$. So, the point (–24, 12) becomes (26, $-2\sqrt{12} + 6$) or (26, $-4\sqrt{3} + 6$).

Section 2.2 Page 89 Question 17

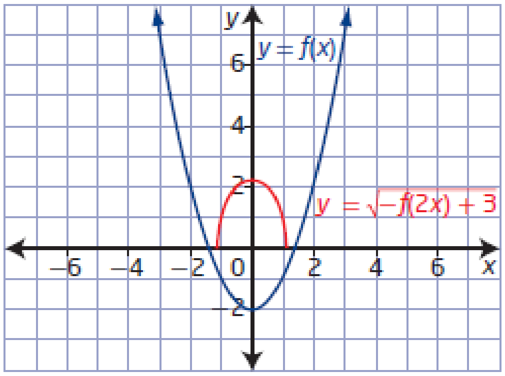
a) For $y = 2\sqrt{f(x)} - 3$, use the mapping $(x, y) \to \left(x, 2\sqrt{y} - 3\right)$.



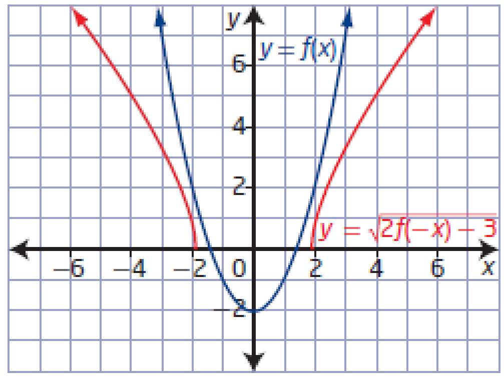
b) For $y = -\sqrt{2f(x-3)}$, use the mapping $(x, y) \to \left(x+3, -\sqrt{2y}\right)$.



c) For $y = \sqrt{-f(2x)+3}$, use the mapping $(x, y) \rightarrow \left(\frac{1}{2}x, \sqrt{-y+3}\right)$.



d) For $y = \sqrt{2f(-x)-3}$, use the mapping $(x, y) \rightarrow \left(-x, \sqrt{2y-3}\right)$.



Section 2.2 Page 89 Question 18

Example: First sketch the graph of $y = 2f(x - 3)$. Next, sketch the graph of $y = \sqrt{2f(x-3)}$. Finally, reflect in the x-axis to get the graph of $y = -\sqrt{2f(x-3)}$.

Section 2.2 Page 89 Question 19

a) A formula for radius as a function of the surface area, SA, of a cylinder with equal diameter and height, $h = 2r$, is

$$SA = 2\pi r^2 + 2\pi rh$$
$$SA = 2\pi r^2 + 2\pi r(2r)$$
$$SA = 6\pi r^2$$
$$\frac{SA}{6\pi} = r^2$$
$$r = \sqrt{\frac{SA}{6\pi}}$$

b) A formula for radius as a function of the surface area, SA, of a cone with height three times its diameter, $h = 6r$, is

$$SA = \pi r\sqrt{h^2 + r^2} + \pi r^2$$
$$SA = \pi r\sqrt{(6r)^2 + r^2} + \pi r^2$$
$$SA = \pi r\sqrt{37r^2} + \pi r^2$$
$$SA = \pi r^2\left(\sqrt{37} + 1\right)$$
$$\frac{SA}{\pi\left(\sqrt{37}+1\right)} = r^2$$

$$r = \sqrt{\frac{SA}{\pi\left(\sqrt{37}+1\right)}}$$

Section 2.2 Page 89 Question C1

Example: Choose key points on the graph of $y = f(x)$. Transform the points using the mapping $(x, y) \rightarrow \left(x, \sqrt{y}\right)$. Plot the new points and smooth out the graph.

Section 2.2 Page 89 Question C2

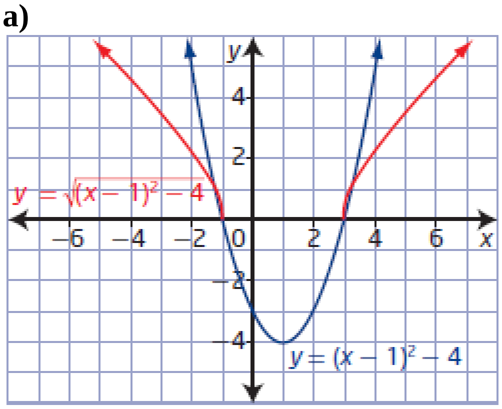
Example: The graph of $y = 16 - 4x$ is a line with slope -4, y-intercept 16, and x-intercept 4. The graph of $y = \sqrt{16-4x}$ only exists when the graph of $y = 16 - 4x$ is on or above the x-axis. The y-intercept is 4 and the x-intercept stays the same. For $x \le 4$, the points are related by the mapping $(x, y) \rightarrow \left(x, \sqrt{y}\right)$.

Section 2.2 Page 89 Question C3

Example: No, it is not possible to completely graph the function $y = f(x)$ given only the graph of $y = \sqrt{f(x)}$. The graph of $y = f(x)$ may exist when $y < 0$ but the graph of $y = \sqrt{f(x)}$ does not.

Section 2.2 Page 89 Question C4

a)



b) Example: The function $y = (x - 1)^2 - 4$ has domain $\{x \mid x \in \mathrm{R}\}$ and range $\{y \mid y \ge -4, y \in \mathrm{R}\}$.

The function $y = \sqrt{(x-1)^2 - 4}$ has domain $\{x \mid x \le -1 \text{ and } x \ge 3, x \in \mathrm{R}\}$ and range $\{y \mid y \ge 0, y \in \mathrm{R}\}$. The points on the graph of $y = (x - 1)^2 - 4$ that are on or above the x-axis become points $\left(x, \sqrt{y}\right)$ on the graph of $y = \sqrt{(x-1)^2 - 4}$.

Section 2.3 Solving Radical Equations Graphically

Section 2.3 Page 96 Question 1

a) The equation $2+\sqrt{x+4}=4$ can be solved by graphing the single function $y=-2+\sqrt{x+4}$: choice **B**.

b) The equation $x-4=\sqrt{x+4}$ can be solved by graphing the single function $y=x-4-\sqrt{x+4}$: choice **A**.

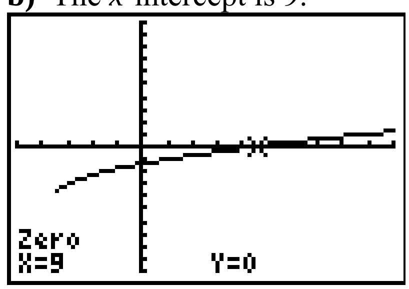
c) The equation $2=\sqrt{x+4}-4$ can be solved by graphing the single function $y=\sqrt{x+4}-6$: choice **D**.

d) The equation $\sqrt{x+4}+2=x+6$ can be solved by graphing the single function $y=\sqrt{x+4}-4-x$: choice **C**.

Section 2.3 Page 96 Question 2

a)
$$\begin{aligned}\sqrt{x+7}-4&=0\\ \sqrt{x+7}&=4\\ x+7&=16\\ x&=9\end{aligned}$$

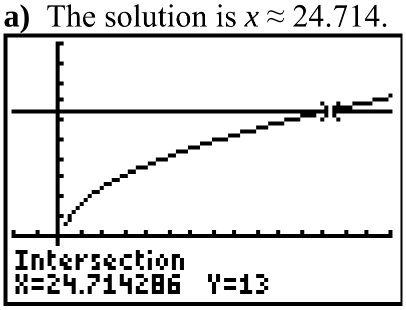
b) The x-intercept is 9.



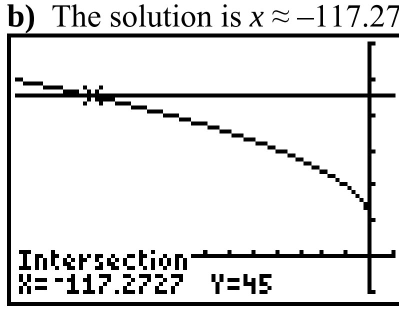
c) The solutions or roots of a radical equation are equivalent to the x-intercept(s) of the graph of the corresponding radical function.

Section 2.3 Page 96 Question 3

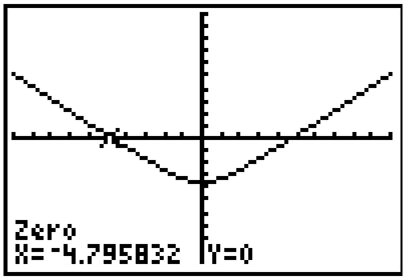
a) The solution is $x \approx 24.714$.

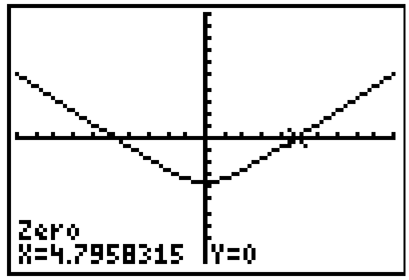


b) The solution is $x \approx -117.273$.

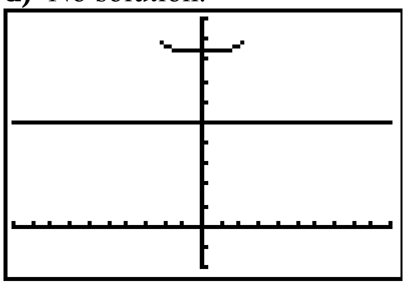


c) The solution is $x \approx \pm 4.796$.





d) No solution.



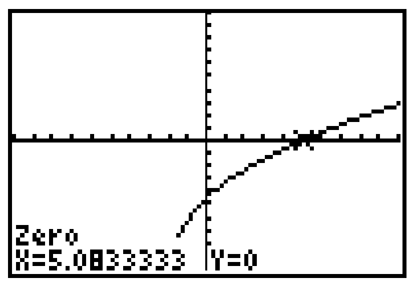
Section 2.3 Page 96 Question 4

a)

$$
\begin{aligned}
2\sqrt{3x+5}+7 &= 16 \\
2\sqrt{3x+5} &= 9 \\
\sqrt{3x+5} &= \frac{9}{2} \\
3x+5 &= \frac{81}{4} \\
3x &= \frac{61}{4} \\
x &= \frac{61}{12}
\end{aligned}
$$

The solution is $x = \frac{61}{12}$ or $x \approx 5.083$.

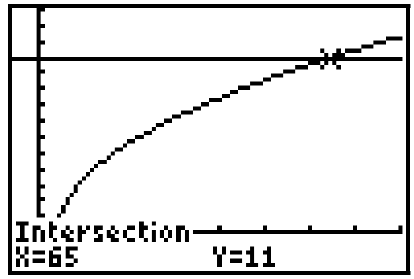
b) The solution will be the *x*-intercept of the graph, $x \approx 5.083$.



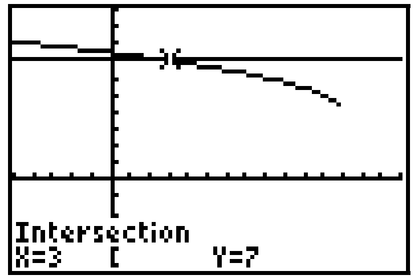
Section 2.3 Page 96 Question 5

a) For $\sqrt{2x-9} = 11$, $x \geq \frac{9}{2}$.

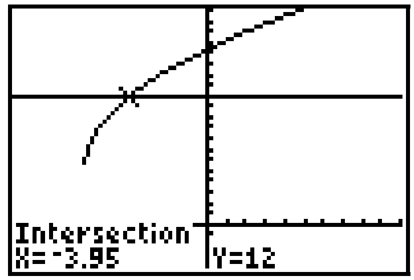
The solution is $x = 65$.



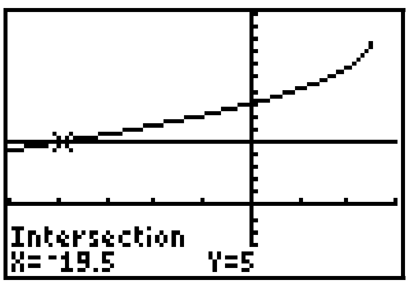
b) For $7 = \sqrt{12 - x} + 4$, $x \le 12$.
The solution is $x = 3$.



c) For $5 + 2\sqrt{5x + 32} = 12$, $x \ge -\dfrac{32}{5}$.
The solution is $x = –3.95$.



d) For $5 = 13 - \sqrt{25 - 2x}$, $x \le \dfrac{25}{2}$. The solution is $x = –19.5$.



Section 2.3 Page 97 Question 6

a) For $\sqrt{5x^2 + 11} = x + 5$, there are no restrictions on the variable.

$$\begin{aligned}
\sqrt{5x^2 + 11} &= x + 5 \\
5x^2 + 11 &= x^2 + 10x + 25 \\
4x^2 - 10x - 14 &= 0 \\
(2x + 2)(2x - 7) &= 0
\end{aligned}$$

$2x + 2 = 0$ or $2x – 7 = 0$

$2x = –2$ $\quad$ $2x = 7$

$x = –1$ $\quad$ $x = \dfrac{7}{2}$

b) For $x+3=\sqrt{2x^2-7}$, the restrictions on the variable are $x \leq -\frac{\sqrt{14}}{2}$ or $x \geq \frac{\sqrt{14}}{2}$.

$$\begin{aligned} x+3 &= \sqrt{2x^2-7} \\ x^2+6x+9 &= 2x^2-7 \\ 0 &= x^2-6x-16 \\ 0 &= (x+2)(x-8) \end{aligned}$$

$x+2=0$ or $x-8=0$

$x=-2$ $\quad$ $x=8$

c) For $\sqrt{13-4x^2}=2-x$, the restrictions on the variable are $-\frac{\sqrt{13}}{2} \leq x \leq \frac{\sqrt{13}}{2}$.

$$\begin{aligned} \sqrt{13-4x^2} &= 2-x \\ 13-4x^2 &= 4-4x+x^2 \\ 0 &= 5x^2-4x-9 \\ 0 &= (5x-9)(x+1) \end{aligned}$$

$5x-9=0$ or $x+1=0$

$5x=9$ $\quad$ $x=-1$

$x=\frac{9}{5}$ or 1.8

d) For $x+\sqrt{-2x^2+9}=3$, the restrictions on the variable are $-\frac{3\sqrt{2}}{2} \leq x \leq \frac{3\sqrt{2}}{2}$.

$$\begin{aligned} x+\sqrt{-2x^2+9} &= 3 \\ \sqrt{-2x^2+9} &= 3-x \\ -2x^2+9 &= 9-6x+x^2 \\ 0 &= 3x^2-6x \\ 0 &= 3x(x-2) \end{aligned}$$

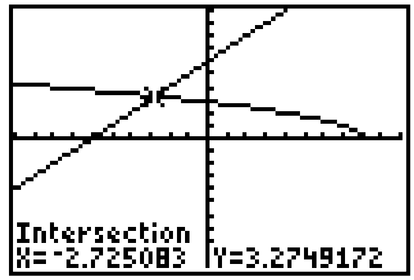
$3x=0$ or $x-2=0$

$x=0$ $\quad$ $x=2$

Section 2.3 Page 97 Question 7

a) For $\sqrt{8-x}=x+6$, the restrictions on the variable are $x \le 8$.

$$\begin{aligned}
\sqrt{8-x} &= x+6 \\
8-x &= x^2+12x+36 \\
0 &= x^2+13x+28 \\
x &= \frac{-13 \pm \sqrt{13^2-4(1)(28)}}{2(1)} \\
x &= \frac{-13 \pm \sqrt{57}}{2}
\end{aligned}$$



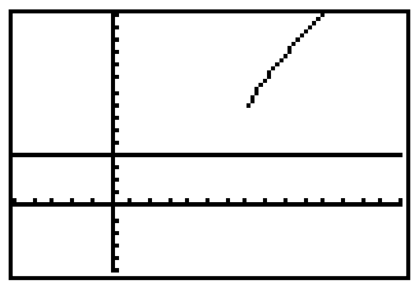
$x = \frac{-13+\sqrt{57}}{2}$ or $x = \frac{-13-\sqrt{57}}{2}$

$x \approx -2.7$ $\quad$ $x \approx -10.3$

The solution is $x \approx -2.7$, as $x \approx -10.3$ is an extraneous root.

b) For $4 = x + 2\sqrt{x-7}$ the restrictions on the variable are $x \ge 7$.

$$\begin{aligned}
4 &= x+2\sqrt{x-7} \\
4-x &= 2\sqrt{x-7} \\
16-8x+x^2 &= 4x^2-14 \\
0 &= 3x^2+8x-30 \\
x &= \frac{-8 \pm \sqrt{8^2-4(3)(-30)}}{2(3)} \\
x &= \frac{-8 \pm \sqrt{424}}{6} \\
x &= \frac{-8 \pm 2\sqrt{106}}{6} \\
x &= \frac{-4 \pm \sqrt{106}}{3}
\end{aligned}$$



$x = \frac{-4+\sqrt{106}}{3}$ or $x = \frac{-4-\sqrt{106}}{3}$

$x \approx 2.1$ $\quad$ $x \approx -4.8$

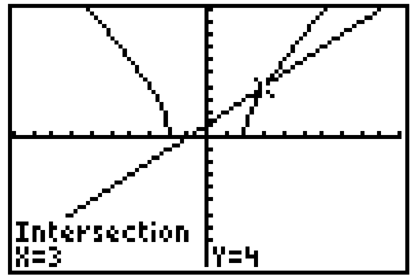
There is no solution, as neither value of x meets the restriction.

c) For $\sqrt{3x^2-11} = x+1$ the restrictions on the variable are $x \le -\frac{\sqrt{33}}{3}$ or $x \ge \frac{\sqrt{33}}{3}$.

$$\sqrt{3x^2-11}=x+1$$
$$3x^2-11=x^2+2x+1$$
$$2x^2-2x-12=0$$
$$x^2-x-6=0$$
$$(x+2)(x-3)=0$$
$x+2=0$ or $x-3=0$

$x=–2$ $\quad$ $x=3$

The solution is $x = 3$, as $x = –2$ is an extraneous root.

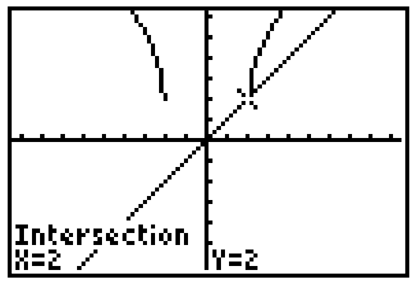


d) For $x=\sqrt{2x^2-8}+2$ the restrictions on the variable are $x \le –2$ or $x \ge 2$.

$$x=\sqrt{2x^2-8}+2$$
$$x-2=\sqrt{2x^2-8}$$
$$x^2-4x+4=2x^2-8$$
$$0=x^2+4x-12$$
$$0=(x+6)(x-2)$$
$x+6=0$ or $x-2=0$

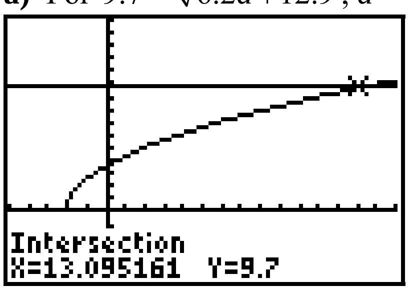
$x=–6$ $\quad$ $x=2$

The solution is $x = 2$, as $x = –6$ is an extraneous root.

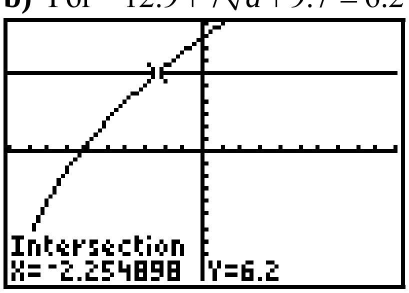


Section 2.3 Page 97 Question 8

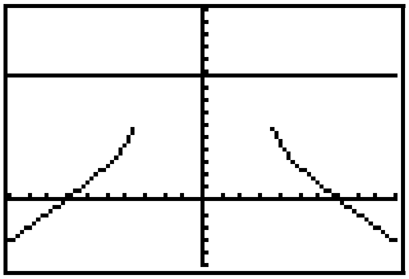
a) For $9.7=\sqrt{6.2a+12.9}$, $a \approx 13.10$.



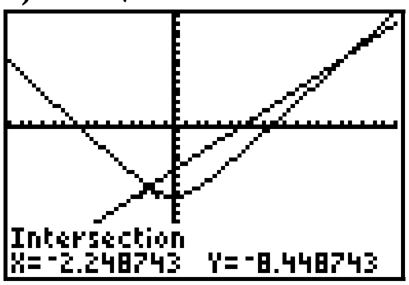
b) For $-12.9+7\sqrt{a+9.7}=6.2$, $a \approx –2.25$.

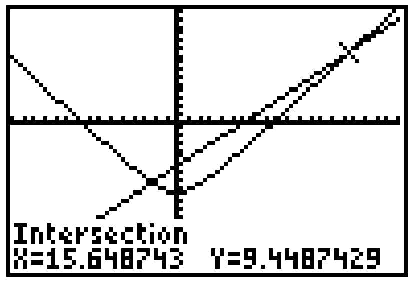


c) For $9.7 = 6.2 - \sqrt{a^2 - 12.9}$, there is no solution.



d) For $\sqrt{2a^2 + 9.7} - 12.9 = a - 6.2$, $a \approx -2.25$ or $a \approx 15.65$.



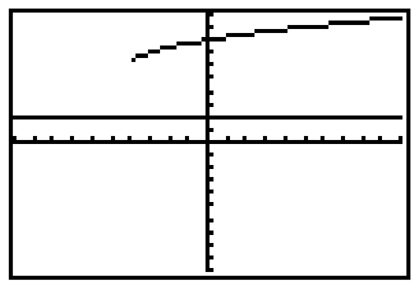


Section 2.3 Page 97 Question 9

a) For $6 + \sqrt{x+4} = 2$, the restrictions on the variable are $x \geq -4$.

$$6 + \sqrt{x+4} = 2$$
$$\sqrt{x+4} = -4$$

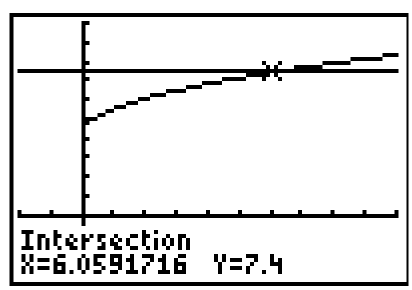
The square root of the expression cannot be negative.
There is no solution.



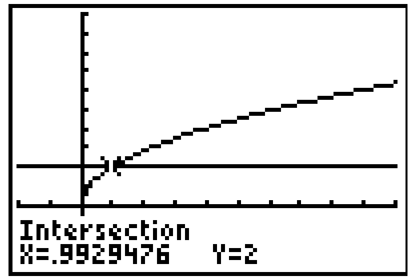
b) Yes, you can tell that this equation has no solutions by examining the equation. As shown in part a), by noticing that isolating the radical expression equates it to a negative value.

Section 2.3 Page 97 Question 10

Determine the point of intersection of the graphs of $N(t) = 1.3\sqrt{t} + 4.2$ and $N(t) = 7.4$.
Greg is correct. The entire population would be affected after 6 years.

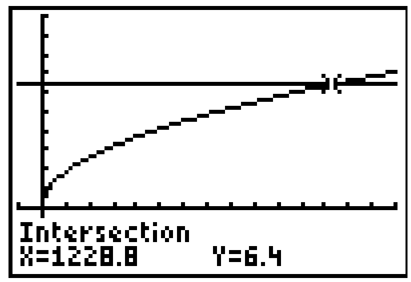


Section 2.3 Page 97 Question 11

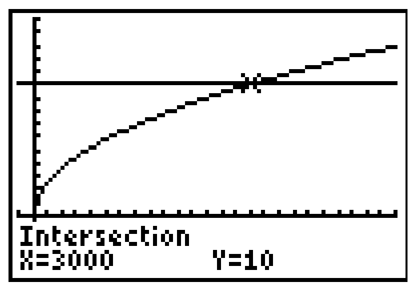


Determine the point of intersection of the graphs of $T = 2\pi\sqrt{\frac{L}{9.8}}$ and $T = 2$. The pendulum should be approximately 0.99 m, or 99 cm, in length.

Section 2.3 Page 97 Question 12



a) Determine the point of intersection of the graphs of $d = \sqrt{\frac{b}{30}}$ and $d = 6.4$. Since the value of the b-coordinate is greater than or equal to 1000, a cable with diameter of 6.4 mm will support a mass of 1000 kg.



b) Determine the point of intersection of the graphs of $d = \sqrt{\frac{b}{30}}$ and $d = 10$. Since the value of the b-coordinate is 3000, a safe working load for a cable with diameter of 10 mm is less than or equal to 3000 kg.

Section 2.3 Page 97 Question 13

Hazeem is incorrect. The equation $\sqrt{x^2} = 9$ has two solutions, while the equation $\left(\sqrt{x}\right)^2 = 9$ has only one.

$$\begin{aligned} \sqrt{x^2} &= 9 \\ x^2 &= 81 \\ x &= \pm 9 \end{aligned} \qquad\qquad \begin{aligned} \left(\sqrt{x}\right)^2 &= 9 \\ x &= 9 \end{aligned}$$

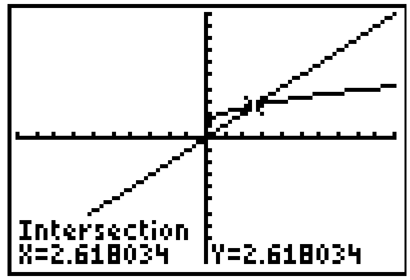
Section 2.3 Page 98 Question 14

Let x represent a real number.

$$\begin{aligned}
x &= \sqrt{x} + 1 \\
x - 1 &= \sqrt{x} \\
x^2 - 2x + 1 &= x \\
x^2 - 3x + 1 &= 0 \\
x &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)} \\
x &= \frac{3 \pm \sqrt{5}}{2}
\end{aligned}$$

$x = \dfrac{3+\sqrt{5}}{2}$ or $x = \dfrac{3-\sqrt{5}}{2}$

$x \approx 2.6$ $\quad\quad$ $x \approx 0.4$



Check:

For $x \approx 2.6$

Left Side	Right Side
x	$\sqrt{x} + 1$
≈ 2.6	$\approx \sqrt{2.6} + 1$
	≈ 2.6

Left Side = Right Side

For $x \approx 0.4$

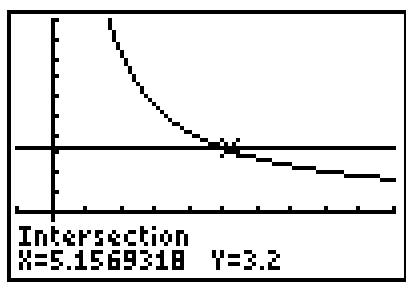
Left Side	Right Side
x	$\sqrt{x} + 1$
≈ 0.4	$\approx \sqrt{0.4} + 1$
	≈ 1.6

Left Side $\neq$ Right Side

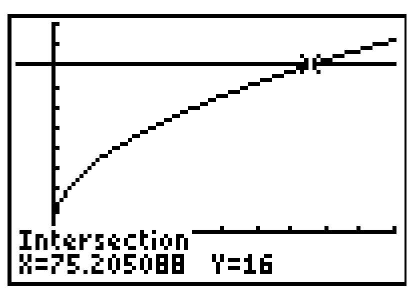
The real number is $\dfrac{3+\sqrt{5}}{2}$.

Section 2.3 Page 98 Question 15

a) Substitute $m = 20$. Determine the point of intersection of the graphs of $d = 3.69\sqrt{\dfrac{20}{v^2}}$ and $d = 3.2$. The landing velocity for a 20-kg object using a parachute that is 3.2 m in diameter is approximately 5 m/s.



b) Substitute $v = 2$. Determine the point of intersection of the graphs of $d = 3.69\sqrt{\dfrac{m}{4}}$ and $d = 16$. The landing velocity for a 2 m/s and a parachute that is 16 m can carry a parachutist with a maximum mass of approximately 75 kg.



Section 2.3 Page 98 Question 16

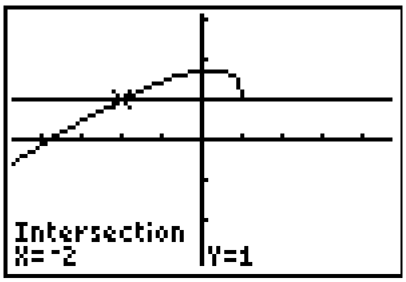
Substitute (–1, 1) into $y = \sqrt{-3(x+c)} + c$.

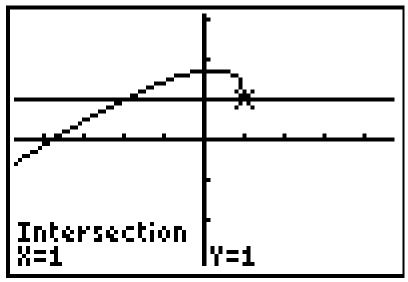
$$\begin{aligned} 1 &= \sqrt{-3(-1+c)} + c \\ 1 - c &= \sqrt{3 - 3c} \\ 1 - 2c + c^2 &= 3 - 3c \\ c^2 + c - 2 &= 0 \\ (c+2)(c-1) &= 0 \end{aligned}$$

$c + 2 = 0$ or $c - 1 = 0$

$c = -2$ $\quad$ $c = 1$

The solutions are $c = -2$ and $c = 1$.





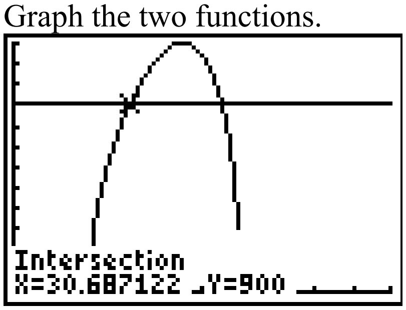
Example: If the function $y = \sqrt{-3(x+c)} + c$ passes through the point (0.25, 0.75), what is the value of c?

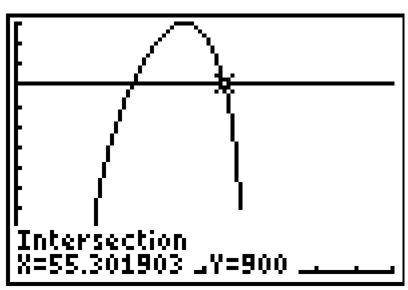
Section 2.3 Page 98 Question 17

Given $A = 900$ and $a = 60$. Let $c = 2b$. Then, $s = 30 + 1.5b$. Substitute into Heron's formula.

$$\begin{aligned} A &= \sqrt{s(s-a)(s-b)(s-c)} \\ 900 &= \sqrt{(30+1.5b)(30+1.5b-60)(30+1.5b-b)(30+1.5b-2b)} \\ 900 &= \sqrt{(30+1.5b)(-30+1.5b)(30+0.5b)(30-0.5b)} \\ 900 &= \sqrt{(2.25b^2 - 900)(900 - 0.25b^2)} \end{aligned}$$

Graph the two functions.





If $b \approx 30.7$, then $c \approx 61.4$. $\quad$ If $b \approx 55.3$, then $c \approx 110.6$.

The lengths of the three side of the triangle are 60 cm, 30.7 cm, and 61.4 cm or 60 cm, 55.3 cm, and 110.6 cm.

Section 2.3 Page 98 Question C1

The x-intercepts of the graph of a function are the solutions to the corresponding equation.
Example: A graph of the function $y = \sqrt{x-1} - 2$ would show that the x-intercept is 5. The equation that corresponds to this function is $0 = \sqrt{x-1} - 2$ and its solution is 5.

Section 2.3 Page 98 Question C2

a) Let v represent the speed, in metres per second. Let d represent the depth of the water, in metres. Then, a function for the speed of a tsunami is $v = \sqrt{9.8d}$

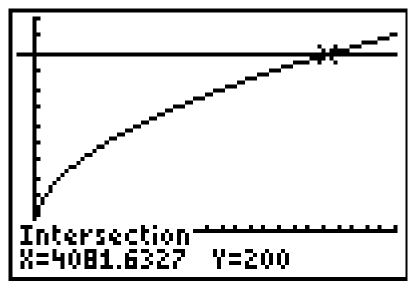
b) Substitute $d = 2500$ into $v = \sqrt{9.8d}$.

$v = \sqrt{(9.8 \text{ m/s}^2)(2500 \text{ m})}$

$v = \sqrt{24\,500 \text{ m}^2/\text{s}^2}$

$v \approx 156.5$ m/s

c) Graphically: Determine the point of intersection of the graphs of
$v = \sqrt{9.8d}$ and $v = 200$.



Algebraically:

$$200 = \sqrt{9.8d}$$
$$40\,000 = 9.8d$$
$$d = \frac{40\,000}{9.8}$$
$$d \approx 4081.6$$

The depth of water that would produce a speed of 200 m/s is approximately 4081.6 m.

d) Example: In this case, I prefer the algebraic method because it is faster than graphing with technology, where I have to adjust window settings to locate the point of intersection.

Section 2.3 Page 98 Question C3

Radical equations only have a solution if the graph of the corresponding function has an x-intercept. For example, $y = \sqrt{x} + 4$ and $y = \sqrt{x+4} + 4$ have no solutions because their graphs have no x-intercepts.

Section 2.3 Page 98 Question C4

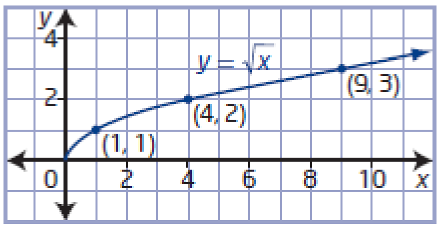
Extraneous roots may occur when solving equations algebraically. For example, a possible solution may not meet the restrictions on the variable in the square root or a check of the solution may yield an extraneous root. An extraneous root can also be identified by graphing.
Extraneous roots of a radical equation may occur anytime an expression is squared. For example, $x^2 = 1$ has two possible solutions, $x = \pm 1$.

Chapter 2 Review

Chapter 2 Review Page 99 Question 1

a)

x	$y = \sqrt{x}$
0	0
1	1
4	2
9	3

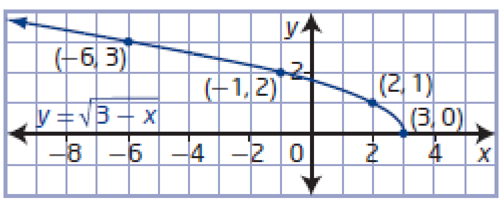


domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

The restriction on the domain is because the radicand must be greater than or equal to zero. All x- and y-values in the table are within the domain and range, respectively. The graph exists in quadrant I, as defined by the domain and range.

b)

x	$y = \sqrt{3 - x}$
3	0
2	1
–1	2
–6	3

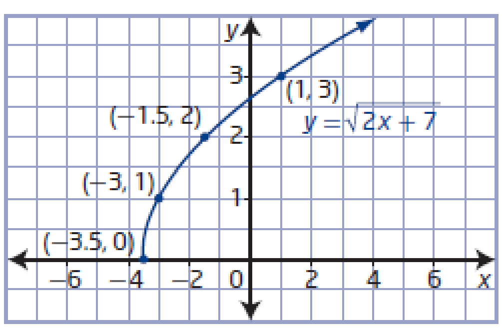


domain
$\{x \mid x \leq 3, x \in \mathrm{R}\}$
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

The restriction on the domain is because the radicand must be greater than or equal to zero. All x- and y-values in the table are within the domain and range, respectively. The graph extends from quadrant I to quadrant II, as defined by the domain and range.

c)

x	$y = \sqrt{2x + 7}$
–3.5	0
–3	1
–1.5	2
1	3



domain
$\{x \mid x \geq -3.5, x \in \mathrm{R}\}$
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

The restriction on the domain is because the radicand must be greater than or equal to zero. All x- and y-values in the table are within the domain and range, respectively. The graph extends from quadrant II to quadrant I, as defined by the domain and range.

Chapter 2 Review Page 99 Question 2

a) For $y = 5\sqrt{x+20}$, $a = 5$, $b = 1$, $h = –20$, and $k = 0$. The graph of $y = \sqrt{x}$ is vertically stretched by a factor of 5 and translated 20 units to the left.
domain $\{x \mid x \geq –20, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

b) For $y = \sqrt{-2x} - 8$, $a = 1$, $b = –2$, $h = 0$, and $k = –8$. The graph of $y = \sqrt{x}$ is horizontally stretched by a factor of 0.5, reflected in the y-axis, and translated 8 units down.
domain $\{x \mid x \leq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq –8, y \in \mathrm{R}\}$

c) For $y = -\sqrt{\frac{1}{6}(x-11)}$, $a = –1$, $b = \frac{1}{6}$, $h = 11$, and $k = 0$. The graph of $y = \sqrt{x}$ is reflected in the x-axis, horizontally stretched by a factor of 6, and translated 11 units to the right.
domain $\{x \mid x \geq 11, x \in \mathrm{R}\}$, range $\{y \mid y \leq 0, y \in \mathrm{R}\}$

Chapter 2 Review Page 99 Question 3

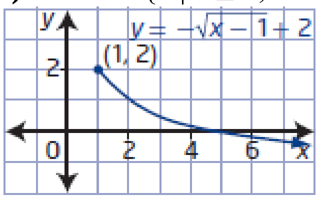
a) For a horizontal stretch by a factor of 10 and a vertical translation of 12 units up, $b = \frac{1}{10}$, $k = 12$, and the equation of the transformed function is $y = \sqrt{\frac{1}{10}x} + 12$.
domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq 12, y \in \mathrm{R}\}$

b) For a vertical stretch by a factor of 2.5, a reflection in the x-axis, and a horizontal translation of 9 units left, $a = –2.5$, $h = –9$, and the equation of the transformed function is $y = -2.5\sqrt{x+9}$.
domain $\{x \mid x \geq –9, x \in \mathrm{R}\}$, range $\{y \mid y \leq 0, y \in \mathrm{R}\}$

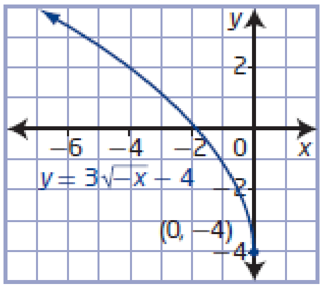
c) For a horizontal stretch by a factor of $\frac{5}{2}$, a vertical stretch by a factor of $\frac{1}{20}$, a reflection in the y-axis, and a translation of 7 units right and 3 units down, $a = \frac{1}{20}$, $b = -\frac{2}{5}$, $h = 7$, $k = –3$, and the equation of the transformed function is
$y = \frac{1}{20}\sqrt{-\frac{2}{5}(x-7)} - 3$.
domain $\{x \mid x \leq 7, x \in \mathrm{R}\}$, range $\{y \mid y \geq –3, y \in \mathrm{R}\}$

Chapter 2 Review Page 99 Question 4

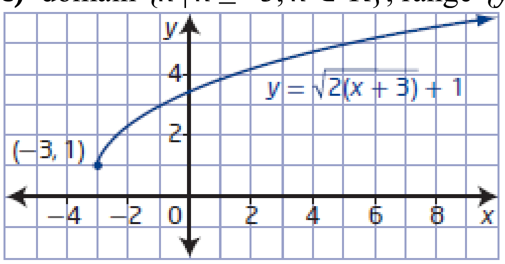
a) domain $\{x \mid x \geq 1, x \in \mathrm{R}\}$, range $\{y \mid y \leq 2, y \in \mathrm{R}\}$



b) domain $\{x \mid x \leq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq -4, y \in \mathrm{R}\}$



c) domain $\{x \mid x \geq -3, x \in \mathrm{R}\}$, range $\{y \mid y \geq 1, y \in \mathrm{R}\}$



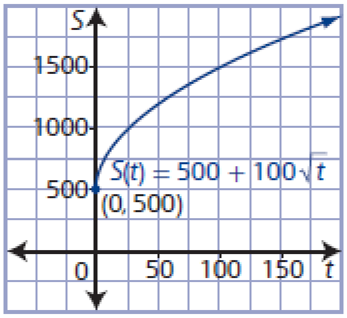
Chapter 2 Review Page 99 Question 5

For $y = -2\sqrt{3(x-4)} + 9$, $a = -2$, $b = 3$, $h = 4$, and $k = 9$. The graph of $y = \sqrt{x}$ is vertically stretched by a factor of 2, reflected in the x-axis, horizontally stretched by a factor of $\frac{1}{3}$, and translated 4 units to the right and 9 units up.
Domain is affected by the values of b and h: $\{x \mid x \geq 4, x \in \mathrm{R}\}$
Range is affected by the values of a and k: $\{y \mid y \leq 9, y \in \mathrm{R}\}$

Chapter 2 Review Page 99 Question 6

a) For $S(t) = 500 + 100\sqrt{t}$, $a = 100$, $b = 1$, $h = 0$, and $k = 500$. The graph of $y = \sqrt{t}$ is vertically stretched by a factor of 100 and translated 500 units up.

b) Since the endpoint of the graph is (0, 500), the minimum sales of the new product is 500 units. Sales will continue to increase with each day.



c) domain $\{t \mid t \geq 0, t \in \mathrm{R}\}$, range $\{S \mid S \geq 500, S \in \mathrm{W}\}$
The domain represent time, so it is non-negative. The range represents number of units sold, which is a minimum of 500 units.

d) Substitute $t = 60$ into $S(t) = 500 + 100\sqrt{t}$.
$S(60) = 500 + 100\sqrt{60}$
$S(60) = 1274.596\ldots$
After 60 days, approximately 1274 units will be sold.

Chapter 2 Review Page 99 Question 7

a) Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph.
$(0, 0) \to (-3, 2)$
$(1, 1) \to (1, 3)$
$(4, 2) \to (13, 4)$
The overall width has changed, so the graph has been horizontally stretched by a factor of 4. From the endpoint, the graph has been translated 3 units to the left and 2 units up.
So, $a = 1$, $b = \frac{1}{4}$, $h = -3$, $k = 2$, and the equation of the transformed graph is
$y = \sqrt{\frac{1}{4}(x+3)} + 2$.

b) Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph.
$(0, 0) \to (-4, 3)$
$(1, 1) \to (-3, 1)$
$(4, 2) \to (0, -1)$
$(9, 3) \to (5, -3)$
The overall height has changed, so the graph has been vertically stretched by a factor of 2. From the general shape, the graph has been reflected in the x-axis. From the endpoint, the graph has been translated 4 units to the left and 3 units up. So, $a = -2$, $b = 1$, $h = -4$, $k = 3$, and the equation of the transformed graph is $y = -2\sqrt{x+4} + 3$.

c) Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph.
$(0, 0) \rightarrow (6, -4)$
$(1, 1) \rightarrow (2, 4)$
$(4, 2) \rightarrow (-3, 8)$
The overall height has changed, so the graph has been vertically stretched by a factor of 4. From the general shape, the graph has been reflected in the *y*-axis. From the endpoint, the graph has been translated 6 units to the right and 4 units down. So, $a = 4$, $b = -1$, $h = 6$, $k = -4$, and the equation of the transformed graph is $y = 4\sqrt{-(x-6)} - 4$.

Chapter 2 Review Page 100 Question 8

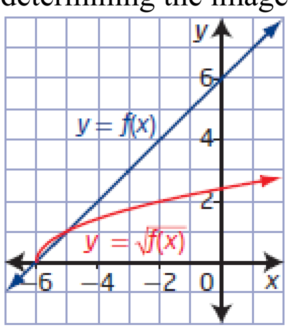
a) For $y = x - 2$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{x-2}$, the domain is $\{x \mid x \geq 2, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{x-2}$ is undefined for $x < 2$.
The ranges differ because $y = \sqrt{x-2}$ is undefined for $y < 0$.

b) For $y = 10 - x$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{10-x}$, the domain is $\{x \mid x \leq 10, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{10-x}$ is undefined for $x > 10$. The ranges differ because $y = \sqrt{10-x}$ is undefined for $y < 0$.

c) For $y = 4x + 11$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{4x+11}$, the domain is $\{x \mid x \geq -\frac{11}{4}, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{4x+11}$ is undefined for $x < -\frac{11}{4}$. The ranges differ because $y = \sqrt{4x+11}$ is undefined for $y < 0$.

Chapter 2 Review Page 100 Question 9

a) I sketched the graph by locating key points, including invariant points, and determining the image points on the graph of the square root of the function.



b) Since the graph of $y = f(x)$ is on or above the x-axis for $x \geq -6$, the graph of $y = \sqrt{f(x)}$ exists only for these values.

c) For $y = f(x)$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{f(x)}$, the domain is $\{x \mid x \geq -6, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{f(x)}$ is undefined for $x < -6$.
The ranges differ because $y = \sqrt{f(x)}$ is undefined for $y < 0$.

Chapter 2 Review Page 100 Question 10

a) For $y = 4 - x^2$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \leq 4, y \in \mathrm{R}\}$.
For $y = \sqrt{4 - x^2}$, the domain is $\{x \mid -2 \leq x \leq 2, x \in \mathrm{R}\}$ and the range is $\{y \mid 0 \leq y \leq 2, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{4 - x^2}$ is undefined for $x < -2$ and $x > 2$. The ranges differ because $y = \sqrt{4 - x^2}$ is undefined for $y < 0$ and has a maximum value of 2.

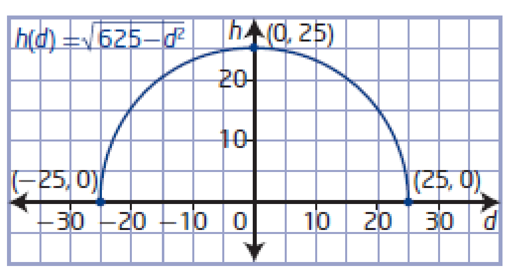
b) For $y = 2x^2 + 24$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 24, y \in \mathrm{R}\}$.
For $y = \sqrt{2x^2 + 24}$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq \sqrt{24}, y \in \mathrm{R}\}$.
The domains are the same because the entire graph of $y = 2x^2 + 24$ is above the x-axis.
The ranges differ because $y = \sqrt{2x^2 + 24}$ has a minimum value of $\sqrt{24}$.

c) For $y = x^2 - 6x$ (or $y = (x - 3)^2 - 9$), the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -9, y \in \mathrm{R}\}$.
For $y = \sqrt{x^2 - 6x}$, the domain is $\{x \mid x \leq 0 \text{ and } x \geq 6, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{x^2 - 6x}$ is undefined for $0 < x < 6$. The ranges differ because $y = \sqrt{x^2 - 6x}$ is undefined for $y < 0$.

Chapter 2 Review Page 100 Question 11

a) Use the Pythagorean theorem because the ladder forms a right triangle with the ground and the wall. An equation to represent h as a function of d is $h = \sqrt{625 - d^2}$.

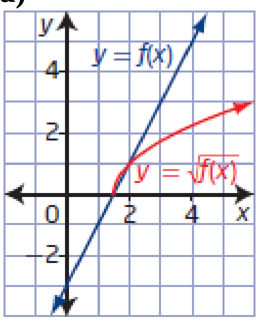
b) The domain is $\{d \mid -25 \leq d \leq 25, d \in \mathrm{R}\}$ and the range is $\{h \mid 0 \leq h \leq 25, h \in \mathrm{R}\}$.



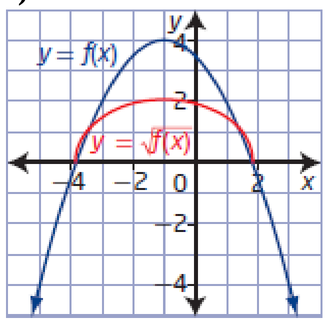
c) Since h and d represent distances, the values must be non-negative. So, the domain becomes $\{d \mid 0 \le d \le 25, d \in \mathrm{R}\}$ to represent the distances of the ladder from the base of the wall. The range represents scenarios from the ladder lying on the ground to flat against the wall.

Chapter 2 Review Page 100 Question 12

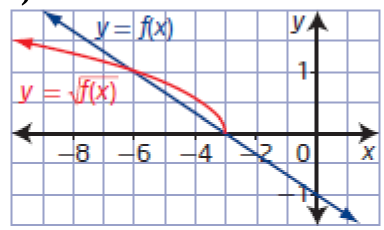
a)



b)

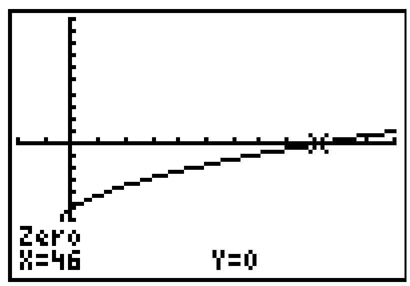


c)



Chapter 2 Review Page 100 Question 13

a)
$$\sqrt{x+3}-7=0$$
$$\sqrt{x+3}=7$$
$$x+3=49$$
$$x=46$$

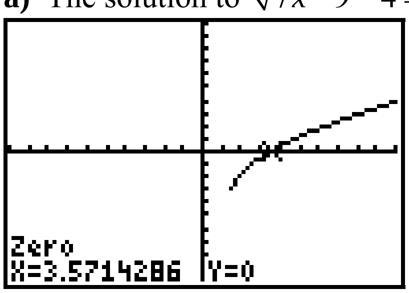


b) The x-intercept is 46.

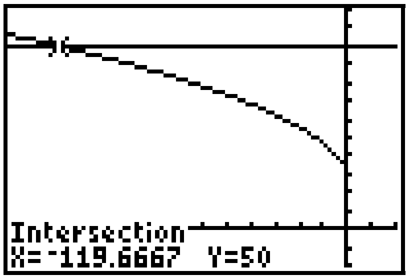
c) The solutions or roots of a radical equation are equivalent to the x-intercepts of the graph of the corresponding radical function.

Chapter 2 Review Page 101 Question 14

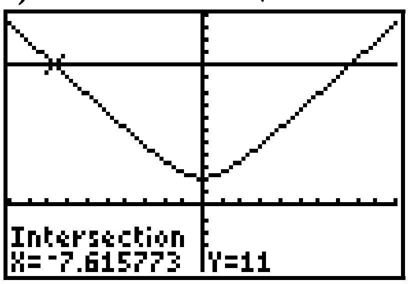
a) The solution to $\sqrt{7x-9}-4=0$ is $x \approx 3.571$.

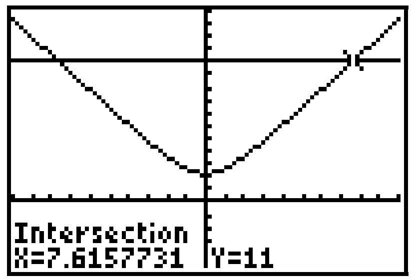


b) The solution to $50 = 12 + \sqrt{8 - 12x}$ is $x \approx -119.667$.



c) The solution to $\sqrt{2x^2 + 5} = 11$ is $x \approx -7.616$ and $x \approx 7.616$.





Chapter 2 Review Page 101 Question 15

Solve $9 = \sqrt{2(9.8)h}$.

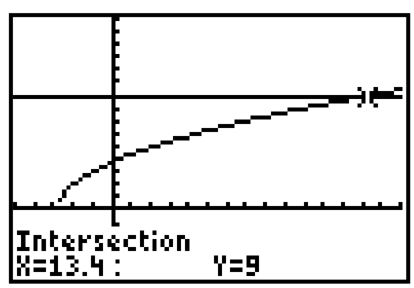
$$\begin{aligned} 9 &= \sqrt{2(9.8)h} \\ 81 &= 19.6h \\ h &= \frac{81}{19.6} \\ h &= 4.132... \end{aligned}$$

At a height of approximately 4.13 m, the water is flowing out at 9 m/s.

Chapter 2 Review Page 101 Question 16

a) For $\sqrt{5x + 14} = 9$, the restrictions on the variable are $x \geq -\frac{14}{5}$.

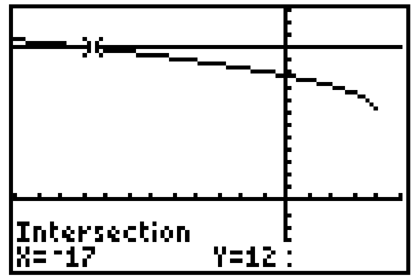
$$\begin{aligned} \sqrt{5x + 14} &= 9 \\ 5x + 14 &= 81 \\ 5x &= 67 \\ x &= \frac{67}{5} \\ x &= 13.4 \end{aligned}$$



The solution is $x = 13.4$.

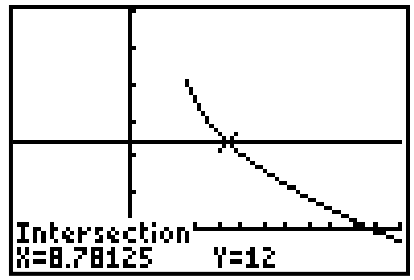
b) For $7 + \sqrt{8 - x} = 12$, the restrictions on the variable are $x \leq 8$.

$$7+\sqrt{8-x}=12$$
$$\sqrt{8-x}=5$$
$$8-x=25$$
$$x=-17$$
The solution is $x = –17$.



c) For $23-4\sqrt{2x-10}=12$, the restrictions on the variable are $x \geq 5$.

$$23-4\sqrt{2x-10}=12$$
$$-4\sqrt{2x-10}=-11$$
$$\sqrt{2x-10}=\frac{11}{4}$$
$$2x-10=\frac{121}{16}$$
$$2x=\frac{281}{16}$$
$$x=\frac{281}{32}$$



The solution is $x = \frac{281}{32}$, or $x \approx 8.781$.

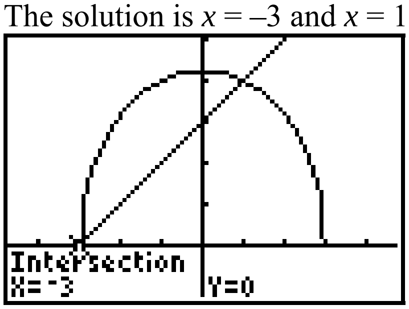
d) For $x+3=\sqrt{18-2x^2}$, the restrictions on the variable are $–3 \leq x \leq 3$.

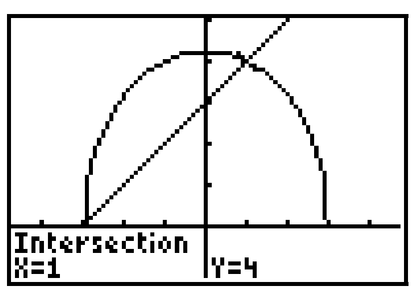
$$x+3=\sqrt{18-2x^2}$$
$$x^2+6x+9=18-2x^2$$
$$3x^2+6x-9=0$$
$$x^2+2x-3=0$$
$$(x+3)(x-1)=0$$

$x + 3 = 0$ or $x – 1 = 0$

$x = –3$ $x = 1$

The solution is $x = –3$ and $x = 1$.





Chapter 2 Review   Page 101   Question 17

a) For $3+\sqrt{x-1}=x$, the restrictions on the variable are $x \geq 1$.

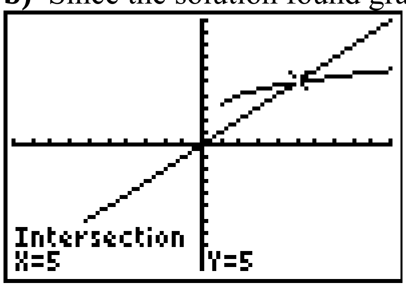
$$
\begin{aligned}
3+\sqrt{x-1} &= x \\
\sqrt{x-1} &= x-3 \\
x-1 &= x^2-6x+9 \\
0 &= x^2-7x+10 \\
0 &= (x-2)(x-5)
\end{aligned}
$$

$x-2=0$ or $x-5=0$

$x=2$ $\quad$ $x=5$

Since the solution found algebraically is $x = 2$ and $x = 5$, Jaime most likely used this approach.

b) Since the solution found graphically is $x = 5$, Carly most likely used this approach.



c) Atid most likely made an error, since $x = 2$ is an extraneous root.

Chapter 2 Review   Page 101   Question 18

a) Substitute $r = 5.2$ into $S(r) = \pi r\sqrt{36+r^2}$.

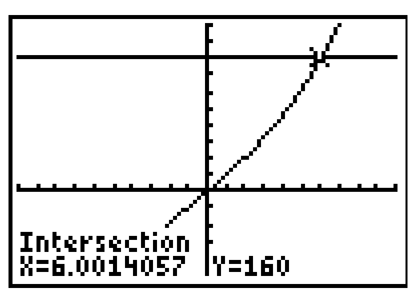
$S(5.2) = \pi(5.2)\sqrt{36+5.2^2}$

$S(5.2) = 5.2\pi\sqrt{63.04}$

$S(5.2) = 129.706\ldots$

For a tipi with radius 5.2 m, the minimum area of canvas required for the walls is 130 m^2, to the nearest square metre.

b) Solve $160 = \pi r\sqrt{36+r^2}$ graphically. If 160 m^2 of canvas form the wall of a tipi, then the radius will be approximately 6.0 m.



Chapter 2 Practice Test

Chapter 2 Practice Test Page 102 Question 1

The points on the graph of $y = \sqrt{f(x)}$, where it exits, have the same x-coordinates as points on the graph of $y = f(x)$ but the y-coordinates are $\sqrt{y}$. If $x = 0$, $f(x) = 1$ and $\sqrt{f(x)} = 1$. The point (0, 1) is on the graph of $y = \sqrt{f(x)}$: choice **B**.

Chapter 2 Practice Test Page 102 Question 2

To solve $\sqrt{2x-5} = 4$ graphically, find determine the x-intercepts of the corresponding graph $y = \sqrt{2x-5} - 4$: choice **A**.

Chapter 2 Practice Test Page 102 Question 3

For a radical function with domain $\{x \mid x \geq 5, x \in \mathrm{R}\}$, $a = 1$ and $h = 5$.
For a radical function with range $\{y \mid y \geq 0, y \in \mathrm{R}\}$, $b = 1$ and $k = 0$.
The equation of this function is $f(x) = \sqrt{x-5}$: choice **A**.

Chapter 2 Practice Test Page 102 Question 4

For a horizontal stretch by a factor of 6, $b = \dfrac{1}{6}$ and the resulting function is $y = \sqrt{\dfrac{1}{6}x}$: choice **C**.

Chapter 2 Practice Test Page 102 Question 5

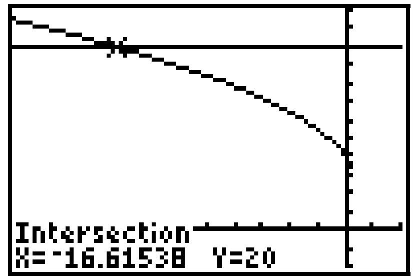
Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph.
$(0, 0) \rightarrow (0, -2)$
$(1, 1) \rightarrow (-1, -1)$
$(4, 2) \rightarrow (-4, 0)$
The overall width and height have not changed, so the graph has not been stretched horizontally or vertically. From the orientation of the graph, it has been reflected in the y-axis. From the endpoint, the graph has been translated 2 units down. So, $a = 1$, $b = -1$, $h = 0$, $k = -2$, and the equation of the transformed graph is $y = \sqrt{-x} - 2$: choice **D**.

Chapter 2 Practice Test Page 102 Question 6

For $y = \sqrt{x}$, the domain is $\{x \mid x \geq 0, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
For $y = \sqrt{5x} + 8$, $a = 1$, $b = 5$, $h = 0$, and $k = 8$. The graph of $y = \sqrt{x}$ is horizontally stretched by a factor of 0.2 and translated 8 units up. The domain remains as $\{x \mid x \geq 0, x \in \mathrm{R}\}$ but the range becomes $\{y \mid y \geq 8, y \in \mathrm{R}\}$: choice **B**.

Chapter 2 Practice Test Page 102 Question 7

The solution is $x \approx -16.62$.



Chapter 2 Practice Test Page 102 Question 8

Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph.
$(0, 0) \rightarrow (0, 0)$
$(1, 1) \rightarrow (1, 4)$
$(4, 2) \rightarrow (4, 8)$
$(9, 3) \rightarrow (9, 12)$
The overall height has changed, so the graph has been vertically stretched by a factor of 4. From the general shape, the graph has not been reflected in an axis. From the endpoint, the graph has not been translated. So, $a = 4$, $b = 1$, $h = 0$, $k = 0$, and an equation of the transformed graph is $y = 4\sqrt{x}$. The equation of the function can also be represented as $y = \sqrt{16x}$, a horizontal stretch by a factor of $\frac{1}{16}$.

Chapter 2 Practice Test Page 102 Question 9

For $y = 7 - x$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{7 - x}$, the domain is $\{x \mid x \le 7, x \in \mathrm{R}\}$ and the range is $\{y \mid y \ge 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{7 - x}$ is undefined for $x > 7$. The ranges differ because $y = \sqrt{7 - x}$ is undefined for $y < 0$.

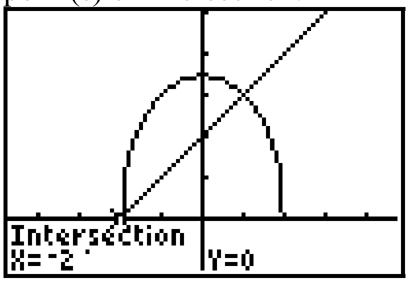
Chapter 2 Practice Test Page 102 Question 10

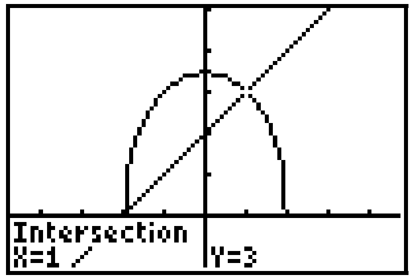
For $f(x) = 8 - 2x^2$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \le 8, y \in \mathrm{R}\}$.
For $y = \sqrt{8 - 2x^2}$, the domain is $\{x \mid -2 \le x \le 2, x \in \mathrm{R}\}$ and the range is $\{y \mid 0 \le y \le \sqrt{8}, y \in \mathrm{R}\}$.

Chapter 2 Practice Test Page 102 Question 11

Solve $\sqrt{12-3x^2} = x+2$ using two graphical methods.

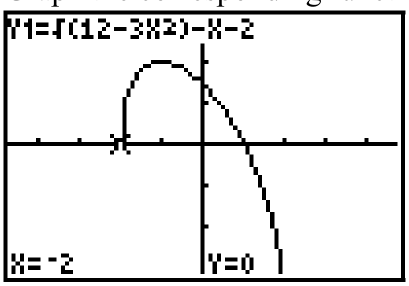
Graph the corresponding functions $y = \sqrt{12-3x^2}$ and $y = x + 2$ and determine the point(s) of intersection.

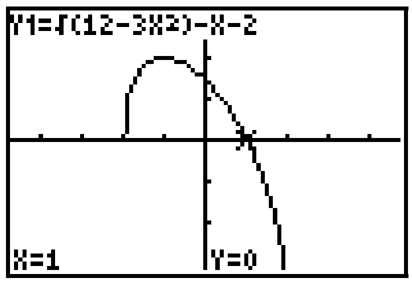




The solution is $x = -2$ and $x = 1$.

Graph the corresponding function $y = \sqrt{12-3x^2} - x - 2$ and determine the x-intercept(s).



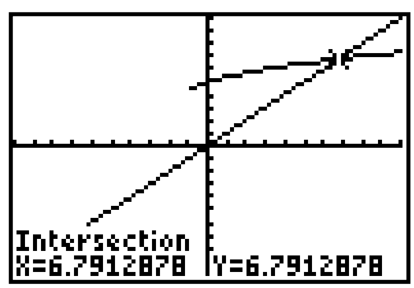


The solution is $x = -2$ and $x = 1$.

Chapter 2 Practice Test Page 102 Question 12

For $4+\sqrt{x+1} = x$, the restrictions on the variable are $x \geq -1$.

$$4+\sqrt{x+1} = x$$
$$\sqrt{x+1} = x-4$$
$$x+1 = x^2-8x+16$$
$$0 = x^2-9x+15$$
$$x = \frac{-(-9)\pm\sqrt{(-9)^2-4(1)(15)}}{2(1)}$$
$$x = \frac{9\pm\sqrt{21}}{2}$$



$x = \dfrac{9+\sqrt{21}}{2}$ or $x = \dfrac{9-\sqrt{21}}{2}$

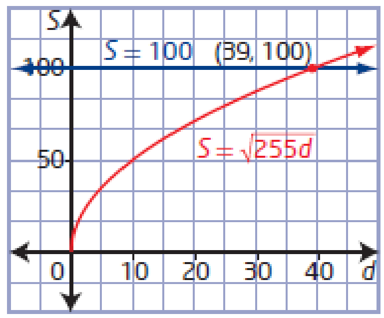
$x \approx 6.8$ $x \approx 2.2$

The solution is $x \approx 6.8$, as $x \approx 2.2$ is an extraneous root.

Chapter 2 Practice Test Page 103 Question 13

a) For $S = \sqrt{255d}$, $a = 1$, $b = 255$, $h = 0$, and $k = 0$. The graph of $S = \sqrt{d}$ is horizontally stretched by a factor of $\frac{1}{255}$.

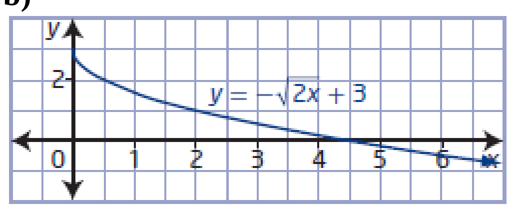
b) The length of skid mark expected from a vehicle travelling at 100 km/h is approximately 39 m.



Chapter 2 Practice Test Page 103 Question 14

a) For $y = -\sqrt{2x} + 3$, $a = -1$, $b = 2$, $h = 0$, and $k = 3$. The graph of $y = \sqrt{x}$ is reflected in the x-axis, horizontally stretched by a factor of $\frac{1}{2}$, and translated 3 units up.

b)



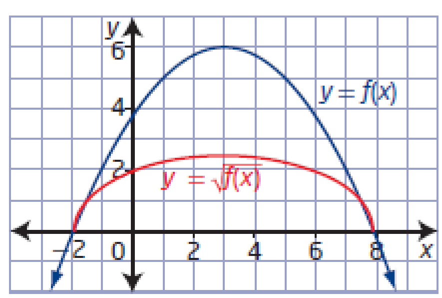
c) domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$, range $\{y \mid y \leq 3, y \in \mathrm{R}\}$

d) The domain remains the same as that for $y = \sqrt{x}$, since $b > 0$ and $h = 0$. The range differs from that for $y = \sqrt{x}$, since $a < 0$ and $k = 3$.

e) The equation $5 + \sqrt{2x} = 8$ can be solved by graphing the corresponding function $y = -\sqrt{2x} + 3$ and determining its x-intercept.

Chapter 2 Practice Test Page 103 Question 15

I sketched the graph by locating key points, including invariant points, and determining the image points on the graph of the square root of the function.



Chapter 2 Practice Test Page 103 Question 16

a) Consider the right half of the roof. The endpoint is (5, 0), so $h = 5$ and $k = 0$. The function is reflected in the y-axis, so $b = -1$. To determine the value of a, substitute the coordinates of the maximum point of the roof, (0, 5), into $y = a\sqrt{-(x-5)}$.

$5 = a\sqrt{-(0-5)}$

$5 = a\sqrt{5}$

$a = \sqrt{5}$

A function where y represents the distance from the base to the roof and x represents the horizontal distance from the centre is $y = \left(\sqrt{5}\right)\sqrt{-(x-5)}$.

b) For this situation, the domain is $\{x \mid 0 \le x \le 5, x \in \mathrm{R}\}$ and the range is $\{y \mid 0 \le y \le 5, y \in \mathrm{R}\}$. The domain cannot be negative nor greater than the radius of the base, or 5. The range cannot be negative nor greater than the height of the roof, or 5.

c) The approximate height of the roof at a point 2 m horizontally from the centre of the roof is 4.58 m.