Chapter 6 Trigonometric Identities

Section 6.1 Reciprocal, Quotient, and Pythagorean Identities

Section 6.1 Page 296 Question 1

a) For $\frac{\cos x}{\sin x}$, non-permissible values occur when sin x = 0.
sin x = 0 at x = 0, π, 2π, …
Therefore, $x \neq \pi n$, where $n \in$ I.

b) For $\frac{\cos x}{\tan x}$, non-permissible values occur when tan x = 0. Since tan x = $\frac{\sin x}{\cos x}$ the non-permissible values occur when cos x = 0 and when sin x = 0.
cos x = 0 at x = $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$ sin x = 0 at x = 0, π, 2π, …
Therefore, $x \neq \frac{\pi}{2} + \pi n$ and $x \neq \pi n$, where $n \in$ I.
These two sets of restrictions can be combined as $x \neq \left(\frac{\pi}{2}\right)n$, where $n \in$ I.

c) For $\frac{\cot x}{1 - \sin x}$, non-permissible values occur, for the denominator, when 1 – sin x = 0 and, for the numerator, when sin x = 0.
For the first restriction, sin x = 1 at x = $\frac{\pi}{2}, \frac{5\pi}{2}, \ldots$. In general, $x \neq \frac{\pi}{2} + 2\pi n$, where $n \in$ I.
For the other restriction, sin x = 0 at x = 0, π, 2π, …. In general, $x \neq \pi n$, where $n \in$ I..

d) For $\frac{\tan x}{\cos x + 1}$, non-permissible values occur, for the denominator, when cos x + 1 = 0 and, for the numerator, when cos x = 0.
For the first restriction, cos x = –1 at x = π, 3π, …. In general, $x \neq \pi + 2\pi n$, , where $n \in$ I.
For the other restriction, cos x = 0 at x = $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$. In general, $x \neq \frac{\pi}{2} + \pi n$, where $n \in$ I.

Section 6.1 Page 296 Question 2

Some identities have non-permissible values because they involve rational expressions and some values of the variable would make the denominator zero. This is not permitted. For example, an identity involving tan x has non-permissible values when cos x = 0.

Section 6.1 Page 296 Question 3

a) $$\begin{aligned}\sec x \sin x &= \left(\frac{1}{\cos x}\right)\sin x\\ &= \tan x\end{aligned}$$

b) $$\begin{aligned}\sec x \cot x \sin^2 x &= \left(\frac{1}{\cos x}\right)\left(\frac{\cos x}{\sin x}\right)\sin^2 x\\ &= \sin x\end{aligned}$$

c) $$\begin{aligned}\frac{\cos x}{\cot x} &= \cos x\left(\frac{\sin x}{\cos x}\right)\\ &= \sin x\end{aligned}$$

Section 6.1 Page 296 Question 4

a) $$\begin{aligned}\left(\frac{\cos x}{\tan x}\right)\left(\frac{\tan x}{\sin x}\right) &= \frac{\cos x}{\sin x}\\ &= \cot x\end{aligned}$$

b) $$\begin{aligned}\csc x \cot x \sec x \sin x &= \left(\frac{1}{\sin x}\right)\left(\frac{\cos x}{\sin x}\right)\left(\frac{1}{\cos x}\right)\sin x\\ &= \frac{1}{\sin x}\\ &= \csc x\end{aligned}$$

c) $$\begin{aligned}\frac{\cos x}{1-\sin^2 x} &= \frac{\cos x}{\cos^2 x}\\ &= \frac{1}{\cos x}\\ &= \sec x\end{aligned}$$

Section 6.1 Page 296 Question 5

a) For $x = 30°$:

$$\text{Left Side} = \frac{\sec x}{\tan x + \cot x}$$
$$= \frac{\sec 30°}{\tan 30° + \cot 30°}$$
$$= \frac{\frac{2}{\sqrt{3}}}{\frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{1}}$$
$$= \frac{2}{\sqrt{3}} \div \frac{1+3}{\sqrt{3}}$$
$$= \frac{2}{4}$$
$$= \frac{1}{2}$$

$$\text{Right Side} = \sin x$$
$$= \sin 30°$$
$$= \frac{1}{2}$$

For $x = \frac{\pi}{4}$:

$$\text{Left Side} = \frac{\sec x}{\tan x + \cot x}$$
$$= \frac{\sec \frac{\pi}{4}}{\tan \frac{\pi}{4} + \cot \frac{\pi}{4}}$$
$$= \frac{\sqrt{2}}{1+1}$$
$$= \frac{\sqrt{2}}{2}$$

$$\text{Right Side} = \sin x$$
$$= \sin \frac{\pi}{4}$$
$$= \frac{1}{\sqrt{2}}$$
$$= \frac{\sqrt{2}}{2}$$

The equation checks for both values.

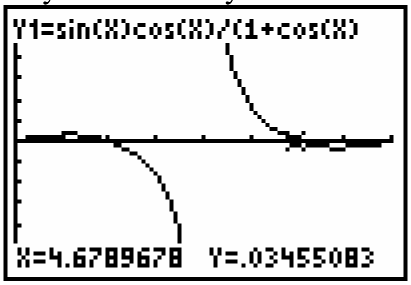
b) The non-permissible values occur for the numerator when $\cos x = 0$ and, for the denominator, when $\sin x = 0$. So, in the domain $0° \le x < 360°$, $x \ne 0°$, 90°, 180°, and 270°.

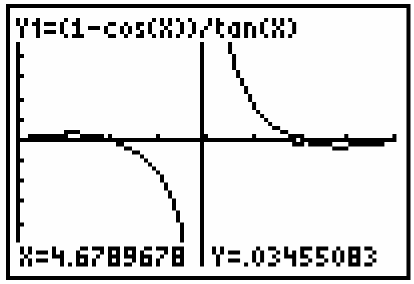
Section 6.1 Page 297 Question 6

a) The non-permissible values occur for the left side of the equation when $1 + \cos x = 0$, or $\cos x = -1$, and, for the right side of the equation, when $\tan x = 0$.

So, in radians, $x \ne \pi + 2\pi n,\ n \in \mathrm{I}$ and $x \ne \frac{\pi}{2} + \pi n,\ n \in \mathrm{I}$.

b) The graph of the left side looks the same as the graph of the right side, so the equation may be an identity.





c) For $\frac{\pi}{4}$:

$$\begin{aligned}
\text{Left Side} &= \frac{\sin x \cos x}{1+\cos x} \\
&= \frac{\sin\frac{\pi}{4}\cos\frac{\pi}{4}}{1+\cos\frac{\pi}{4}} \\
&= \frac{\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)}{1+\left(\frac{1}{\sqrt{2}}\right)} \\
&= \left(\frac{1}{2}\right)\div\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right) \\
&= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{\sqrt{2}+1}\right)\left(\frac{\sqrt{2}-1}{\sqrt{2}-1}\right) \\
&= \left(\frac{1}{2}\right)\left(\frac{2-\sqrt{2}}{1}\right) \\
&= \frac{2-\sqrt{2}}{2}
\end{aligned}$$

$$\begin{aligned}
\text{Right Side} &= \frac{1-\cos x}{\tan x} \\
&= \frac{1-\cos\frac{\pi}{4}}{\tan\frac{\pi}{4}} \\
&= 1-\frac{1}{\sqrt{2}} \\
&= 1-\frac{\sqrt{2}}{2} \\
&= \frac{2-\sqrt{2}}{2}
\end{aligned}$$

Both sides have the same value, so the equation is true when $x = \frac{\pi}{4}$.

Section 6.1 Page 297 Question 7

a) Using the Pythagorean identity, $\sin^2 \theta + \cos^2 \theta = 1$, an equivalent expression for $1 - \sin^2 \theta$ is $\cos^2 \theta$.

b) When $\theta = \frac{\pi}{6}$, the fraction of light lost is given by

$$\cos^2\theta = \cos^2\frac{\pi}{6} = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

c) When θ = 60°, the fraction of light lost is given by

$$\cos^2\theta = \cos^2 60° = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

As a percent, the amount lost is 25%.

Section 6.1 Page 297 Question 8

a) When $x = \frac{\pi}{3}$:

$$\sin x = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\sqrt{1-\cos^2 x} = \sqrt{1-\cos^2\left(\frac{\pi}{3}\right)} = \sqrt{1-\left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

So, for $x = \frac{\pi}{3}$, $\sin x = \sqrt{1-\cos^2 x}$.

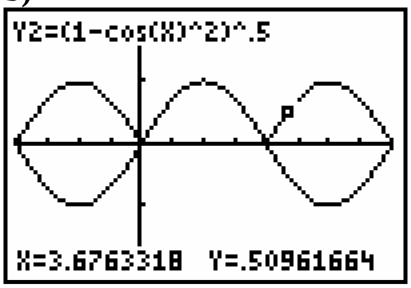
When $x = \frac{5\pi}{6}$:

$$\sin x = \sin\frac{5\pi}{6} = \frac{1}{2}$$

$$\sqrt{1-\cos^2 x} = \sqrt{1-\cos^2\left(\frac{5\pi}{6}\right)} = \sqrt{1-\left(-\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

So, for $x = \frac{5\pi}{6}$, $\sin x = \sqrt{1-\cos^2 x}$.

b)



c)
The equation is not an identity. As the graph shows, $y = \sqrt{1-\cos^2 x}$ only has positive values in its range, whereas $y = \sin x$ has all values from –1 to 1 in its range.

Section 6.1 Page 297 Question 9

a)

$$\begin{aligned}\sec\theta &= \frac{I}{ER^2}\\ \frac{1}{\cos\theta} &= \frac{I}{ER^2}\\ E &= \frac{I\cos\theta}{R^2}\end{aligned}$$

b)

$$\begin{aligned}E &= \frac{I\cot\theta}{R^2\csc\theta}\\ E &= \frac{I\left(\frac{\cos\theta}{\sin\theta}\right)}{R^2\left(\frac{1}{\sin\theta}\right)}\\ E &= \left(\frac{I\cos\theta}{\sin\theta}\right)\left(\frac{\sin\theta}{R^2}\right)\\ E &= \frac{I\cos\theta}{R^2}\end{aligned}$$

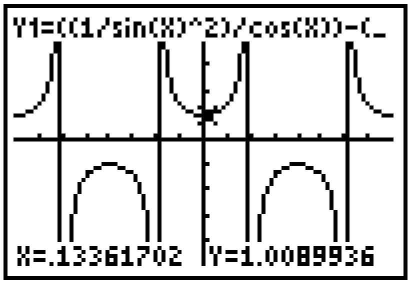
Section 6.1 Page 297 Question 10

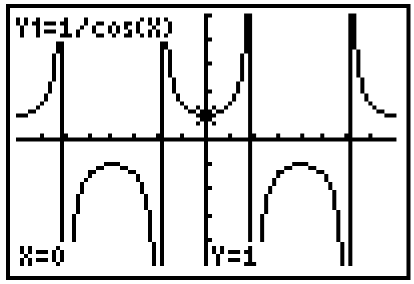
$$\begin{aligned}\frac{\csc x}{\tan x + \cot x} &= \left(\frac{1}{\sin x}\right) \div \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)\\ &= \left(\frac{1}{\sin x}\right) \div \left(\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}\right)\\ &= \left(\frac{1}{\sin x}\right)\left(\frac{\cos x \sin x}{1}\right)\\ &= \cos x\end{aligned}$$

Division by $\sin x$ and by $\cos x$ occurs, so $\sin x \neq 0$ and $\cos x \neq 0$.
In the domain $0 \le \theta < 2\pi$, $x \neq 0, \frac{\pi}{2}, \pi$, and $\frac{3\pi}{2}$.

Section 6.1 Page 298 Question 11

a) The graph of $y = \dfrac{\csc^2 x - \cot^2 x}{\cos x}$ appears to be equivalent to the graph of $y = \sec x$.





b) Division by sin x and by cos x occurs, so $\sin x \neq 0$ and $\cos x \neq 0$. So in general, in radians, $x \neq \dfrac{\pi}{2} + \pi n,\ n \in \mathrm{I}$.

c)

$$\begin{aligned}\frac{\csc^2 x - \cot^2 x}{\cos x} &= \frac{\dfrac{1}{\sin^2 x} - \dfrac{\cos^2 x}{\sin^2 x}}{\cos x} \\ &= \frac{\dfrac{1 - \cos^2 x}{\sin^2 x}}{\cos x} \\ &= \frac{\dfrac{\sin^2 x}{\sin^2 x}}{\cos x} \\ &= \frac{1}{\cos x} \\ &= \sec x\end{aligned}$$

Section 6.1 Page 298 Question 12

a) Substitute $x = \dfrac{\pi}{4}$:

$$\begin{aligned}\text{Left Side} &= \frac{\cot x}{\sec x} + \sin x \\ &= \frac{\cot \frac{\pi}{4}}{\sec \frac{\pi}{4}} + \sin \frac{\pi}{4} \\ &= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \\ &= \frac{2}{\sqrt{2}} \text{ or } \sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{Right Side} &= \csc x \\ &= \csc \frac{\pi}{4} \\ &= \sqrt{2}\end{aligned}$$

Since the right side equals the left side for this one value, $\frac{\cot x}{\sec x} + \sin x = \csc x$ may be an identity.

b) $\frac{\cot x}{\sec x} + \sin x = \frac{\cos x}{\sin x} \div \frac{1}{\cos x} + \sin x$

$$= \frac{\cos^2 x}{\sin x} + \sin x$$

$$= \frac{\cos^2 x + \sin^2 x}{\sin x}$$

$$= \csc x$$

Section 6.1 Page 298 Question 13

a) Substitute $x = 0$:

Left Side $= \sin x + \cos x$
$= \sin 0 + \cos 0$
$= 0 + 1$
$= 1$

Right Side $= \tan x + 1$
$= \tan 0 + 1$
$= 0 + 1$
$= 1$

For $x = 0$, Left Side = Right Side = 1.

b) Substitute $x = \frac{\pi}{2}$:

Left Side $= \sin x + \cos x$
$= \sin \frac{\pi}{2} + \cos \frac{\pi}{2}$
$= 1 + 0$
$= 1$

Right Side $= \tan x + 1$
$= \tan \frac{\pi}{2} + 1$
$=$ undefined $+ 1$

For $x = \frac{\pi}{2}$, Left Side = 1 but the right side is undefined.

c) Lisa's choice for x is not permissible because $\cos \frac{\pi}{2} = 0$, so in the right side of the equation, in $\tan x$, you would be dividing by 0 which is not permitted.

d) Substitute $x = \frac{\pi}{4}$:

$$\text{Left Side} = \sin x + \cos x \qquad \text{Right Side} = \tan x + 1$$

$$\begin{aligned} \text{Left Side} &= \sin x + \cos x \\ &= \sin\frac{\pi}{4} + \cos\frac{\pi}{4} \\ &= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \\ &= \frac{2}{\sqrt{2}} \end{aligned} \qquad \begin{aligned} \text{Right Side} &= \tan x + 1 \\ &= \tan\frac{\pi}{4} + 1 \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

For $x = \frac{\pi}{4}$, the left side = $\frac{2}{\sqrt{2}}$ but the right side = 2.

e) Yes, the three students have enough information to conclude that the equation in not an identity. Giselle has found a permissible value of x for which the left and right sides do not have the same value.

Section 6.1 Page 298 Question 14

$$\begin{aligned} &(\sin x + \cos x)^2 + (\sin x - \cos x)^2 \\ &= \sin^2 x + 2\sin x \cos x + \cos^2 x + \sin^2 x - 2\sin x \cos x + \cos^2 x \\ &= 2\sin^2 x + 2\cos^2 x \\ &= 2(\sin^2 x + \cos^2 x) \\ &= 2 \end{aligned}$$

Section 6.1 Page 298 Question 15

Given $\csc^2 x + \sin^2 x = 7.89$.

$$\begin{aligned} \frac{1}{\csc^2 x} + \frac{1}{\sin^2 x} &= \frac{1}{\frac{1}{\sin^2 x}} + \csc^2 x \\ &= \sin^2 x + \csc^2 x \\ &= 7.89 \end{aligned}$$

Section 6.1 Page 298 Question 16

$$\begin{aligned} \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} &= \frac{1-\sin\theta+1+\sin\theta}{(1-\sin\theta)(1+\sin\theta)} \\ &= \frac{2}{(1-\sin^2\theta)} \\ &= 2\sec^2\theta \end{aligned}$$

Section 6.1 Page 298 Question 17

$$\begin{aligned}\frac{2-\cos^2 x}{\sin x} &= \frac{1+1-\cos^2 x}{\sin x}\\ &= \frac{1+\sin^2 x}{\sin x}\\ &= \frac{1}{\sin x}+\sin x\\ &= \csc x+\sin x\end{aligned}$$

So, $\dfrac{2-\cos^2 x}{\sin x} = m + \sin x$ is an identity if $m = \csc x$.

Section 6.1 Page 298 Question C1

$$\begin{aligned}\cot^2 x+1 &= \frac{\cos^2 x}{\sin^2 x}+\frac{\sin^2 x}{\sin^2 x}\\ &= \frac{\cos^2 x+\sin^2 x}{\sin^2 x}\\ &= \frac{1}{\sin^2 x}\\ &= \csc^2 x\end{aligned}$$

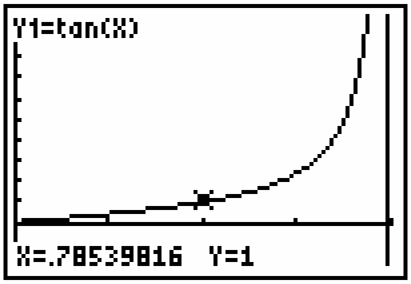
Section 6.1 Page 298 Question C2

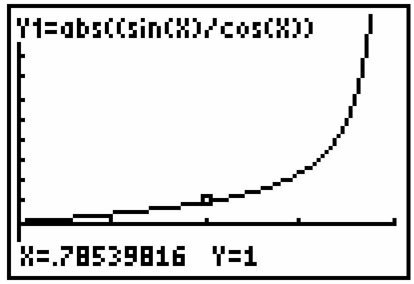
$$\begin{aligned}\left(\frac{\sin\theta}{1+\cos\theta}\right)\left(\frac{1-\cos\theta}{1-\cos\theta}\right) &= \frac{\sin\theta-\sin\theta\cos\theta}{1-\cos^2\theta}\\ &= \frac{\sin\theta-\sin\theta\cos\theta}{\sin^2\theta}\\ &= \frac{1-\cos\theta}{\sin\theta}\end{aligned}$$

It helps to simplify by creating an opportunity to use the Pythagorean identity.

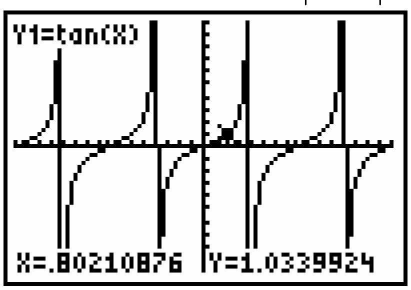
Section 6.1 Page 298 Question C3

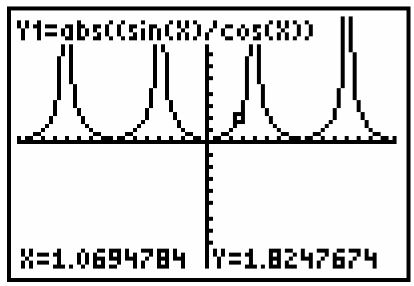
Step 1 For the domain $0 \le x < \dfrac{\pi}{2}$, the graphs $y = \tan x$ and $y = \left|\dfrac{\sin x}{\cos x}\right|$ are the same. In this domain, $\tan x = \left|\dfrac{\sin x}{\cos x}\right|$ is an identity.





Step 2 For the domain $-2\pi < x \le 2\pi$, the graphs $y = \tan x$ and $y = \left|\frac{\sin x}{\cos x}\right|$ are not the same. In this domain, $\tan x = \left|\frac{\sin x}{\cos x}\right|$ is not an identity.





Step 3 Example: $y = \cot\theta$ and $y = \left|\frac{\cos\theta}{\sin\theta}\right|$ are identities over the domain $0 \le \theta \le \frac{\pi}{2}$ but not over the domain $-2\pi \le \theta \le 2\pi$.

Step 4 The weakness of using a graphical or numerical approach is that for some equations you may think it is an identity when really it is only an identity over a restricted domain.

Section 6.2 Sum, Difference, and Double-Angle Identities

Section 6.2 Page 306 Question 1

a) $\cos 43° \cos 27° - \sin 43° \sin 27° = \cos (43° + 27°)$
$= \cos 70°$

b) $\sin 15° \cos 20° + \cos 15° \sin 20° = \sin (15° + 20°)$
$= \sin 35°$

c) $\cos^2 19° - \sin^2 19° = \cos 2(19°)$
$= \cos 38°$

d) $\sin\frac{3\pi}{2}\cos\frac{5\pi}{4}-\cos\frac{3\pi}{2}\sin\frac{5\pi}{4}=\sin\left(\frac{3\pi}{2}-\frac{5\pi}{4}\right)$

$$=\sin\frac{\pi}{4}$$

e) $8\sin\frac{\pi}{3}\cos\frac{\pi}{3}=4\sin 2\left(\frac{\pi}{3}\right)$

$$=4\sin\frac{2\pi}{3}$$

Section 6.2 Page 306 Question 2

a) $\cos 40° \cos 20° - \sin 40° \sin 20° = \cos (40° + 20°)$
$= \cos 60°$
$= 0.5$

b) $\sin 20° \cos 25° + \cos 20° \sin 25° = \sin (20° + 25°)$
$= \sin 45°$
$= \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

c) $\cos^2\frac{\pi}{6}-\sin^2\frac{\pi}{6}=\cos 2\left(\frac{\pi}{6}\right)$

$$=\cos\frac{\pi}{3}$$

$$=0.5$$

d) $\cos\frac{\pi}{2}\cos\frac{\pi}{3}-\sin\frac{\pi}{2}\sin\frac{\pi}{3}=\cos\left(\frac{\pi}{2}+\frac{\pi}{3}\right)$

$$=\cos\frac{5\pi}{6}$$

$$=-\frac{\sqrt{3}}{2}$$

Section 6.2 Page 306 Question 3

Substitute $\cos 2x = 1 - 2\sin^2 x$.
$1 - \cos 2x = 1 - (1 - 2\sin^2 x)$
$= 1 - 1 + 2\sin^2 x$
$= 2\sin^2 x$

Section 6.2 Page 306 Question 4

a) $2\sin\frac{\pi}{4}\cos\frac{\pi}{4} = \sin 2\left(\frac{\pi}{4}\right)$

$= \sin\frac{\pi}{2}$

b) $(6\cos^2 24° - 6\sin^2 24°)\tan 48° = 6\cos 2(24°)\tan 48°$

$= 6\cos 48° \tan 48°$

$= 6\cos 48° \left(\frac{\sin 48°}{\cos 48°}\right)$

$= 6\sin 48°$

c) $\frac{2\tan 76°}{1-\tan^2 76°} = \tan 2(76°)$

$= \tan 152°$

d) $2\cos^2\frac{\pi}{6} - 1 = \cos 2\left(\frac{\pi}{6}\right)$

$= \cos\frac{\pi}{3}$

e) $1 - 2\cos^2\frac{\pi}{12} = -\left(2\cos^2\frac{\pi}{12} - 1\right)$

$= -\cos 2\left(\frac{\pi}{12}\right)$

$= -\cos\frac{\pi}{6}$

Section 6.2 Page 306 Question 5

a) $\frac{\sin 2\theta}{2\cos\theta} = \frac{2\sin\theta\cos\theta}{2\cos\theta}$

$= \sin\theta$

b) $\cos 2x\cos x + \sin 2x\sin x = (\cos^2 x - \sin^2 x)\cos x + 2\sin x\cos x\sin x$

$= \cos^3 x - \sin^2 x\cos x + 2\sin^2 x\cos x$

$= \cos^3 x + \sin^2 x\cos x$

$= \cos x(\cos^2 x + \sin^2 x)$

$= \cos x$

c) $\dfrac{\cos 2\theta + 1}{2\cos\theta} = \dfrac{2\cos^2\theta - 1 + 1}{2\cos\theta}$

$= \dfrac{2\cos^2\theta}{2\cos\theta}$

$= \cos\theta$

d) $\dfrac{\cos^3 x}{\cos 2x + \sin^2 x} = \dfrac{\cos x(1 - \sin^2 x)}{(1 - 2\sin^2 x) + \sin^2 x}$

$= \dfrac{\cos x(1 - \sin^2 x)}{1 - \sin^2 x}$

$= \cos x$

Section 6.2 Page 306 Question 6

Use a counterexample to show that $\sin(x - y) \neq \sin x - \sin y$.
Substitute $x = 60°$ and $y = 30°$:

Left Side $= \sin(60° - 30°)$
$= \sin 30°$
$= 0.5$

Right Side $= \sin 60° - \sin 30°$
$= \dfrac{\sqrt{3}}{2} - 0.5$
≈ 0.366

Left Side ≠ Right Side

Section 6.2 Page 306 Question 7

$\cos(90° - x) = \cos 90° \cos x + \sin 90° \sin x$
$= 0(\cos x) + 1 \sin x$
$= \sin x$

Section 6.2 Page 306 Question 8

a) $\cos 75° = \cos(45° + 30°)$

$= \cos 45° \cos 30° - \sin 45° \sin 30°$

$= \dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{3}}{2}\right) - \dfrac{1}{\sqrt{2}}\left(\dfrac{1}{2}\right)$

$= \dfrac{\sqrt{3} - 1}{2\sqrt{2}}$ or $\dfrac{\sqrt{6} - \sqrt{2}}{4}$

b) $\tan 165° = \tan(120° + 45°)$

$= \dfrac{\tan 120° + \tan 45°}{1 - \tan 120° \tan 45°}$

$= \dfrac{-\sqrt{3} + 1}{1 - (-\sqrt{3})(1)}$

$= \dfrac{-\sqrt{3} + 1}{1 + \sqrt{3}}$ or $\dfrac{(1 - \sqrt{3})(1 - \sqrt{3})}{(1 + \sqrt{3})(1 - \sqrt{3})} = \dfrac{4 - 2\sqrt{3}}{-2}$ or $\sqrt{3} - 2$

c) $\sin\frac{7\pi}{12} = \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)$

$= \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right) + \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{1+\sqrt{3}}{2\sqrt{2}}$ or $\frac{\sqrt{2}+\sqrt{6}}{4}$

d) cos 195° = cos (60° + 135°)

= cos 60° cos 135° – cos 60° sin 135°

$= \frac{1}{2}\left(-\frac{1}{\sqrt{2}}\right) - \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{-1-\sqrt{3}}{2\sqrt{2}}$ or $\frac{\sqrt{2}-\sqrt{6}}{4}$

e)

$\csc\frac{\pi}{12} = \csc\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$

$= \csc\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= 1 \div \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= 1 \div \left[\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)\right]$

$= 1 \div \left[\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)\right]$

$= 1 \div \left[\frac{\sqrt{3}-1}{2\sqrt{2}}\right]$

$= \frac{2\sqrt{2}}{\sqrt{3}-1}$ or $\frac{\left(2\sqrt{2}\right)\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)} = \sqrt{2}\left(\sqrt{3}+1\right)$

f) $\sin\left(-\frac{\pi}{12}\right) = \sin\left(\frac{3\pi}{12} - \frac{4\pi}{12}\right)$

$$= \sin\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$$

$$= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{1-\sqrt{3}}{2\sqrt{2}} \text{ or } \frac{\sqrt{2}-\sqrt{6}}{4}$$

Section 6.2 Page 306 Question 9

a) $P = 1000 (\sin x \cos 113.5° + \cos 113.5° \sin x)$
$= 1000 \sin (x + 113.5°)$

b) i) For Whitehorse, Yukon, 60.7° N, substitute $x = 60.7°$.
$P = 1000 \sin (60.7° + 113.5°)$
$= 1000 \sin 174.2°$
≈ 101.056
The amount of power received from the sun on the winter solstice at Whitehorse is approximately 101.056 W/m^2.

ii) For Victoria, British Columbia, 48.4° N, substitute $x = 48.4°$.
$P = 1000 \sin (48.4° + 113.5°)$
$= 1000 \sin 161.9°$
≈ 310.676
The amount of power received from the sun on the winter solstice at Victoria is approximately 310.676 W/m^2.

iii) For Igloolik, Nunavut, 69.4° N, substitute $x = 69.4°$.
$P = 1000 \sin (69.4° + 113.5°)$
$= 1000 \sin 182.9°$
≈ -50.593
The amount of power received from the sun on the winter solstice at Iglooik is approximately –50.593 W/m^2.

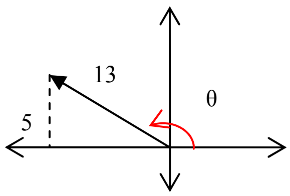
c) On the winter solstice, Igloolik receives no sunlight, so no warmth from the sun. The land is losing heat.
When $x = 66.5°$
$P = 1000 \sin (66.5° + 113.5°)$
$= 1000 \sin 180°$
$= 0$
At latitute 66.5° N, the power received is 0 W/m^2.

Section 6.2 Page 307 Question 10

$\cos(\pi + x) + \cos(\pi - x)$
$= \cos \pi \cos x - \sin \pi \sin x + \cos \pi \cos x + \sin \pi \sin x$
$= 2 \cos \pi \cos x$
$= 2(-1) \cos x$
$= -2 \cos x$

Section 6.2 Page 307 Question 11

$\sin \theta = \frac{5}{13}$ and θ is in quadrant II.
Using the Pythagorean theorem, $x = -12$.



a) $\cos 2\theta = 1 - 2\sin^2 \theta$
$= 1 - 2\left(\frac{5}{13}\right)^2$
$= 1 - 2\left(\frac{25}{169}\right)$
$= \frac{119}{169}$

b) $\sin 2\theta = 2 \cos \theta \sin \theta$
$= 2\left(\frac{-12}{13}\right)\left(\frac{5}{13}\right)$
$= -\frac{120}{169}$

c) $\sin\left(\theta + \frac{\pi}{2}\right) = \sin\theta\cos\left(\frac{\pi}{2}\right) + \cos\theta\sin\left(\frac{\pi}{2}\right)$
$= \left(\frac{5}{13}\right)0 + \left(\frac{-12}{13}\right)1$
$= -\frac{12}{13}$

Section 6.2 Page 307 Question 12

a) Substitute $x = \frac{\pi}{6}$.
Left Side $= \tan 2x$
$= \tan 2\left(\frac{\pi}{6}\right)$
$= \tan\left(\frac{\pi}{3}\right)$
$= \sqrt{3}$

$$\text{Right Side} = \frac{2\tan x}{1-\tan^2 x}$$
$$= \frac{2\tan\left(\frac{\pi}{6}\right)}{1-\tan^2\left(\frac{\pi}{6}\right)}$$
$$= 2\left(\frac{1}{\sqrt{3}}\right) \div \left(1-\left(\frac{1}{\sqrt{3}}\right)^2\right)$$
$$= \left(\frac{2}{\sqrt{3}}\right)\left(\frac{3}{2}\right)$$
$$= \sqrt{3}$$

Since Left Side = Right Side, the equation $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$ is true for $x = \frac{\pi}{6}$.

b) Substitute $x = \frac{\pi}{6}$.

From part a),
Left Side = tan 2*x*
$= \sqrt{3}$

$$\text{Right Side} = \frac{\sin 2x}{\cos 2x}$$
$$= \frac{\sin 2\left(\frac{\pi}{6}\right)}{\cos 2\left(\frac{\pi}{6}\right)}$$
$$= \frac{\sin\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{3}\right)}$$
$$= \left(\frac{\sqrt{3}}{2}\right) \div \left(\frac{1}{2}\right)$$
$$= \sqrt{3}$$

Since Left Side = Right Side, the equation $\tan 2x = \frac{\sin 2x}{\cos 2x}$ is true for $x = \frac{\pi}{6}$.

c) $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

$= \dfrac{2\tan x}{1-\tan^2 x}\left(\dfrac{\cos^2 x}{\cos^2 x}\right)$

$= \dfrac{2\left(\dfrac{\sin x}{\cos x}\right)(\cos^2 x)}{\left(1-\dfrac{\sin^2 x}{\cos^2 x}\right)\cos^2 x}$

$= \dfrac{2\sin x\cos x}{\cos^2 x-\sin^2 x}$

Section 6.2 Page 307 Question 13

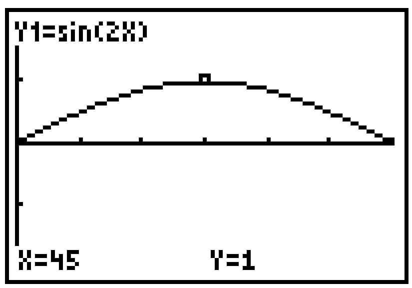
a) $d = \dfrac{2v^2\sin\theta\cos\theta}{g}$

$d = \dfrac{v^2\sin 2\theta}{g}$

b) Since the values of sine increases from 0 to 1 as θ increases from 0° to 90°, it is reasonable that the maximum distance occurs when θ = 45°, or when the value of sin 2*x* is its maximum value, 1.

Graphing the function *y* = sin 2*x* confirms this.

The maximum distance would be $\dfrac{v_0}{9.8}$ metres.



c) It is easier after applying the double-angle identity since there is only one trigonometric function to consider.

Section 6.2 Page 307 Question 14

$(\sin x + \cos x)^2 = k$

$\sin^2 x + 2\sin x\cos x + \cos^2 x = k$

$1 + \sin 2x = k$

Therefore, $\sin 2x = k - 1$.

Section 6.2 Page 307 Question 15

a) $\cos^4 x - \sin^4 x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$
$= \cos^2 x - \sin^2 x$
$= \cos 2x$

b)

$$\frac{\csc^2 x - 2}{\csc^2 x} = 1 - \frac{2}{\csc^2 x}$$
$$= 1 - 2\sin^2 x$$
$$= \cos 2x$$

Section 6.2 Page 307 Question 16

a) $$\frac{1-\cos 2x}{2} = \frac{1-(1-2\sin^2 x)}{2}$$
$$= \sin^2 x$$

b) $$\frac{4-8\sin^2 x}{2\sin x\cos x} = \frac{4\cos 2x}{\sin 2x}$$
$$= \frac{4}{\tan 2x}$$

Section 6.2 Page 307 Question 17

For the point (2, 5), $x = 2$, $y = 5$ and so $r = \sqrt{29}$.
$\cos(\pi + x) = \cos \pi \cos x - \sin \pi \sin x$

$$= -1\left(\frac{2}{\sqrt{29}}\right) - 0\left(\frac{5}{\sqrt{29}}\right)$$
$$= -\frac{2}{\sqrt{29}}$$

This answer can also be obtained by reasoning that the value of $\cos(\pi + x)$ will be numerically the same as the value of $\cos x$, but negative because the angle is in quadrant III.

Section 6.2 Page 307 Question 18

$\sin 5x \cos x + \cos 5x \sin x = \sin(5x + x)$
$= \sin 6x$
$= 2 \sin 3x \cos 3x$

The equation $\sin 5x \cos x + \cos 5x \sin x = 2 \sin kx \cos kx$ is true when $k = 3$.

Section 6.2 Page 307 Question 19

a) Given $\cos\theta = \frac{3}{5}$ and $0 < \theta < 2\pi$, θ may be in quadrant I or quadrant IV.

In quadrant I: $\sin\theta = \frac{4}{5}$

$$\begin{aligned}\sin\left(\theta + \frac{\pi}{6}\right) &= \sin\theta\cos\left(\frac{\pi}{6}\right) + \cos\theta\sin\left(\frac{\pi}{6}\right)\\ &= \left(\frac{4}{5}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{3}{5}\right)\left(\frac{1}{2}\right)\\ &= \frac{4\sqrt{3}+3}{10} \approx 0.9928\end{aligned}$$

In quadrant IV: $\sin\theta = \frac{-4}{5}$

$$\begin{aligned}\sin\left(\theta + \frac{\pi}{6}\right) &= \sin\theta\cos\left(\frac{\pi}{6}\right) + \cos\theta\sin\left(\frac{\pi}{6}\right)\\ &= \left(\frac{-4}{5}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{3}{5}\right)\left(\frac{1}{2}\right)\\ &= \frac{-4\sqrt{3}+3}{10} \approx -0.3928\end{aligned}$$

b) Given $\sin\theta = -\frac{2}{3}$ and $\frac{3\pi}{2} < \theta < 2\pi$, θ must be in quadrant IV.

In quadrant IV, $\cos\theta = \frac{\sqrt{5}}{3}$,

$$\begin{aligned}\cos\left(\theta + \frac{\pi}{3}\right) &= \cos\theta\cos\left(\frac{\pi}{3}\right) - \sin\theta\sin\left(\frac{\pi}{3}\right)\\ &= \left(\frac{\sqrt{5}}{3}\right)\left(\frac{1}{2}\right) - \left(-\frac{2}{3}\right)\left(\frac{\sqrt{3}}{2}\right)\\ &= \frac{\sqrt{5}+2\sqrt{3}}{6} \approx 0.9500\end{aligned}$$

Section 6.2 Page 307 Question 20

Given $\sin A = \frac{4}{5}$, then $\cos A = \frac{3}{5}, \tan A = \frac{4}{3}$

$\cos B = \frac{12}{13}$, then $\sin B = \frac{5}{13}, \tan A = \frac{5}{12}$

a) cos (A – B) = cos A cos B + sin A sin B

$$= \left(\frac{3}{5}\right)\left(\frac{12}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{5}{13}\right)$$
$$= \frac{36+20}{65}$$
$$= \frac{56}{65}$$

b) sin (A + B) = sin A cos B + cos A sin B

$$= \left(\frac{4}{5}\right)\left(\frac{12}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{5}{13}\right)$$
$$= \frac{48+15}{65}$$
$$= \frac{63}{65}$$

c) cos 2A = $\cos^2$ A – $\sin^2$ A

$$= \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2$$
$$= \frac{9}{25} - \frac{16}{25}$$
$$= \frac{-7}{25}$$

d) sin 2A = 2 sin A cos A

$$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$$
$$= \frac{24}{25}$$

Section 6.2 Page 307 Question 21

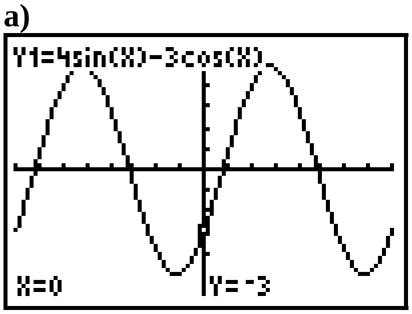
a)
$$\frac{\boxed{?}\sin 2x}{2-2\cos^2 x}=\cos x$$
$$\frac{\boxed{?}2\sin x\cos x}{2(1-\cos^2 x)}=\cos x$$
$$\frac{\boxed{?}2\sin x\cos x}{2(\sin^2 x)}=\cos x$$
$$\frac{\boxed{?}2\cos x}{2\sin x}=\cos x$$
The equation will be true if the missing ratio is sin *x*.

Section 6.2 Page 307 Question 22

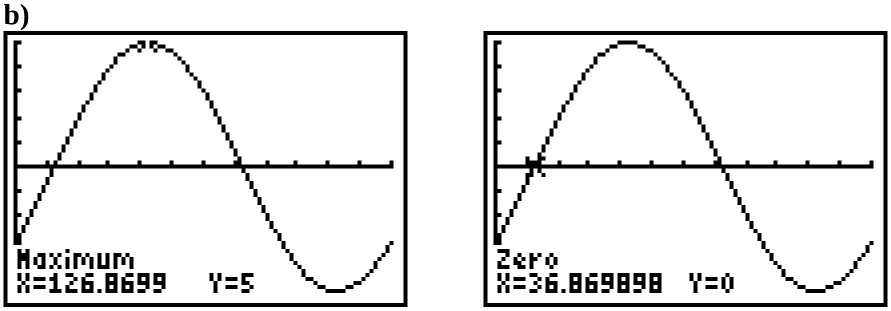
$$\cos x=2\cos^2\left(\frac{x}{2}\right)-1$$
$$\frac{\cos x+1}{2}=\cos^2\left(\frac{x}{2}\right)$$
$$\pm\sqrt{\frac{\cos x+1}{2}}=\cos\frac{x}{2}$$

Section 6.2 Page 308 Question 23

a)



b)



By analysing the graph using technology, the maximum is 5 and the horizontal shift is approximately 37. For the curve in the form $y = a \sin (x - c)$, $a = 5$ and $c \approx 37°$.

c) $4 \sin x - 3 \cos x = 5\left[\sin x\left(\frac{4}{5}\right) - \cos x\left(\frac{3}{5}\right)\right]$

$$= 5\left[\sin\left(x - \cos^{-1}\left(\frac{4}{5}\right)\right)\right]$$

$$\approx 5\sin(x - 36.87)$$

Section 6.2 Page 308 Question 24

$y = 6 \sin x \cos^3 x + 6 \sin^3 x \cos x - 3$
$y = 6 \sin x \cos x (\cos^2 x + \sin^2 x) - 3$
$y = 3 (2 \sin x \cos x) - 3$
$y = 3 \sin 2x - 3$

Section 6.2 Page 308 Question C1

a) If $\cos x = -\frac{5}{13}$ and $\pi < x < \frac{3\pi}{2}$ then $\sin x = -\frac{12}{13}$

i) Since $\cos x = -\frac{5}{13}$ and x in in quadrant III, $x \approx 4.3176$. Then,

$\sin x = -\cos\left(x + \frac{\pi}{2}\right)$

$\sin 2x = -\cos\left(2x + \frac{\pi}{2}\right)$

$\sin 2x \approx -\cos\left(2(4.3176) + \frac{\pi}{2}\right)$

$\sin 2x \approx 0.7101$

ii) $\sin 2x = 2 \sin x \cos x$

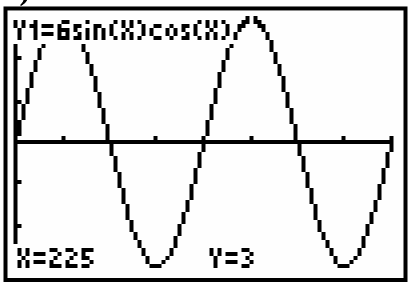
$$= 2\left(-\frac{12}{13}\right)\left(-\frac{5}{13}\right)$$

$$= \frac{120}{169} \approx 0.7101$$

b) Using the double-angle identity is more straightforward.

Section 6.2 Page 308 Question C2

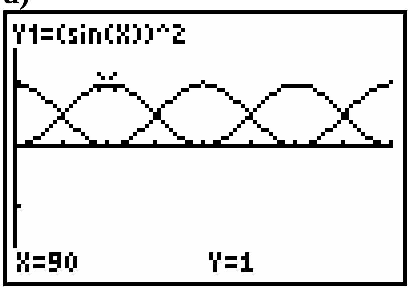
a)



b) To find the sine function from the graph you compare the amplitude and the period to that of a base sine curve. The alternative equation is $y = 3 \sin 2x$. This equation is found directly from the given equation $y = 6 \sin x \cos x$ using the the double-angle identity.

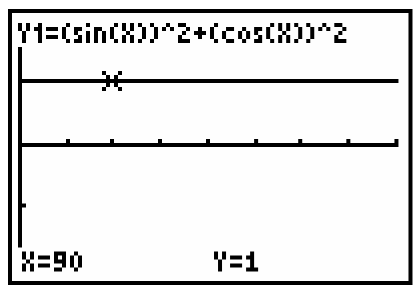
Section 6.2 Page 308 Question C3

a)

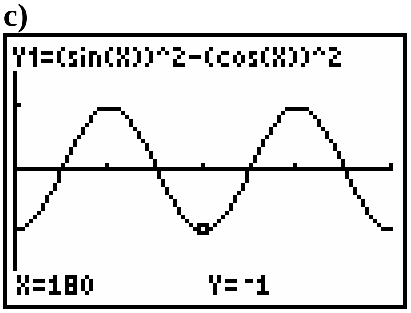


The graphs have the same shape and may be related by a reflection in the line $y = 0.5$ or by a translation of 90° to the right.

b) I predict that $y_1 + y_2$ will be a horizontal line passing through (0, 1), because the two functions increase and decrease from 0 to 1 relative to each other, and their sum is always 1.



c)



The resulting graph is a cosine function reflected over the x-axis and the period becomes π.

d) Using trigonometric identities,

$$\begin{aligned}\sin^2 x - \cos^2 x &= 1 - \cos^2 x - \cos^2 x \\ &= 1 - 2\cos^2 x \\ &= -\cos 2x\end{aligned}$$

So in the form $f(x) = a \cos bx$, the function is $f(x) = -\cos 2x$.

Section 6.3 Proving Identities

Section 6.3 Page 314 Question 1

a) $$\frac{\sin x - \sin x\cos^2 x}{\sin^2 x} = \frac{\sin x(1-\cos^2 x)}{\sin^2 x}$$
$$= \frac{\sin x(\sin^2 x)}{\sin^2 x}$$
$$= \sin x$$

b) $$\frac{\cos^2 x - \cos x - 2}{6\cos x - 12} = \frac{(\cos x - 2)(\cos x + 1)}{6(\cos x - 2)}$$
$$= \frac{\cos x + 1}{6}$$

c) $$\frac{\sin x\cos x - \sin x}{\cos^2 x - 1} = \frac{\sin x(\cos x - 1)}{(\cos x - 1)(\cos x + 1)}$$
$$= \frac{\sin x}{\cos x + 1}$$

d) $$\frac{\tan^2 x - 3\tan x - 4}{\sin x\tan x + \sin x} = \frac{(\tan x - 4)(\tan x + 1)}{\sin x(\tan x + 1)}$$
$$= \frac{\tan x - 4}{\sin x} \text{ or } \sec x - 4\csc x$$

Section 6.3 Page 314 Question 2

a) Left Side = $\cos x + \cos x\tan^2 x$
$$= \cos x + \frac{\sin^2 x}{\cos x}$$
$$= \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x}$$
$$= \frac{1}{\cos x}$$
$$= \sec x$$
= Right Side

b) Left Side = $\dfrac{\sin^2 x - \cos^2 x}{\sin x + \cos x}$
$$= \frac{(\sin x - \cos x)(\sin x + \cos x)}{\sin x + \cos x}$$
$$= \sin x - \cos x$$
= Right Side

c) Left Side $= \dfrac{\sin x \cos x - \sin x}{\cos^2 x - 1}$

$$= \frac{\sin x \cos x - \sin x}{-\sin^2 x}$$
$$= \frac{-\sin x(1 - \cos x)}{-\sin^2 x}$$
$$= \frac{1 - \cos x}{\sin x}$$
$$= \text{Right Side}$$

d) Left Side $= \dfrac{1 - \sin^2 x}{1 + 2\sin x - 3\sin^2 x}$

$$= \frac{(1 - \sin x)(1 + \sin x)}{(1 - \sin x)(1 + 3\sin x)}$$
$$= \frac{1 + \sin x}{1 + 3\sin x}$$
$$= \text{Right Side}$$

Section 6.3 Page 314 Question 3

a) $\dfrac{\sin x}{\cos x} + \sec x = \dfrac{\sin x}{\cos x} + \dfrac{1}{\cos x}$

$$= \frac{\sin x + 1}{\cos x}$$

b) $\dfrac{1}{\sin x - 1} + \dfrac{1}{\sin x + 1} = \dfrac{\sin x + 1 + \sin x - 1}{(\sin x - 1)(\sin x + 1)}$

$$= \frac{2\sin x}{\sin^2 x - 1}$$
$$= \frac{2\sin x}{-\cos^2 x}$$
$$= \frac{-2\tan x}{\cos x}$$

c) $\dfrac{\sin x}{1 + \cos x} + \dfrac{\cos x}{\sin x} = \dfrac{\sin^2 x + \cos x(1 + \cos x)}{(1 + \cos x)\sin x}$

$$= \frac{\sin^2 x + \cos x + \cos^2 x}{(1 + \cos x)\sin x}$$
$$= \frac{1 + \cos x}{(1 + \cos x)\sin x}$$
$$= \frac{1}{\sin x}$$
$$= \csc x$$

d) $\dfrac{\cos x}{\sec x - 1} + \dfrac{\cos x}{\sec x + 1} = \dfrac{\cos x(\sec x + 1) + \cos x(\sec x - 1)}{(\sec x - 1)(\sec x + 1)}$

$$= \frac{\cos x\left(\dfrac{1}{\cos x} + 1\right) + \cos x\left(\dfrac{1}{\cos x} - 1\right)}{\sec^2 x - 1}$$

$$= \frac{1 + \cos x + 1 - \cos x}{\tan^2 x}$$

$$= \frac{2}{\tan^2 x}$$

$$= 2\cot^2 x$$

Section 6.3 Page 314 Question 4

a) $\dfrac{\sec x - \cos x}{\tan x} = \dfrac{\sec x}{\tan x} - \dfrac{\cos x}{\tan x}$

$$= \left(\frac{1}{\cos x}\right)\left(\frac{\cos x}{\sin x}\right) - \cos x\left(\frac{\cos x}{\sin x}\right)$$

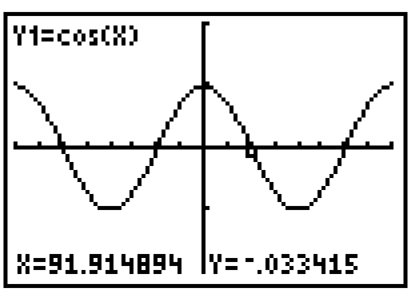
$$= \frac{1}{\sin x} - \frac{\cos^2 x}{\sin x}$$

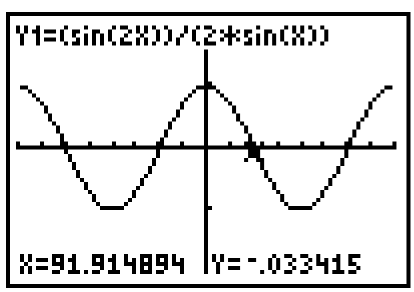
b) $\dfrac{1}{\sin x} - \dfrac{\cos^2 x}{\sin x} = \dfrac{1 - \cos^2 x}{\sin x}$

$$= \frac{\sin^2 x}{\sin x}$$

$$= \sin x$$

Section 6.3 Page 314 Question 5





From the graphs, $\cos x = \dfrac{\sin 2x}{2\sin x}$ appears to be an identity.

$$\frac{\sin 2x}{2\sin x} = \frac{2\sin x \cos}{2\sin x}$$

$$= \cos x$$

To allow division by sin x, $x \neq \pi n;\ n \in \mathrm{I}$.

Section 6.3 Page 314 Question 6

$$\begin{aligned}(\sec x - \tan x)(\sin x + 1) &= \sec x \sin x + \sec x - \tan x \sin x - \tan x \\ &= \left(\frac{1}{\cos x}\right)\sin x + \frac{1}{\cos x} - \left(\frac{\sin x}{\cos x}\right)\sin x - \frac{\sin x}{\cos x} \\ &= \frac{1}{\cos x} - \frac{\sin^2 x}{\cos x} \\ &= \frac{1-\sin^2 x}{\cos x} \\ &= \frac{\cos^2 x}{\cos x} \\ &= \cos x\end{aligned}$$

Section 6.3 Page 314 Question 7

a) $$\begin{aligned}\frac{\csc x}{2\cos x} &= \frac{1}{2\sin x\cos x} \\ &= \frac{1}{\sin 2x} \\ &= \csc 2x\end{aligned}$$

b) $$\begin{aligned}\sin x + \cos x \cot x &= \sin x + \cos x\left(\frac{\cos x}{\sin x}\right) \\ &= \frac{\sin^2 x + \cos^2 x}{\sin x} \\ &= \frac{1}{\sin x} \\ &= \csc x\end{aligned}$$

Section 6.3 Page 314 Question 8

Hanna's Method:

$$\begin{aligned}\text{Left Side} &= \frac{\cos 2x - 1}{\sin 2x} \\ &= \frac{1-2\sin^2 x - 1}{2\sin x\cos x} \\ &= \frac{-2\sin^2 x}{2\sin x\cos x} \\ &= \frac{-\sin x}{\cos x} \\ &= -\tan x \\ &= \text{ Right Side}\end{aligned}$$

Chloe's Method:

$$\begin{aligned}\text{Left Side} &= \frac{\cos 2x - 1}{\sin 2x} \\ &= \frac{2\cos^2 x - 1 - 1}{2\sin x\cos x} \\ &= \frac{2(\cos^2 x - 1)}{2\sin x\cos x} \\ &= \frac{2(-\sin^2 x)}{2\sin x\cos x} \\ &= \frac{-\sin x}{\cos x} \\ &= -\tan x \\ &= \text{ Right Side}\end{aligned}$$

Hanna's method is a bit simpler and leads to a shorter proof.

Section 6.3 Page 314 Question 9

a) Substitute $v_0 = 21$, $\theta = 55°$, and $g = 9.8$.

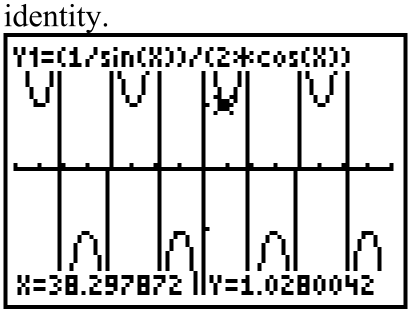
$$d = \frac{v_0^2 \sin 2\theta}{g}$$
$$= \frac{21^2 \sin 2(55°)}{9.8}$$
$$\approx 42.3$$

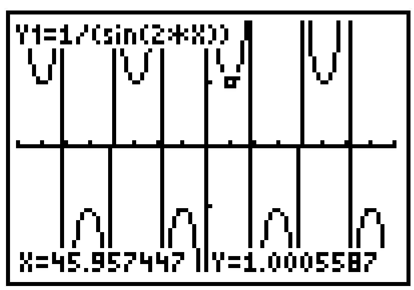
The ball will travel approximately 42.3 m.

b)
$$\frac{v_o^{\,2} \sin 2\theta}{g} = \frac{v_o^{\,2}\, 2\sin\theta\cos\theta}{g}$$
$$= \frac{2v_o^{\,2} \sin^2\theta\cos\theta}{g\sin\theta}$$
$$= \frac{2v_o^{\,2}\sin^2\theta}{g\tan\theta}$$
$$= \frac{2v_o^{\,2}(1-\cos^2\theta)}{g\tan\theta}$$

Section 6.3 Page 314 Question 10

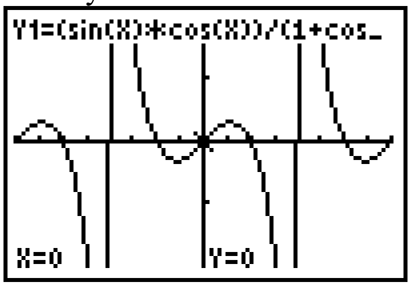
a) The graphs of each side appear to be the same, so the equation is potentially an identity.

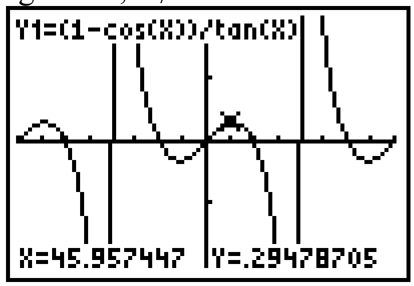




$$\text{Left Side} = \frac{\csc x}{2\cos x}$$
$$= \frac{1}{2\sin x\cos x}$$
$$= \frac{1}{\sin 2x}$$
$$= \csc 2x$$
$$= \text{Right Side}$$

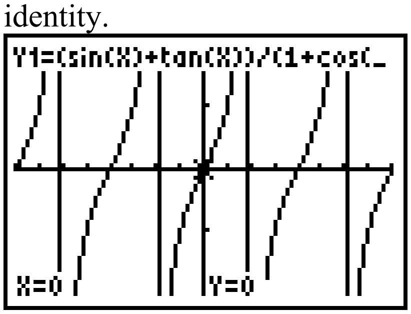
b) The graphs of each side appear to be the same, so the equation is potentially an identity. There is a restriction on the right side, $x \neq 0° + 180°n$.

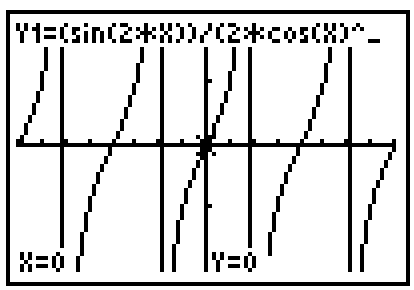




$$\text{Left Side} = \frac{\sin x \cos x}{1+\cos x}$$
$$= \frac{(\sin x \cos x)(1-\cos x)}{(1+\cos x)(1-\cos x)}$$
$$= \frac{\sin x \cos x - \sin x \cos^2 x}{\sin^2 x}$$
$$= \frac{\cos x - \cos^2 x}{\sin x}$$
$$= \frac{1-\cos x}{\tan x}$$
$$= \text{Right Side}$$

c) The graphs of each side appear to be the same, so the equation is potentially an identity.





$$\text{Left Side} = \frac{\sin x + \tan x}{1+\cos x}$$
$$= \left(\frac{\sin x}{1} + \frac{\sin x}{\cos x}\right) \div (1+\cos x)$$
$$= \left(\frac{\sin x \cos x + \sin x}{\cos x}\right)\left(\frac{1}{1+\cos x}\right)$$
$$= \left(\frac{\sin x(1+\cos x)}{\cos x}\right)\left(\frac{1}{1+\cos x}\right)$$
$$= \frac{\sin x}{\cos x}$$

Section 6.3 Page 314 Question 11

a)

$$\begin{aligned}\text{Left Side} &= \frac{\sin 2x}{\cos x}+\frac{\cos 2x}{\sin x}\\ &= \frac{2\sin x\cos x}{\cos x}+\frac{1-2\sin^2 x}{\sin x}\\ &= 2\sin x+\csc x-2\sin x\\ &= \csc x\\ &= \text{ Right Side}\end{aligned}$$

b)

$$\begin{aligned}\text{Left Side} &= \csc^2 x+\sec^2 x\\ &= \frac{1}{\sin^2 x}+\frac{1}{\cos^2 x}\\ &= \frac{\sin^2 x+\cos^2 x}{\sin^2 x\cos^2 x}\\ &= \frac{1}{\sin^2 x\cos^2 x}\\ &= \csc^2 x\sec^2 x\\ &= \text{ Right Side}\end{aligned}$$

c)

$$\begin{aligned}\text{Left Side} &= \frac{\cot x-1}{1-\tan x}\\ &= \frac{1-\tan x}{\tan x(1-\tan x)}\\ &= \frac{1}{\tan x}\\ &= \frac{\csc x}{\sec x}\\ &= \text{ Right Side}\end{aligned}$$

Section 6.3 Page 315 Question 12

a) Left Side = sin (90° + θ)
= sin 90° cos θ + cos 90° sin θ
= cos θ
Right Side = sin (90° – θ)
= sin 90° cos θ – cos 90° sin θ
= cos θ
Left Side = Right Side, so sin (90° + θ) = sin (90° – θ).

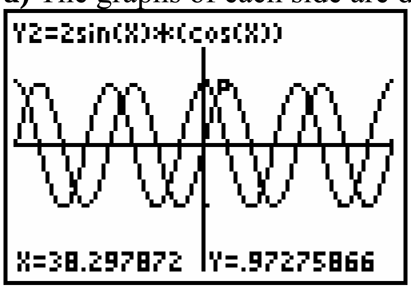
b) Left Side = sin (2π – θ)
= sin (2π) cos (θ) – cos (2π) sin (θ)
= –sin θ
= Right Side
So, sin (2π – θ) = –sin θ.

Section 6.3 Page 315 Question 13

Left Side = 2 cos x cos y
Right Side = cos(x + y) + cos(x – y)
= cos x cos y – sin x sin y + cos x cos y + sin x sin y
= 2 cos x cos y

Section 6.3 Page 315 Question 14

a) The graphs of each side are different so the equation is not an identity.



b) Try $x = 30°$.

Left Side = cos 2x
= cos 2(30°)
= cos 60°
= 0.5

Right Side = 2 sin x cos x
= 2 sin 30° cos 30°
$= 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$
$= \frac{\sqrt{3}}{2}$

Left Side ≠ Right Side, so cos 2x = 2 sin x sec x is not an identity.

Section 6.3 Page 315 Question 15

a) For $\frac{\sin 2x}{1-\cos 2x} = \cot x$:
The left side denominator cannot be zero.
So, cos 2x ≠ 1, or 2x ≠ 0°, x ≠ 0° + 180°n.
For the right side, $\cot x = \frac{\cos x}{\sin x}$, so sin x ≠ 0, or x ≠ 0°.
In general, the non-permissible values are $x \neq 180°n$, $n \in$ I.

b) $\text{Left Side} = \dfrac{\sin 2x}{1-\cos 2x}$

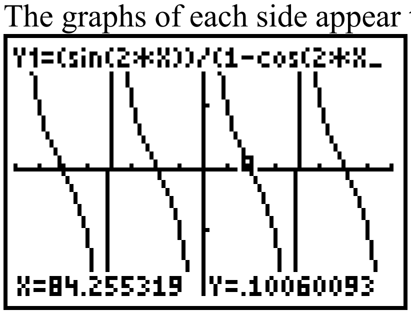
$$= \frac{2\sin x\cos x}{1-1+2\sin^2 x}$$
$$= \frac{\cos x}{\sin x}$$
$$= \cot x$$
$$= \text{ Right Side}$$

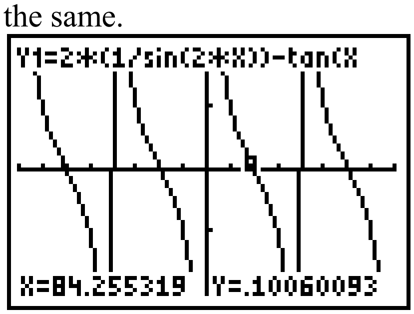
Section 6.3 Page 315 Question 16

$$\text{Right Side} = \frac{\sin 4x - \sin 2x}{\cos 4x + \cos 2x}$$
$$= \frac{2\sin 2x\cos 2x - 2\sin x\cos x}{\cos 4x + 2\cos^2 x - 1}$$
$$= \frac{2(2\sin x\cos x)(2\cos^2 x - 1) - 2\sin x\cos x}{2\cos^2 2x - 1 + 2\cos^2 x - 1}$$
$$= \frac{(2\sin x\cos x)(2(2\cos^2 x - 1) - 1)}{2(2\cos^2 x - 1)^2 + 2\cos^2 x - 2}$$
$$= \frac{(2\sin x\cos x)(4\cos^2 x - 3)}{2(4\cos^4 x - 4\cos^2 x + 1) + 2\cos^2 x - 2}$$
$$= \frac{(2\sin x\cos x)(4\cos^2 x - 3)}{8\cos^4 x - 6\cos^2 x}$$
$$= \frac{2\sin x\cos x}{2\cos^2 x}$$
$$= \tan x$$
$$= \text{ Left Side}$$

Section 6.3 Page 315 Question 17

The graphs of each side appear to be the same.





$$
\begin{aligned}
\text{Left Side} &= \frac{\sin 2x}{1-\cos 2x} \\
&= \frac{\sin 2x + \sin 2x \cos 2x}{1-\cos^2 2x} \\
&= \frac{\sin 2x + \sin 2x \cos 2x}{\sin^2 2x} \\
&= \frac{1}{\sin 2x} + \frac{\cos 2x}{\sin 2x} \\
&= \frac{1}{\sin 2x} + \frac{1-2\sin^2 x}{\sin 2x} \\
&= \frac{2}{\sin 2x} - \frac{2\sin^2 x}{\sin 2x} \\
&= 2\csc 2x - \frac{2\sin^2 x}{2\sin x \cos x} \\
&= 2\csc 2x - \tan x \\
&= \text{ Right Side}
\end{aligned}
$$

Section 6.3 Page 315 Question 18

$$
\begin{aligned}
\text{Left Side} &= \frac{1-\sin^2 x - 2\cos x}{\cos^2 x - \cos x - 2} \\
&= \frac{\cos^2 x - 2\cos x}{\cos^2 x - \cos x - 2} \\
&= \frac{\cos x(\cos x - 2)}{(\cos x - 2)(\cos x + 1)} \\
&= \frac{\cos x}{(\cos x + 1)} \\
&= \frac{1}{1+\sec x} \\
&= \text{ Right Side}
\end{aligned}
$$

Section 6.3 Page 315 Question 19

a) $n_1 \sin \theta_i = n_2 \sin \theta_t$

$$\frac{n_1 \sin \theta_i}{n_2} = \sin \theta_t$$

b) Using $\sin^2 \theta + \cos^2 \theta = 1$, substitute $\cos\theta = \sqrt{1-\sin^2\theta}$.

$$R = \left(\frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}\right)^2$$
$$= \left(\frac{n_1 \cos\theta_i - n_2\sqrt{1-\sin^2\theta_t}}{n_1 \cos\theta_i + n_2\sqrt{1-\sin^2\theta_t}}\right)^2$$

c) From part a) substitute $\sin\theta_t = \dfrac{n_1 \sin\theta_i}{n_2}$, or $\sin^2\theta_t = \left(\dfrac{n_1}{n_2}\right)^2 \sin^2\theta_i$, in

$$\left(\frac{n_1 \cos\theta_i - n_2\sqrt{1-\sin^2\theta_t}}{n_1 \cos\theta_i + n_2\sqrt{1-\sin^2\theta_t}}\right)^2.$$

$$\left(\frac{n_1 \cos\theta_i - n_2\sqrt{1-\sin^2\theta_t}}{n_1 \cos\theta_i + n_2\sqrt{1-\sin^2\theta_t}}\right)^2 = \left(\frac{n_1 \cos\theta_i - n_2\sqrt{1-\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_i}}{n_1 \cos\theta_i + n_2\sqrt{1-\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_i}}\right)^2$$

Section 6.3 Page 315 Question C1

The graphs may appear to be the same but there may be some values that one function does not take; there may be discontinuities in the graph. Only pure luck would identify these values either by graphing or by checking numerically.

Section 6.3 Page 315 Question C2

$$\text{Left Side} = \cos\left(\frac{\pi}{2} - x\right)$$
$$= \cos\left(\frac{\pi}{2}\right)\cos x + \sin\left(\frac{\pi}{2}\right)\sin x$$
$$= \sin x$$
$$= \text{ Right Side}$$

Section 6.3 Page 315 Question C3

a) In the equation, the radical must be positive and the radicand cannot be negative. So, $\cos x \geq 0$ and $1 - \sin^2 x \geq 0$. The second condition is always true.
From the first condition, x is in quadrant I or IV. The non-permissible values are any values of x in quadrant II or III. In general, the non-permissible values are
$\dfrac{\pi}{2} + 2\pi n < x < \dfrac{3\pi}{2} + 2\pi n,\ n \in \mathrm{I}$

b) The equation is true for $x = 1$.

Left Side = $\cos 1 \approx 0.5403$ Right Side = $\sqrt{1-\sin^2(1)} \approx 0.5403$

c) The equation is not true for $x = \pi$.

Left Side = $\cos \pi = -1$ Right Side = $\sqrt{1-\sin^2(\pi)} = 1$

d) An identity is always true for all values for which each side of the equation is defined. An equation may be true for a restricted domain.

Section 6.4 Solving Trigonometric Equations Using Identities

Section 6.4 Page 320 Question 1

In the domain $0 \le x < 2\pi$:

a) $\tan^2 x - \tan x = 0$
$\tan x (\tan x - 1) = 0$
$\tan x = 0$ or $\tan x = 1$
$x = 0, \pi$, or $x = \frac{\pi}{4}, \frac{5\pi}{4}$

b) $\sin 2x - \sin x = 0$
$2 \sin x \cos x - \sin x = 0$
$\sin x (2 \cos x - 1) = 0$
$\sin x = 0$ or $\cos x = \frac{1}{2}$
$x = 0, \pi$, or $x = \frac{\pi}{3}, \frac{5\pi}{3}$

c) $\sin^2 x - 4 \sin x = 5$
$\sin^2 x - 4 \sin x - 5 = 0$
$(\sin x - 5)(\sin x + 1) = 0$
$\sin x = 5$ or $\sin x = -1$
The first value for $\sin x$ is impossible.
$x = \frac{3\pi}{2}$

d) $\cos 2x = \sin x$
$1 - 2 \sin^2 x = \sin x$
$2 \sin^2 x + \sin x - 1 = 0$
$(2 \sin x - 1)(\sin x + 1) = 0$
$\sin x = \frac{1}{2}$ or $\sin x = -1$
$x = \frac{\pi}{6}, \frac{5\pi}{6}$ or $x = \frac{3\pi}{2}$

Section 6.4 Page 320 Question 2

In the domain $0° \le x < 360°$:

a) $\cos x - \cos 2x = 0$
$\cos x - (2 \cos^2 x - 1) = 0$
$2 \cos^2 x - \cos x - 1 = 0$
$(2 \cos x + 1)(\cos x - 1) = 0$
$\cos x = -\frac{1}{2}$ or $\cos x = 1$
$x = 120°, 240°$ or $x = 0°$

b) $\sin^2 x - 3 \sin x = 4$
$\sin^2 x - 3 \sin x - 4 = 0$
$(\sin x - 4)(\sin x + 1) = 0$
$\sin x = 4$ or $\sin x = -1$
The first value for $\sin x$ is impossible.
$x = 270°$

c) $\tan x \cos x \sin x - 1 = 0$

$$\left(\frac{\sin x}{\cos x}\right)\cos x \sin x - 1 = 0$$

$$\sin^2 x - 1 = 0$$

$$(\sin x - 1)(\sin x + 1) = 0$$

$$\sin x = 1 \text{ or } \sin x = -1$$

$x = 90°$ or $270°$

However, the initial equation has restrictions $\cos 90º = 0$ and $\cos 270º = 0$ are not permissible.

So, there is no solution.

d) $\tan^2 x + \sqrt{3}\tan x = 0$

$$\tan x(\tan x + \sqrt{3}) = 0$$

$$\tan x = 0 \text{ or } \tan x = -\sqrt{3}$$

$x = 0°, 180°$, or $x = 120°, 300°$

Section 6.4 Page 320 Question 3

In the domain $0 \le x < 2\pi$:

a) $\cos 2x - 3 \sin x = 2$

$$1 - 2\sin^2 x - 3\sin x = 2$$

$$2\sin^2 x + 3\sin x + 1 = 0$$

$$(2\sin x + 1)(\sin x + 1) = 0$$

$$\sin x = -\frac{1}{2} \text{ or } \sin x = -1$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6} \text{ or } x = \frac{3\pi}{2}$$

b) $2\cos^2 x - 3\sin x - 3 = 0$

$$2(1 - \sin^2 x) - 3\sin x - 3 = 0$$

$$2\sin^2 x + 3\sin x + 1 = 0$$

$$(2\sin x + 1)(\sin x + 1) = 0$$

$$\sin x = -\frac{1}{2} \text{ or } \sin x = -1$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6} \text{ or } x = \frac{3\pi}{2}$$

c) $3\csc x - \sin x = 2$

$$3\left(\frac{1}{\sin x}\right) - \sin x = 2$$

$$3 - \sin^2 x = 2\sin x$$

$$\sin^2 x + 2\sin x - 3 = 0$$

$$(\sin x + 3)(\sin x - 1) = 0$$

$$\sin x = -3 \text{ or } \sin x = 1$$

The first value for $\sin x$ is impossible.

$$x = \frac{\pi}{2}$$

d) $\tan^2 x + 2 = 0$

$$\frac{\sin^2 x}{\cos^2 x} + 2 = 0$$

$$\sin^2 x + 2\cos^2 x = 0$$

$$\sin^2 x + 2(1 - \sin^2 x) = 0$$

$$2 - \sin^2 x = 0$$

$\sin^2 x = 2$ is impossible, so the equation has no solution.

Section 6.4 Page 320 Question 4

$4 \sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\frac{1}{2}$

In the domain $-180° \le x < 180°$: $x = -150°, -30°, 30°, 150°$

Section 6.4 Page 320 Question 5

$2 \tan^2 x + 3 \tan x - 2 = 0$

$(2 \tan x - 1)(\tan x + 2) = 0$

$\tan x = \frac{1}{2}$ or $\tan x = -2$

In the domain $0 \le x < 2\pi$: $x \approx 0.4636, 3.6052$, or $x \approx 2.0344, 5.1760$

Section 6.4 Page 321 Question 6

Sanesh should not have divided both sides by cos *x*. Some solutions were lost by doing that.

$2\cos^2 x = \sqrt{3}\cos x$

$2\cos^2 x - \sqrt{3}\cos x = 0$

$\cos x(2\cos x - \sqrt{3}) = 0$

$\cos x = 0$ or $\cos x = \frac{\sqrt{3}}{2}$

$x = 90° + 360°n$ and $x = 270° + 360°n$, or $x = 30° + 360°n$ and $x = 330° + 360°n$, where $n \in$ I.

Section 6.4 Page 321 Question 7

a) $\sin 2x = 0.5$

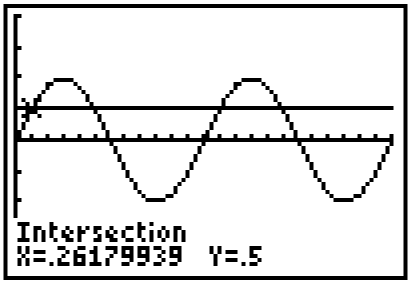
$2x = \sin^{-1}(0.5)$

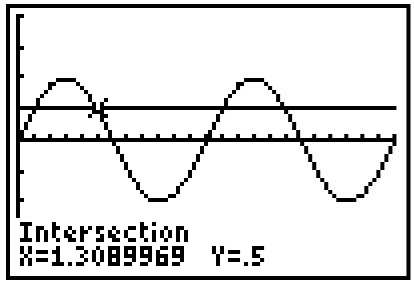
$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \ldots$

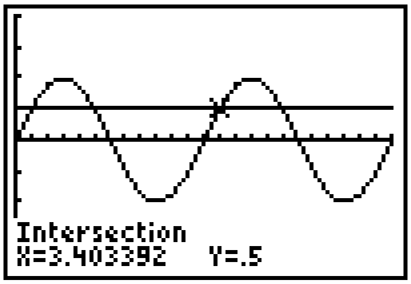
In the domain $0 \le x < 2\pi$:

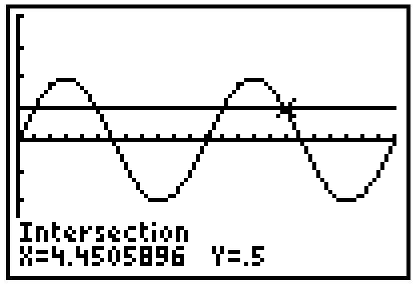
$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

b) Graph $y = \sin 2x$ and $y = 0.5$ over the domain $0 \leq \theta\ \pi$ and find the point(s) of intersection.









Section 6.4 Page 321 Question 8

$\sin^2 x = \cos^2 x + 1$
$\sin^2 x = 1 - \sin^2 x + 1$
$2 \sin^2 x - 2 = 0$
$2 (\sin^2 x - 1) = 0$
$\sin^2 x = 1$
$\sin x = \pm 1$
$x = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

In general, $x = \frac{\pi}{2} + \pi n,\ n \in \mathrm{I}$

Section 6.4 Page 321 Question 9

$\cos x \sin 2x - 2 \sin x = -2$
$\cos x\,(2 \sin x \cos x) - 2 \sin x + 2 = 0$
$2 \sin x \cos^2 x - 2 \sin x + 2 = 0$
$\sin x(1 - \sin^2 x) - \sin x + 1 = 0$
$\sin x - \sin^3 x - \sin x + 1 = 0$
$\sin^3 x = 1$
$\sin x = 1$

The general solution, in radians, is $x = \frac{\pi}{2} + 2\pi n,\ n \in \mathrm{I}$.

Section 6.4 Page 321 Question 10

The equation $(7 \sin x + 2)(3 \cos x + 3)(\tan^2 x - 2) = 0$ will have 7 solutions over the interval $0° < x \le 360°$. The first factor yields two solutions, one each is in quadrants II and IV where x is $\sin^{-1}\left(-\frac{2}{7}\right)$. The second factor yields one solution, $\cos^{-1}(-1)$ and the third factor yields four solutions, one in each quadrant for $\tan^{-1}(\pm 2)$.

Section 6.4 Page 321 Question 11

$$\sqrt{3}\cos x \csc x = -2\cos x$$
$$2\cos x + \sqrt{3}\cos x \csc x = 0$$
$$\cos x(2 + \sqrt{3}\csc x) = 0$$
$$\cos x = 0 \quad \text{or} \quad \csc x = -\frac{2}{\sqrt{3}}$$
$$\sin x = -\frac{\sqrt{3}}{2}$$

Over the domain $0 \le x < 2\pi$:

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad \text{or} \quad x = \frac{4\pi}{3}, \frac{5\pi}{3}$$

Section 6.4 Page 321 Question 12

Given that $\cos x = \frac{2}{3}$ and $\cos x = -\frac{1}{3}$ are the solutions for a trigonometric equation, then the equation has the form

$(3 \cos x - 2)(3 \cos x + 1) = 0$

$9 \cos^2 x - 3 \cos x - 2 = 0$

So, in the form $9 \cos^2 x - B \cos x - C = 0$, $B = -3$ and $C = -2$.

Section 6.4 Page 321 Question 13

Example: Give a general solution, in degrees to the following equation.

$\sin 2x + \sin 2x \cos x = 0$

$\sin 2x(1 + \cos x) = 0$

$\sin 2x = 0$ or $\cos x = -1$

$2x = 0°, 180°, 360°, \ldots$ or $x = 180°, 540°, \ldots$

$x = 0°, 90°, 180°, 270°, 360°, \ldots$

In general, $x = 90°n$, $n \in \mathrm{I}$.

Section 6.4 Page 321 Question 14

$$\sin 2x = 2 \cos x \cos 2x$$
$$2 \sin x \cos x - 2 \cos x \cos 2x = 0$$
$$2 \sin x \cos x - 2 \cos x (1 - 2 \sin^2 x) = 0$$
$$2 \sin x \cos x - 2 \cos x + 4 \cos x \sin^2 x = 0$$
$$2 \cos x (2 \sin^2 x + \sin x - 1) = 0$$
$$2 \cos x (2 \sin x - 1)(\sin x + 1) = 0$$
$$\cos x = 0 \text{ or } \sin x = \frac{1}{2} \text{ or } \sin x = -1$$

So, $x = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$ or $x = \frac{\pi}{6}, \frac{5\pi}{6}, \ldots$ or $x = \frac{3\pi}{2}, \ldots$

The general solution, in radians, is $x = \left(\frac{\pi}{2}\right)(2n+1)$ or $x = \left(\frac{\pi}{6}\right) + 2\pi n$ or $\left(\frac{5\pi}{6}\right) + 2\pi n$, where $n \in$ I.

Section 6.4 Page 321 Question 15

Over the domain $-360° < x \leq 360°$:

$$\cos 2x \cos x - \sin 2x \sin x = 0$$
$$(1 - 2 \sin^2 x) \cos x - 2 \sin x \cos x \sin x = 0$$
$$\cos x - 4 \sin^2 x \cos x = 0$$
$$\cos x (1 - 4 \sin^2 x) = 0$$
$$\cos x = 0 \text{ or } \sin x = \pm\frac{1}{2}$$

$x = -270°, -90°, 90°, 270°$ or $x = -330°, -210°, -150°, -30°, 30°, 150°, 210°, 330°$
There are 12 solutions in the given domain.

Section 6.4 Page 321 Question 16

$$\sec x + \tan^2 x - 3 \cos x = 2$$
$$\frac{1}{\cos x} + \frac{\sin^2 x}{\cos^2 x} - 3\cos x = 2$$
$$\cos x + \sin^2 x - 3\cos^3 x - 2\cos^2 x = 0$$
$$\cos x + 1 - \cos^2 x - 3\cos^3 x - 2\cos^2 x = 0$$
$$3\cos^3 x + 3\cos^2 x - \cos x - 1 = 0$$

Checking the equation, considering it as $3x^3 + 3x^2 - x - 1 = 0$, with the factor theorem reveals that $\cos x + 1$ is one factor.

$(\cos x + 1)(3\cos^2 x - 1) = 0$

$\cos x = -1$ or $\cos x = \pm\sqrt{\frac{1}{3}}$

The general solution, in radians, is $x = \pi + 2\pi n$, or $x \approx \pm 0.9553 + \pi n$, where $n \in$ I.

Section 6.4 Page 321 Question 17

$$4\sin^2 x = 3\tan^2 x - 1$$
$$4(1-\cos^2 x) = 3\left(\frac{\sin^2 x}{\cos^2 x}\right) - 1$$
$$4\cos^2 x - 4\cos^4 x = 3\sin^2 x - \cos^2 x$$
$$4\cos^2 x - 4\cos^4 x = 3(1-\cos^2 x) - \cos^2 x$$
$$4\cos^2 x - 4\cos^4 x = 3 - 4\cos^2 x$$
$$4\cos^4 x - 8\cos^2 x + 3 = 0$$
$$(2\cos^2 x - 3)(2\cos^2 x - 1) = 0$$
$$\cos^2 x = \frac{3}{2} \quad \text{or} \quad \cos^2 x = \frac{1}{2}$$

The first equation is impossible, so $\cos x = \pm\frac{1}{\sqrt{2}}$.

Then, the general solution, in radians, is $x = \pm\frac{\pi}{4} + \pi n$, $n \in \text{I}$.

Section 6.4 Page 321 Question 18

$$\frac{1-\sin^2 x - 2\cos x}{\cos^2 x - \cos x - 2} = -\frac{1}{3}$$
$$3 - 3\sin^2 x - 6\cos x = -\cos^2 x + \cos x + 2$$
$$3\cos^2 x + \cos^2 x - 7\cos x - 2 = 0$$
$$4\cos^2 x - 7\cos x - 2 = 0$$
$$(4\cos x + 1)(\cos x - 2) = 0$$

The second factor does not yield any possible solutions.
From the first factor, $\cos x = -0.25$.
In the domain $-\pi \le x \le \pi$, $x \approx 1.8235$ or $x = -1.8235$.

Section 6.4 Page 321 Question 19

$$4(16^{\cos^2 x}) = 2^{6\cos x}$$
$$2^2(2^{4\cos^2 x}) = 2^{6\cos x}$$
$$2^{2+4\cos^2 x} = 2^{6\cos x}$$
$$\text{Then, } 2 + 4\cos^2 x = 6\cos x$$
$$2\cos^2 x - 3\cos x + 1 = 0$$
$$(2\cos x - 1)(\cos x - 1) = 0$$

So, $\cos x = \frac{1}{2}$ or $\cos x = 1$.

The general solution, in radians, is $x = \pm\frac{\pi}{3} + 2\pi n$ or $x = 2\pi n$, $n \in I$.

Section 6.4 Page 321 Question 20

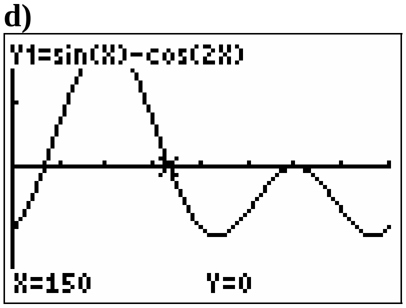
$\sin^2 \alpha + \cos^2 \beta = m^2$ ①
$\cos^2 \alpha + \sin^2 \beta = m$ ②
Add ① + ②.
$\sin^2 \alpha + \cos^2 \alpha + \cos^2 \beta + \sin^2 \beta = m^2 + m$
$2 = m^2 + m$
$0 = m^2 + m - 2$
$0 = (m + 2)(m - 1)$
$m = -2$ or $m = 1$

Section 6.4 Page 321 Question C1

a) To express the equation $\sin x - \cos 2x = 0$ in terms of one trigonometric function, sine, use the identity $\cos 2x = 1 - 2\sin^2 x$.

b) Substitute $\cos 2x = 1 - 2\sin^2 x$.
$\sin x - \cos 2x = 0$
$\sin x - (1 - 2\sin^2 x) = 0$
$2\sin^2 x + \sin x - 1 = 0$
$(2\sin x - 1)(\sin x + 1) = 0$

c) $2\sin x - 1 = 0$ or $\sin x + 1 = 0$
$\sin x = \frac{1}{2}$ or $\sin x = -1$
For the domain $0° \le x < 360°$,
$x = 30°, 150°, 270°$

d)


Section 6.4 Page 321 Question C2

a) It is not possible to factor 3 $\cos^2 x + \cos x - 1$ because there are no two integers with a sum of 1 and a product of 3.

b)
$$\begin{aligned}\cos x &= \frac{-1 \pm \sqrt{1^2 - 4(3)(-1)}}{2(3)}\\ &= \frac{-1 \pm \sqrt{13}}{6}\\ &\approx -0.7676 \text{ or } 0.4343\end{aligned}$$

c) Over the domain $0° \le x < 720°$, $x \approx 140.14°$, 219.86°, 500.14°, 579.86°, or 64.26°, 295.74°, 424.26°, 655.74°.

Section 6.4 Page 321 Question C3

Example: $\sin 2x \cos x + \cos x = 0$. This is not an identity because it is not true for all value of x. For example, when $x = 0°$ the left side has value 1 and the right side has value 0.

$\sin 2x \cos x + \cos x = 0$
$2\sin x \cos x + \cos x = 0$
$\cos x(2\sin x + 1) = 0$
$\cos x = 0$ or $\sin x = -\frac{1}{2}$
In general, $x = 90° + 180°n$, or $x = 135° + 180°n$, where $n \in \mathrm{I}$.

Chapter 6 Review

Chapter 6 Review Page 322 Question 1

In each, the denominator cannot be 0.

a) For the expression $\frac{3\sin x}{\cos x}$, $\cos x \neq 0$. So the restriction is $x \neq \frac{\pi}{2} + \pi n,\ n \in \mathrm{I}$.

b) For the expression $\frac{\cos x}{\tan x}$, $\tan x \neq 0$. So the restriction is $x \neq \left(\frac{\pi}{2}\right)n,\ n \in \mathrm{I}$.

c) For the expression $\frac{\sin x}{1 - 2\cos x}$, $1 - 2\cos x \neq 0$. Then, $\cos x \neq \frac{1}{2}$. So, the restriction is $x \neq \pm\frac{\pi}{3} + 2\pi n,\ n \in \mathrm{I}$.

d) For the expression $\dfrac{\cos x}{\sin^2 x - 1}$, $\sin^2 x - 1 \neq 0$. Then, $\sin x = \pm 1$. So, the restriction is $x \neq \dfrac{\pi}{2} + \pi n,\ n \in \mathrm{I}$.

Chapter 6 Review Page 322 Question 2

a)
$$\begin{aligned}\frac{\sin x}{\tan x} &= \sin x \div \left(\frac{\sin x}{\cos x}\right)\\ &= \sin x\left(\frac{\cos x}{\sin x}\right)\\ &= \cos x\end{aligned}$$

b)
$$\begin{aligned}\frac{\sec x}{\csc x} &= \frac{1}{\cos x} \div \left(\frac{1}{\sin x}\right)\\ &= \frac{\sin x}{\cos x}\\ &= \tan x\end{aligned}$$

c)
$$\begin{aligned}\frac{\sin x + \tan x}{1 + \cos x} &= \frac{\sin x + \dfrac{\sin x}{\cos x}}{1 + \cos x}\\ &= \frac{\sin x(\cos x + 1)}{\cos x} \div (1 + \cos x)\\ &= \tan x\end{aligned}$$

d)
$$\begin{aligned}\frac{\csc x - \sin x}{\cot x} &= \left(\frac{1}{\sin x} - \sin x\right) \div \left(\frac{\cos x}{\sin x}\right)\\ &= \left(\frac{1 - \sin^2 x}{\sin x}\right)\left(\frac{\sin x}{\cos x}\right)\\ &= \frac{\cos^2 x}{\cos x}\\ &= \cos x\end{aligned}$$

Chapter 6 Review Page 322 Question 3

a)
$$\begin{aligned}\tan x \cot x &= \left(\frac{\sin x}{\cos x}\right)\left(\frac{\cos x}{\sin x}\right)\\ &= 1\end{aligned}$$

b)
$$\begin{aligned}\frac{1}{\csc^2 x} + \frac{1}{\sec^2 x} &= \sin^2 x + \cos^2 x\\ &= 1\end{aligned}$$

c)
$$\begin{aligned}\sec^2 x - \tan^2 x &= \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}\\ &= \frac{1 - \sin^2 x}{\cos^2 x}\\ &= \frac{\cos^2 x}{\cos^2 x}\\ &= 1\end{aligned}$$

Chapter 6 Review Page 322 Question 4

When $x = 30°$:

Left Side $= \dfrac{\cos x}{1 - \sin x}$

$= \dfrac{\cos 30°}{1 - \sin 30°}$

$= \dfrac{\sqrt{3}}{2} \div \left(1 - \dfrac{1}{2}\right)$

$= \dfrac{\sqrt{3}}{2}(2)$

$= \sqrt{3}$

Right Side $= \dfrac{1 + \sin x}{\cos x}$

$= \dfrac{1 + \sin 30°}{\cos 30°}$

$= \left(1 + \dfrac{1}{2}\right) \div \dfrac{\sqrt{3}}{2}$

$= \left(\dfrac{3}{2}\right)\left(\dfrac{2}{\sqrt{3}}\right)$

$= \sqrt{3}$

Left Side = Right Side

When $x = \dfrac{\pi}{4}$:

Left Side $= \dfrac{\cos x}{1 - \sin x}$

$= \dfrac{\cos \frac{\pi}{4}}{1 - \sin \frac{\pi}{4}}$

$= \dfrac{1}{\sqrt{2}} \div \left(1 - \dfrac{1}{\sqrt{2}}\right)$

$= \dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{2}}{\sqrt{2} - 1}\right)$

$= \dfrac{1}{\sqrt{2} - 1}$ or $\sqrt{2} + 1$

Right Side $= \dfrac{1 + \sin x}{\cos x}$

$= \dfrac{1 + \sin \frac{\pi}{4}}{\cos \frac{\pi}{4}}$

$= \left(1 + \dfrac{1}{\sqrt{2}}\right) \div \dfrac{1}{\sqrt{2}}$

$= \left(\dfrac{\sqrt{2} + 1}{\sqrt{2}}\right)\left(\dfrac{\sqrt{2}}{1}\right)$

$= \sqrt{2} + 1$

Left Side = Right Side

Chapter 6 Review Page 322 Question 5

a) Examples: When $x = \frac{\pi}{4}$:

Left Side $= \sqrt{\tan^2\left(\frac{\pi}{4}\right)+1}$
$= \sqrt{1+1}$
$= \sqrt{2}$

Right Side $= \frac{1}{\cos\left(\frac{\pi}{4}\right)}$
$= \sqrt{2}$

Left Side = Right Side

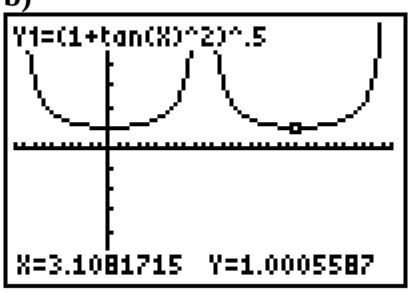
When $x = 1$:

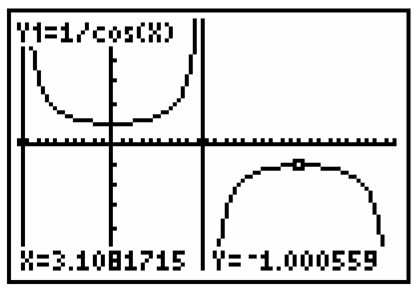
Left Side $= \sqrt{\tan^2(1)+1}$
≈ 1.8508

Right Side $= \frac{1}{\cos 1}$
≈ 1.8508

Left Side = Right Side

b)





The graphs appear to be the same for $-\frac{\pi}{2} < x < \frac{\pi}{2}$ but differ for $\frac{\pi}{2} \le x < \frac{3\pi}{2}$.

c) The graph shows that both sides of the equation do not have the same value for all values in the given domain. There are many values of *x* for which the two sides of the equation have different values.

Chapter 6 Review Page 322 Question 6

a) $f(x) = \sin x + \cos x + \sin 2x + \cos 2x$
$f(0) = \sin 0 + \cos 0 + \sin 2(0) + \cos 2(0)$
$= 0 + 1 + 0 + 1$
$= 2$

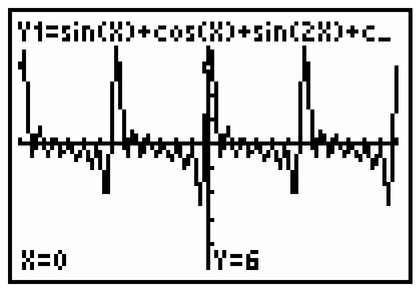
$$f\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{6}\right) + \sin 2\left(\frac{\pi}{6}\right) + \cos 2\left(\frac{\pi}{6}\right)$$
$$= \frac{1}{2} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \frac{1}{2}$$
$$= 1 + \sqrt{3}$$

b) $f(x) = \sin x + \cos x + \sin 2x + \cos 2x$
$= \sin x + \cos x + 2 \sin x \cos x + (1 - 2 \sin^2 x)$
$= \sin x + \cos x + 2 \sin x \cos x - 2 \sin^2 x + 1$

c) This Fourier series cannot be written using only sine or only cosine because the three terms $\sin x + \cos x + 2 \sin x \cos x$ cannot be expressed in terms of one of the ratios.

d) $f(x) = \sin x + \cos x + \sin 2x + \cos 2x + \sin 3x + \cos 3x$
$f(x) = \sin x + \cos x + \sin 2x + \cos 2x + \sin 3x + \cos 3x + \sin 4x + \cos 4x$
$f(x) = \sin x + \cos x + \sin 2x + \cos 2x + \sin 3x + \cos 3x + \sin 4x + \cos 4x + \sin 5x + \cos 5x$
$f(x) = \sin x + \cos x + \sin 2x + \cos 2x + \sin 3x + \cos 3x + \sin 4x + \cos 4x + \sin 5x + \cos 5x + \sin 6x + \cos 6x$

The curved does not smooth out perfectly, but the last equation above gives a reasonable approximation, as shown.



Chapter 6 Review Page 322 Question 7

a) $\sin 25° \cos 65° + \cos 25° \sin 65°$
$= \sin (25° + 65°)$
$= \sin 90°$
$= 1$

b) $\sin 54° \cos 24° - \cos 54° \sin 24°$
$= \sin (54° - 24°)$
$= \sin 30°$
$= \frac{1}{2}$

c)
$$\cos\frac{\pi}{4}\cos\frac{\pi}{12} + \sin\frac{\pi}{4}\sin\frac{\pi}{12} = \cos\left(\frac{\pi}{4} - \frac{\pi}{12}\right)$$
$$= \cos\left(\frac{3\pi - \pi}{12}\right)$$
$$= \cos\frac{\pi}{6}$$
$$= \frac{\sqrt{3}}{2}$$

d) $\cos\frac{\pi}{6}\cos\frac{\pi}{12} - \sin\frac{\pi}{6}\sin\frac{\pi}{12} = \cos\left(\frac{\pi}{6} + \frac{\pi}{12}\right)$

$$= \cos\frac{3\pi}{12}$$

$$= \cos\frac{\pi}{4}$$

$$= \frac{1}{\sqrt{2}}$$

Chapter 6 Review Page 323 Question 8

a) $\sin 15° = \sin (45° - 30°)$

$$= \sin 45° \cos 30° - \cos 30° \sin 45°$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}} \text{ or } \frac{\sqrt{6}-\sqrt{2}}{4}$$

b) $\cos\left(-\frac{\pi}{12}\right) = \cos\left(\frac{3\pi}{12} - \frac{4\pi}{12}\right)$

$$= \cos\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$$

$$= \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{1+\sqrt{3}}{2\sqrt{2}} \text{ or } \frac{\sqrt{2}+\sqrt{6}}{4}$$

c)

tan 165° = tan (120° + 45°)

$$= \frac{\tan 120° + \tan 45°}{1 - \tan 120° \tan 45°}$$

$$= \frac{-\sqrt{3}+1}{1-(-\sqrt{3})}$$

$$= \frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\frac{\left(1-\sqrt{3}\right)}{\left(1-\sqrt{3}\right)}$$

$$= \frac{4-2\sqrt{3}}{-2}$$

$$= \sqrt{3}-2$$

d)

$$\sin\frac{5\pi}{12} = \sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$$

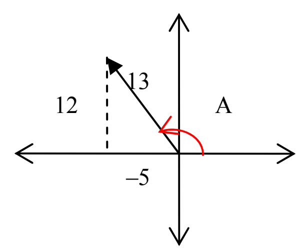
$$= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right)+\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}} \text{ or } \frac{\sqrt{6}+\sqrt{2}}{4}$$

Chapter 6 Review Page 323 Question 9

Given $\cos A = -\frac{5}{13}$, $\frac{\pi}{2} \le A \le \pi$, A is in quadrant II and sin A = $\frac{12}{13}$.



a) $\cos\left(A-\frac{\pi}{4}\right) = \cos A\cos\frac{\pi}{4}+\sin A\sin\frac{\pi}{4}$

$$= \left(-\frac{5}{13}\right)\left(\frac{1}{\sqrt{2}}\right)+\left(\frac{12}{13}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{7}{13\sqrt{2}} \text{ or } \frac{7\sqrt{2}}{26}$$

b) $\sin\left(A+\frac{\pi}{3}\right) = \sin A\cos\left(\frac{\pi}{3}\right)+\cos A\sin\left(\frac{\pi}{3}\right)$

$$= \left(\frac{12}{13}\right)\left(\frac{1}{2}\right)+\left(\frac{-5}{13}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{12-5\sqrt{3}}{26}$$

c) sin 2A = 2 sin A cos A

$$= 2\left(\frac{12}{13}\right)\left(\frac{-5}{13}\right)$$

$$= -\frac{120}{169}$$

Chapter 6 Review Page 323 Question 10

$$\left(\sin\frac{\pi}{8}+\cos\frac{\pi}{8}\right)^2 = \sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}+2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$$
$$= 1+\sin 2\left(\frac{\pi}{8}\right)$$
$$= 1+\sin\frac{\pi}{4}$$
$$= 1+\frac{1}{\sqrt{2}}$$

Chapter 6 Review Page 323 Question 11

$$\frac{\cos^2 x-\cos 2x}{0.5\sin 2x} = \frac{\cos^2 x-(\cos^2 x-\sin^2 x)}{0.5(2\sin x\cos x)}$$
$$= \frac{\sin^2 x}{\sin x\cos x}$$
$$= \frac{\sin x}{\cos x}$$
$$= \tan x$$

Chapter 6 Review Page 323 Question 12

a) $$\frac{1-\sin^2 x}{\cos x\sin x-\cos x} = \frac{\cos^2 x}{\cos x(\sin x-1)}$$
$$= \frac{\cos x}{\sin x-1}$$

b) $\tan^2 x - \cos^2 x\tan^2 x = \tan^2 x(1 - \cos^2 x)$
$= \tan^2 x\sin^2 x$

Chapter 6 Review Page 323 Question 13

a) Left Side = $1 + \cot^2 x$
$$= 1+\frac{\cos^2 x}{\sin^2 x}$$
$$= \frac{\sin^2 x+\cos^2 x}{\sin^2 x}$$
$$= \frac{1}{\sin^2 x}$$
$= \csc^2 x$
= Right Side

b) Right Side $= \csc 2x - \cot 2x$
$$= \frac{1}{\sin 2x}-\frac{\cos 2x}{\sin 2x}$$
$$= \frac{1-(2\cos^2 x-1)}{2\sin x\cos x}$$
$$= \frac{2\sin^2 x}{2\sin x\cos x}$$
$= \tan x$
= Left Side

c) Left Side $= \sec x + \tan x$

$$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$$
$$= \frac{1 + \sin x}{\cos x}$$
$$= \frac{1 - \sin^2 x}{(1 - \sin x)\cos x}$$
$$= \frac{\cos x}{1 - \sin x}$$
$= $ Right Side

d) Left Side $= \frac{1}{1 + \cos x} + \frac{1}{1 - \cos x}$

$$= \frac{1 - \cos x}{1 - \cos^2 x} + \frac{1 + \cos x}{1 - \cos^2 x}$$
$$= \frac{2}{\sin^2 x}$$
$$= 2\csc^2 x$$
$= $ Right Side

Chapter 6 Review Page 323 Question 14

a) When $x = \frac{\pi}{4}$:

Left Side = sin 2x

$$= \sin 2\left(\frac{\pi}{4}\right)$$
$$= \sin \frac{\pi}{2}$$
$$= 1$$

Right Side $= \frac{2\tan x}{1 + \tan^2 x}$

$$= \frac{2\tan\left(\frac{\pi}{4}\right)}{1 + \tan^2\left(\frac{\pi}{4}\right)}$$
$$= \frac{2(1)}{1 + 1}$$
$$= 1$$

Left Side = Right Side

The fact that an equation is true for one particular value does not prove that it is an identity. An identity is true for all permissible values.

b) The non-permissible values occur when tan x is undefined.

$$x = \frac{\pi}{2} + n\pi, \ n \in \mathrm{I}$$

c) Left Side $= \sin 2x$

$$= 2\sin x \cos x$$
$$= \frac{2\sin x \cos^2 x}{\cos x}$$
$$= \frac{2\tan x}{\sec^2 x}$$
$$= \frac{2\tan x}{1 + \tan^2 x}$$
$= $ Right Side

Chapter 6 Review Page 323 Question 15

a) Left Side $= \dfrac{\cos x + \cot x}{\sec x + \tan x}$

$$= \frac{\cos x + \dfrac{\cos x}{\sin x}}{\dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}}$$

$$= \frac{\dfrac{\sin x \cos^2 x}{\sin x} + \dfrac{\cos^2 x}{\sin x}}{1 + \sin x}$$

$$= \frac{\dfrac{(\sin x + 1)\cos^2 x}{\sin x}}{1 + \sin x}$$

$$= \frac{\cos x \cos x}{\sin x}$$

$= \cos x \cot x$

$=$ Right Side

b) Left Side $= \sec x + \tan x$

$$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

$$= \frac{1 + \sin x}{\cos x}$$

$$= \frac{1 - \sin^2 x}{(1 - \sin x)\cos x}$$

$$= \frac{\cos x}{1 - \sin x}$$

$=$ Right Side

Chapter 6 Review Page 323 Question 16

a) To prove that cos 2*x* = 2 sin *x* sec *x* is an identity, you might use algebraic reasoning or compare the graphs of each side. It is definitely not an identity if you find a value for which the left side is not equal to the right side.

b) When *x* = 0, the left side has value 1 and the right side has value 0. So, the equation is not an identity.

Chapter 6 Review Page 323 Question 17

Use the domain $0 \le x < 2\pi$.

a)

$\sin 2x + \sin x = 0$

$2 \sin x \cos x + \sin x = 0$

$\sin x (2 \cos x + 1) = 0$

$\sin x = 0$ or $\cos x = -\dfrac{1}{2}$

$x = 0, \pi$, or $\dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

b)

$\cot x + \sqrt{3} = 0$

$\cot x = -\sqrt{3}$

$\tan x = -\dfrac{1}{\sqrt{3}}$

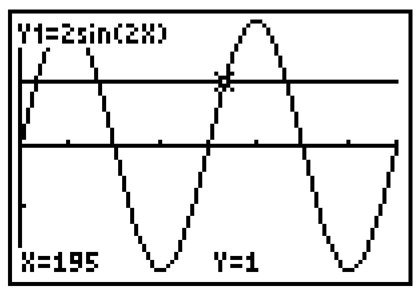
$x = \dfrac{5\pi}{6}, \dfrac{11\pi}{6}$

c) $2\sin^2 x - 3\sin x - 2 = 0$
$(2\sin x + 1)(\sin x - 2) = 0$
$\sin x = -\frac{1}{2}$ or $\sin x = 2$ (which is not possible)
$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

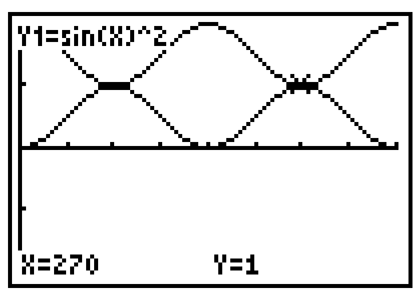
d) $\sin^2 x = \cos x - \cos 2x$
$1 - \cos^2 x = \cos x - (2\cos^2 x - 1)$
$\cos^2 x - \cos x = 0$
$\cos x(\cos x - 1) = 0$
$\cos x = 0$ or $\cos x = 1$
$x = \frac{\pi}{2}, \frac{3\pi}{2}$ or 0

Chapter 6 Review Page 323 Question 18

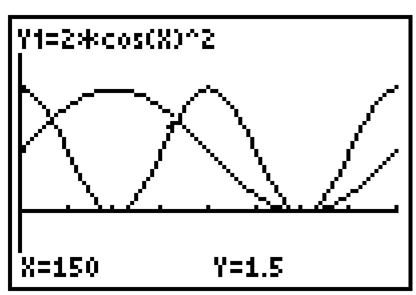
a) $2\sin 2x = 1$
$\sin 2x = \frac{1}{2}$
$2x = 30°$ or $150°$, or $390°$, $510°$, …
So, in the domain $0° \le x < 360°$,
$x = 15°, 75°$, or $195°, 255°$.
Graph $y = 2\sin 2x$ and $y = 1$ to verify the four values.



b) $\sin^2 x = 1 + \cos^2 x$
$\sin^2 x - \cos^2 x = 1$
$-\cos 2x = 1$
$\cos 2x = -1$
$2x = 180°, 540°, …$
So, in the domain $0° \le x < 360°$,
$x = 90°, 270°$.
Graph $y = \sin^2 x$ and $y = 1 + \cos^2 x$ to verify the two values.



c) $2\cos^2 x = \sin x + 1$
$2(1 - \sin^2 x) = \sin x + 1$
$2\sin^2 x + \sin x - 1 = 0$
$(2\sin x - 1)(\sin x + 1) = 0$
$\sin x = \frac{1}{2}$ or $\sin x = -1$
$x = 30°, 150°$, or $x = 270°$
Graph $y = 2\cos^2 x$ and $y = \sin x + 1$ to verify the three values.



d) $\cos x \tan x - \sin^2 x = 0$

$$\cos x\left(\frac{\sin x}{\cos x}\right) - \sin^2 x = 0$$

$$\sin x - \sin^2 x = 0$$

$$\sin x(1 - \sin x) = 0$$

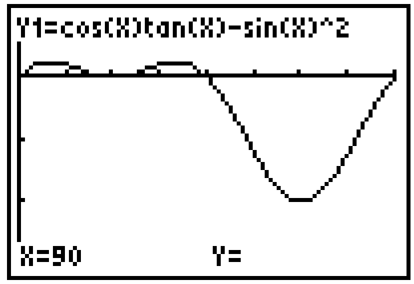
$\sin x = 0$ or $1 - \sin x = 0$

$\sin x = 1$

$x = 0°, 180°$ $\quad$ $x = 90°$

Graph $y = \cos x \tan x - \sin^2 x$ to verify the roots. The other possible root, $x = 90°$, does not check.

The solution is $x = 0°, 180°$.



Chapter 6 Review Page 323 Question 19

$$4\cos^2 x - 1 = 0$$

$$\cos^2 x = \frac{1}{4}$$

$$\cos x = \pm\frac{1}{2}$$

The general solution, in radians, is $x = \pm\frac{\pi}{3} + n\pi, n \in \mathrm{I}$

Chapter 6 Review Page 323 Question 20

$$2\cos^2 x + \sin^2 x = \frac{41}{25}$$

$$2\cos^2 x + 1 - \cos^2 x = \frac{41}{25}$$

$$\cos^2 x = \frac{41}{25} - 1$$

$$\cos^2 x = \frac{16}{25}$$

$$\cos x = \pm\frac{4}{5}$$

Chapter 6 Review Page 323 Question 21

$$2\sin x \cos x = 3\sin x$$

$$2\sin x \cos x - 3\sin x = 0$$

$$\sin x(2\cos x - 3) = 0$$

$\sin x = 0$ or $\cos x = \frac{3}{2}$

The second equation is impossible.
So, in the domain $-2\pi \le x \le 2\pi$, $x = -2\pi, -\pi, 0, \pi, 2\pi$.

Chapter 6 Practice Test Page 324 Question 1

$$\begin{aligned}\frac{\cos 2x - 1}{\sin 2x} &= \frac{-2\sin^2 x}{2\sin x\cos x} \\ &= -\frac{\sin x}{\cos x} \\ &= -\tan x\end{aligned}$$

The correct answer is **A**.

Chapter 6 Practice Test Page 324 Question 2

$$\begin{aligned}\cot\theta + \tan\theta &= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \\ &= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} \\ &= \frac{1}{\sin\theta\cos\theta}\end{aligned}$$

The correct answer is **A**.

Chapter 6 Practice Test Page 324 Question 3

$$\begin{aligned}&\tan^2\theta\csc\theta + \frac{1}{\sin\theta} \\ &= \left(\frac{\sin^2\theta}{\cos^2\theta}\right)\left(\frac{1}{\sin\theta}\right) + \frac{1}{\sin\theta} \\ &= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin\theta} \\ &= \frac{1}{\cos^2\theta\sin\theta} \\ &= \sec^2\theta\csc\theta\end{aligned}$$

The correct answer is **D**.

Chapter 6 Practice Test Page 324 Question 4

$$\cos\frac{\pi}{5}\cos\frac{\pi}{6}-\sin\frac{\pi}{5}\sin\frac{\pi}{6}$$
$$=\cos\left(\frac{\pi}{5}+\frac{\pi}{6}\right)$$
$$=\cos\frac{11\pi}{30}$$

The correct answer is **D**.

Chapter 6 Practice Test Page 324 Question 5

$$4\cos^2 x - 2 = 2(2\cos^2 x - 1)$$
$$= 2\cos 2x$$

The correct answer is **A**.

Chapter 6 Practice Test Page 324 Question 6

If $\sin\theta = c$ and $0 \le \theta < \frac{\pi}{2}$,

$\cos(\pi + \theta)$ must be in quadrant III, with $y = -\sqrt{1-c^2}$.

So $\cos(\pi + \theta) = -\sqrt{1-c^2}$.

The correct answer is **D**.

Chapter 6 Practice Test Page 324 Question 7

a) $\cos 105° = \cos(60° + 45°)$

$$= \cos 60°\cos 45° - \sin 60°\sin 45°$$
$$=\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)-\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$
$$=\frac{1-\sqrt{3}}{2\sqrt{2}} \text{ or } \frac{\sqrt{2}-\sqrt{6}}{4}$$

b) $\sin\frac{5\pi}{12} = \sin\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$ or $\frac{\sqrt{6}+\sqrt{2}}{4}$

Chapter 6 Practice Test Page 324 Question 8

Left Side = $\cot\theta - \tan\theta$

$= \frac{1}{\tan\theta} - \tan\theta$

$= \frac{1-\tan^2\theta}{\tan\theta}$

$= 2\cot 2\theta$

= Right Side

$\theta = \left(\frac{\pi}{2}\right)n,\ n \in \mathrm{I}$

Chapter 6 Practice Test Page 324 Question 9

Theo's Formula = $I_0 \cos^2\theta$

$= I_0 - I_0 \sin^2\theta$

$= I_0 - \frac{I_0}{\csc^2\theta}$

= Sany's Formula

Chapter 6 Practice Test Page 324 Question 10

a) $\sec A + 2 = 0$

$\frac{1}{\cos A} = -2$

$\cos A = -\frac{1}{2}$

The reference angle for A = $\frac{\pi}{3}$ and A is in quadrant II or III.

So, $\text{A} = \frac{2\pi}{3} + 2\pi n,$ or $\frac{4\pi}{3} + 2\pi n,$ where $n \in \text{I}.$

b)

$$2 \sin \text{B} = 3 \tan^2 \text{B}$$
$$2\sin \text{B} = 3\left(\frac{\sin^2 \text{B}}{\cos^2 \text{B}}\right)$$
$$2\sin \text{B}\cos^2 \text{B} - 3\sin^2 \text{B} = 0$$
$$2\sin \text{B}(1-\sin^2 \text{B}) - 3\sin^2 \text{B} = 0$$
$$2\sin \text{B} - 2\sin^3 \text{B} - 3\sin^2 \text{B} = 0$$
$$\sin \text{B}(2 - 3\sin \text{B} - 2\sin^2 \text{B}) = 0$$
$$\sin \text{B}(2+\sin \text{B})(1-2\sin \text{B}) = 0$$
$$\sin \text{B} = 0 \text{ or } \sin \text{B} = \frac{1}{2}$$

So, $\text{B} = \pi n,\ n \in \text{I}$ or $\text{B} = \frac{\pi}{6} + 2\pi n,$ or $\frac{5\pi}{6} + 2\pi n,\ n \in \text{I}.$

c)

$$\sin 2\theta \sin \theta + \cos^2 \theta = 1$$
$$2 \sin \theta \cos \theta \sin \theta + \cos^2 \theta - 1 = 0$$
$$2 \sin^2 \theta \cos \theta - \sin^2 \theta = 0$$
$$\sin^2 \theta(2 \cos \theta - 1) = 0$$
$$\sin \theta = 0 \text{ or } \cos \theta = \frac{1}{2}$$

So, $\theta = \pi n,\ n \in \text{I}$ or $\theta = \pm\frac{\pi}{3} + 2\pi n,\ n \in \text{I}$

Chapter 6 Practice Test Page 324 Question 11

$$\sin 2x + 2 \cos x = 0$$
$$2 \sin x \cos x + 2 \cos x = 0$$
$$2 \cos x (\sin x + 1) = 0$$
$$\cos x = 0 \text{ or } \sin x = -1$$

So, $x = \frac{\pi}{2}$ or $x = \frac{3\pi}{2}$

The general solution, in radians, is $x = \frac{\pi}{2} + \pi n,\ n \in \text{I}.$

Chapter 6 Practice Test Page 324 Question 12

Given $\sin\theta = -\frac{4}{5}$ and θ is in quadrant III, then $\cos\theta = -\frac{3}{5}$.

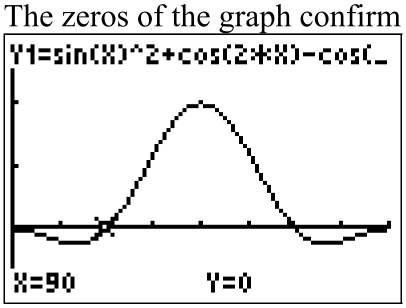
$$\cos\left(\theta - \frac{\pi}{6}\right) = \cos\theta\cos\frac{\pi}{6} + \sin\theta\sin\frac{\pi}{6}$$
$$= \left(-\frac{3}{5}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{4}{5}\right)\left(\frac{1}{2}\right)$$
$$= \frac{-3\sqrt{3} - 4}{10}$$

Chapter 6 Practice Test Page 324 Question 13

$2 \tan x \cos^2 x = 1$
$$2\left(\frac{\sin x}{\cos x}\right)\cos^2 x = 1$$
$$2\sin x\cos x = 1$$
$$\sin 2x = 1$$
$$2x = \frac{\pi}{2}$$
$$x = \frac{\pi}{4}$$
So, in the domain $0 \le x < 2\pi$, $x = \dfrac{\pi}{4}$ or $\dfrac{5\pi}{4}$.

Chapter 6 Practice Test Page 324 Question 14

$\sin^2 x + \cos 2x - \cos x = 0$
$\sin^2 x + \cos^2 x - \sin^2 x - \cos x = 0$
$\cos^2 x - \cos x = 0$
$\cos x (\cos x - 1) = 0$
$\cos x = 0$ or $\cos x = 1$
In the domain $0° \le x < 360°$, $x = 0°, 90°, 270°$.
The zeros of the graph confirm these three values.



Chapter 6 Practice Test Page 324 Question 15

a) $$\text{Left Side} = \frac{\cot x}{\csc x - 1}$$
$$= \frac{\cot x(\csc x + 1)}{\csc^2 x - 1}$$
$$= \frac{\cot x(\csc x + 1)}{1 + \cot^2 x - 1}$$
$$= \frac{(\csc x + 1)}{\cot x}$$
$$= \text{Right Side}$$

b) Left Side = $\sin (x + y) \sin (x - y)$
$= (\sin x \cos y + \sin y \cos x)(\sin x \cos y - \sin y \cos x)$
$= \sin^2 x \cos^2 y - \sin^2 y \cos^2 x$
$= \sin^2 x(1 - \sin^2 y) - \sin^2 y (1 - \sin^2 x)$
$= \sin^2 x - \sin^2 y$
= Right Side

Chapter 6 Practice Test Page 324 Question 16

$2 \cos^2 x + 3 \sin x - 3 = 0$
$2(1 - \sin^2 x) + 3 \sin x - 3 = 0$
$2 - 2 \sin^2 x + 3 \sin x - 3 = 0$
$2 \sin^2 x - 3 \sin x + 1 = 0$
$(2 \sin x - 1)(\sin x - 1) = 0$
$\sin x = \frac{1}{2}$ or $\sin x = 1$

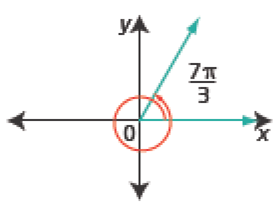
The general solution, in radians, is $x = \frac{\pi}{6} + 2\pi n,\ \frac{5\pi}{6} + 2\pi n,\ n \in \text{I}$,

or $x = \frac{\pi}{2} + 2\pi n,\ n \in \text{I}$.

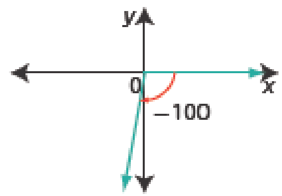
Cumulative Review, Chapters 4-6

Cumulative Review, Chapters 4-6 Page 326 Question 1

a) $\frac{7\pi}{3} \pm 2\pi n, n \in \text{N}$



b) $-100° \pm (360°)n,\ n \in \mathrm{N}$



Cumulative Review, Chapters 4-6 Page 326 Question 2

a) $\pi = 180°$

$$\text{So, } 4 = 4\left(\frac{180°}{\pi}\right) \approx 229°$$

b) $$\frac{-5\pi}{3} = \frac{-5(180°)}{3} = -300°$$

Cumulative Review, Chapters 4-6 Page 326 Question 3

a) $$210° = \frac{210\pi}{180} = \frac{7\pi}{6}$$

b) $$-500° = \frac{-500\pi}{180} = -\frac{25\pi}{9}$$

Cumulative Review, Chapters 4-6 Page 326 Question 4

a) $$\text{arc length} = \frac{\text{circumference}}{42} = \frac{\pi(175)}{42} \approx 13.1$$

The arc length between each gondola is 13.1 ft, to the nearest tenth of a foot.

b) $$\frac{\text{arc length}}{\text{circumference}} = \frac{70}{360}$$

$$\text{arc length} = \pi(175)\left(\frac{70}{360}\right) \approx 106.9$$

In rotating through 70°, the gondola travels 106.9 ft, to the nearest tenth of a foot.

Cumulative Review, Chapters 4-6 Page 326 Question 5

a) Substitute $r = 5$ in $x^2 + y^2 = r^2$.

$x^2 + y^2 = 5^2$

$x^2 + y^2 = 25$

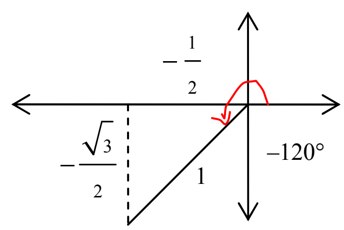
b) Substitute $x = 3$ and $y = \sqrt{7}$ in $x^2 + y^2 = r^2$.

$$3^2 + \left(\sqrt{7}\right)^2 = r^2$$
$$9 + 7 = r^2$$
$$r^2 = 16$$

The equation of the circle through P(3, $\sqrt{7}$) is $x^2 + y^2 = 16$.

Cumulative Review, Chapters 4-6 Page 326 Question 6

a) $P(\theta) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$, then θ terminates in quadrant III.



b) The reference angle of θ is $\frac{\pi}{3}$. In the interval $-2\pi \le \theta \le 2\pi$, $\theta = -\frac{2\pi}{3}, \frac{4\pi}{3}$.

c) The coordinates of $P\left(\theta + \frac{\pi}{2}\right)$ are $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$. When the given quadrant III angle is rotated through $\frac{\pi}{2}$, its terminal arm is in quadrant IV and its coordinates are switched and the signs adjusted.

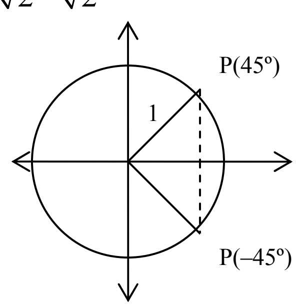
d) The coordinates of $P(\theta - \pi)$ are $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. When the given quadrant III angle is rotated through $-\pi$, its terminal arm is in quadrant I and its coordinates are the same but the signs adjusted.

Cumulative Review, Chapters 4-6 Page 326 Question 7

a) 45° is in an isosceles right triangle with sides $\frac{1}{\sqrt{2}} : \frac{1}{\sqrt{2}} : 1$.

$$P(-45°) = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$
$$P(45°) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

The points have the same *x*-coordinates but opposite *y*-coordinates.



b) 675° = 360° + 315°, so P(675°) is coterminal with P(315°).
This angle has a reference angle of 45° and terminates in quadrant IV.

P(675°) = $\left(\dfrac{1}{\sqrt{2}}, -\dfrac{1}{\sqrt{2}}\right)$.

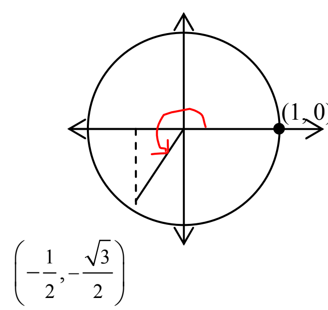
765° = 720° + 45°, so P(765°) is coterminal with P(45°).
This angle has a reference angle of 45° and terminates in quadrant I.

P(765°) = $\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$.

The points have the same *x*-coordinates but opposite *y*-coordinates.

Cumulative Review, Chapters 4-6 Page 326 Question 8

a) A rotation of $\dfrac{4\pi}{3}$ takes the terminal arm of the angle into quadrant III, with reference angle $\dfrac{\pi}{3}$ as shown.



$$\begin{aligned}\sin\frac{4\pi}{3} &= \frac{y}{r}\\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

b) A rotation of 300° takes the terminal arm of the angle into quadrant IV, with reference angle 60°. In quadrant IV, cosine is positive.

cos 300° = $\dfrac{1}{2}$

c) A rotation of –570° is –(360° + 210°) and takes the terminal arm of the angle into quadrant II, with reference angle 30°. In quadrant II, tangent is negative.

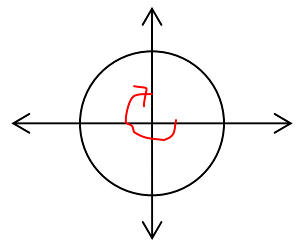
tan(–570°) = $-\dfrac{1}{\sqrt{3}}$

d) A rotation of 135° takes the terminal arm of the angle into quadrant II, with reference angle 45°. In quadrant II, sine and cosecant are positive.

$$\begin{aligned}\csc 135° &= \frac{1}{\sin 45°}\\ &= \sqrt{2}\end{aligned}$$

e) A rotation of $-\dfrac{3\pi}{2}$ is on the *y*-axis above the origin.

$$\sec\left(-\frac{3\pi}{2}\right) = \frac{1}{\cos\left(-\frac{3\pi}{2}\right)}$$
$$= \frac{1}{0}$$



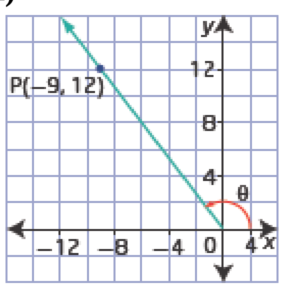
This is undefined.

f) A rotation of $\frac{23\pi}{6} = 2\pi + \frac{11\pi}{6}$ and takes the terminal arm of the angle into quadrant IV, with reference angle $\frac{\pi}{6}$. In quadrant IV, tangent and cotangent are negative.

$$\cot\frac{23\pi}{6} = \frac{1}{\tan\frac{23\pi}{6}}$$
$$= \frac{1}{-\frac{1}{\sqrt{3}}}$$
$$= -\sqrt{3}$$

Cumulative Review, Chapters 4-6 Page 326 Question 9

a)



b) First determine r. Substitute $x = –9$ and $y = 12$ into $x^2 + y^2 = r^2$.

$$x^2 + y^2 = r^2$$
$$(-9)^2 + 12^2 = r^2$$
$$r^2 = 225$$
$$r = 15$$

$$\sin\theta = \frac{y}{r} = \frac{12}{15} = \frac{4}{5}$$

$$\cos\theta = \frac{x}{r} = \frac{-9}{15} = -\frac{3}{5}$$

$$\tan\theta = \frac{y}{x} = \frac{12}{-9} = -\frac{4}{3}$$

Then, $\csc\theta = \frac{5}{4}$, $\sec\theta = -\frac{5}{3}$, $\cot\theta = -\frac{3}{4}$

c) Since θ is in quadrant II, and the reference angle is $\sin^{-1}(0.8)$,
$\theta = 126.87° + 360°n$, where $n \in I$.

Cumulative Review, Chapters 4-6 Page 326 Question 10

a) For $\sin\theta = -\frac{1}{2}$ the reference angle is $\frac{\pi}{6}$ and θ is in quadrant III or IV.

So, in the domain $-2\pi \le \theta \le 2\pi$, $\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$.

b) For $\sec\theta = \frac{2\sqrt{3}}{3}$, or $\cos\theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$, the reference angle is 30° and θ is in quadrant I or IV.
So, in the domain $-180° \le \theta \le 180°$, $\theta = -30°$ or $30°$.

c) For $\tan\theta = -1$ the reference angle is $\frac{\pi}{4}$ and θ is in quadrant II or IV.

So, in the domain $0 \le \theta \le 2\pi$, $\theta = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$.

Cumulative Review, Chapters 4-6 Page 326 Question 11

a) For $\cos\theta = -\frac{\sqrt{2}}{2}$ the reference angle is $\frac{\pi}{4}$ and θ is in quadrant II or III.

Then, the general solution is $\theta = \frac{3\pi}{4} + 2\pi n$ or $\frac{5\pi}{4} + 2\pi n$, $n \in I$.

b) For $\csc\theta = 1$, or $\sin\theta = 1$, θ is $\frac{\pi}{2}$. Then, the general solution is $\theta = \frac{\pi}{2} + 2\pi n$, $n \in I$.

c) For $\cot\theta = 0$, or $\tan\theta$ is undefined, θ is $\frac{\pi}{2}$ and $\frac{3\pi}{2}$. Then, the general solution is

$\theta = \frac{\pi}{2} + \pi n$, $n \in I$.

Cumulative Review, Chapters 4-6 Page 326 Question 12

a)
$$\sin\theta = \sin\theta\tan\theta$$
$$\sin\theta - \sin\theta\tan\theta = 0$$
$$\sin\theta\,(1 - \tan\theta) = 0$$

$\sin\theta = 0$ or $\tan\theta = 1$

In the domain $0 \le \theta \le 2\pi$, $\theta = 0, \pi, 2\pi$ or $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$.

b) $2\cos^2\theta + 5\cos\theta + 2 = 0$
$(2\cos\theta + 1)(\cos\theta + 2) = 0$

$\cos\theta = -\frac{1}{2}$ or $\cos\theta = -2$ (which is impossible)

In the domain $0 \le \theta \le 2\pi$, $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$.

Cumulative Review, Chapters 4-6 Page 326 Question 13

a) $4\tan^2\theta - 1 = 0$
$(2\tan\theta - 1)(2\tan\theta + 1) = 0$

$\tan\theta = \frac{1}{2}$ or $\tan\theta = -\frac{1}{2}$

In the domain $0° \le \theta \le 360°$, $\theta \approx 27°, 153°, 207°, 333°$.

b) $3\sin^2\theta - 2\sin\theta = 1$
$3\sin^2\theta - 2\sin\theta - 1 = 0$
$(3\sin\theta + 1)(\sin\theta - 1) = 0$

$\sin\theta = -\frac{1}{3}$ or $\sin\theta = 1$

In the domain $0° \le \theta \le 360°$, $\theta \approx 199°, 341°$ or $\theta = 90°$.

Cumulative Review, Chapters 4-6 Page 326 Question 14

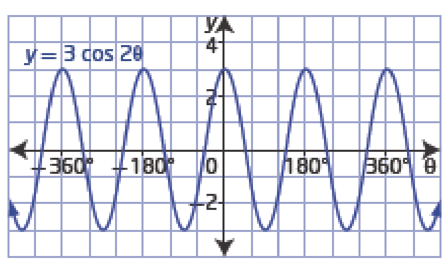
For the sine function in the form $y = a\sin b(x - c) + d$, the amplitude $a = 3$, the period $b = \frac{1}{2}$, and the horizontal shift is $c = -\frac{\pi}{4}$.

The equation is $y = 3\sin\frac{1}{2}\left(x + \frac{\pi}{4}\right)$.

Cumulative Review, Chapters 4-6 Page 327 Question 15

a) $y = 3\cos 2\theta$

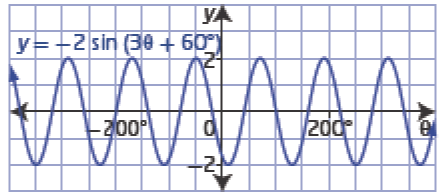
amplitude is 3, period is $\frac{360°}{2}$ or $180°$, there is no phase shift or vertical displacement



b) $y = -2\sin(3\theta + 60°)$

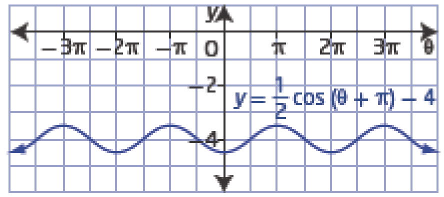
$y = -2\sin 3(\theta + 20°)$

amplitude is 2, period is 120°, the phase shift is 20° to the left, there is no vertical displacement



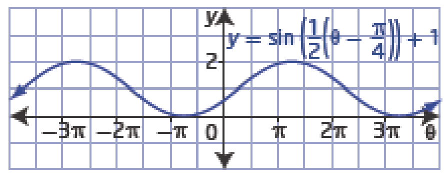
c) $y = \frac{1}{2}\cos(\theta + \pi) - 4$

amplitude is $\frac{1}{2}$, period is 2π, phase shift is π units to the left, and the vertical displacement is 4 units down.



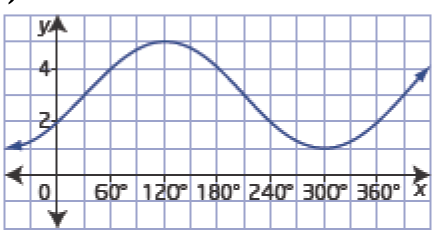
d) $y = \sin\left(\frac{1}{2}\left(\theta - \frac{\pi}{4}\right)\right) + 1$

amplitude is 1, period is 4π, phase shift is $\frac{\pi}{4}$ units to the right and the vertical displacement is 1 unit up.



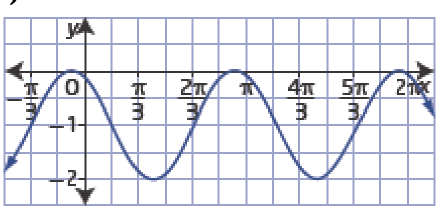
Cumulative Review, Chapters 4-6 Page 327 Question 16

a)



From the graph, the amplitude is 2,the period is 360°, the phase shift is 30° to the right and the vertical displacement is 3 units up.So, in the equation $a = 2$, $b = 1$, $c = 30°$, and $d = 3$. The equation is $y = 2\sin(x - 30°) + 3$ or $y = 2\cos(x - 120°) + 3$.

b)



From the graph, the amplitude is 1,the period is π, the phase shift is $\frac{\pi}{3}$ to the left and the vertical displacement is 1 unit down. So, in the equation $a = 1$, $b = 2$, $c = -\frac{\pi}{3}$, and $d = 1$.

The equation is $y = \sin 2\left(x + \frac{\pi}{3}\right) - 1$ or

$y = \cos 2\left(x + \frac{\pi}{12}\right) - 1$.

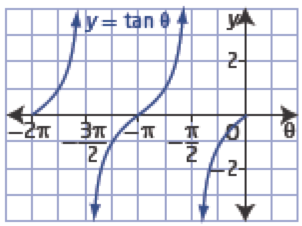
Cumulative Review, Chapters 4-6 Page 327 Question 17

$a = 4$, $b = \dfrac{360°}{300°} = 1.2$, $c = –30°$, $d = –3$

The equation of this cosine function is $y = 4 \cos 1.2 (x + 30°) – 3$.

Cumulative Review, Chapters 4-6 Page 327 Question 18

a)



b) The equations of the asymptotes in the domain $–2\pi \le \theta \le 0$ are

$x = -\dfrac{3\pi}{2}$ and $x = -\dfrac{\pi}{2}$.

Cumulative Review, Chapters 4-6 Page 327 Question 19

a) Assume that the horizontal axis passes through the centre of the wheel. The height of the wheel varies 25 m above and below the centre, so the amplitude is 25. The centre of the wheel is 26 m above the ground so the vertical displacement is 1. The wheel rotates twice in 22 min, so the period is $\dfrac{4\pi}{22}$ or $\dfrac{2\pi}{11}$. For a passenger starting at the lowest point on the Ferris wheel, an equation representing their motion is

$h(x) = -25\cos\dfrac{2\pi}{11}x + 26$.

b) Determine x when $h(x) = 30$.

$$30 = -25\cos\frac{2\pi}{11}x + 26$$

$$25\cos\frac{2\pi}{11}x = 26 - 30$$

$$\cos\frac{2\pi}{11}x = -\frac{4}{25}$$

$$\frac{2\pi}{11}x = \cos^{-1}\left(-\frac{4}{25}\right)$$

$$x = \frac{11}{2\pi}\cos^{-1}\left(-\frac{4}{25}\right)$$

$$x \approx 3.03$$

The passenger is 30 m above the ground after 3.0 min, to the nearest tenth of a minute.

Cumulative Review, Chapters 4-6 Page 327 Question 20

a) For $\frac{1-\cos^2\theta}{\cos^2\theta}$, $\cos^2\theta \neq 0$. So, the non-permissible values are $\theta \neq \frac{\pi}{2} + \pi n,\ n \in \mathrm{I}$.

$$\begin{aligned}\frac{1-\cos^2\theta}{\cos^2\theta} &= \frac{\sin^2\theta}{\cos^2\theta}\\ &= \tan^2\theta\end{aligned}$$

b) For sec x csc x tan x, sin $x \neq 0$ and cos $x \neq 0$. So, the non-permissible values are $x \neq \left(\frac{\pi}{2}\right)n,\ n \in \mathrm{I}$.

$$\begin{aligned}\sec x \csc x \tan x &= \left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right)\left(\frac{\sin x}{\cos x}\right)\\ &= \frac{1}{\cos^2 x}\\ &= \sec^2 x\end{aligned}$$

Cumulative Review, Chapters 4-6 Page 327 Question 21

a)
$$\begin{aligned}\sin 195° &= \sin(45° + 150°)\\ &= \sin 45° \cos 150° + \cos 45° \sin 150°\\ &= \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)\\ &= \frac{1-\sqrt{3}}{2\sqrt{2}} \text{ or } \frac{\sqrt{2}-\sqrt{6}}{4}\end{aligned}$$

b)
$$\begin{aligned}\cos\left(-\frac{5\pi}{12}\right) &= \cos\left(\frac{3\pi}{12}-\frac{8\pi}{12}\right)\\ &= \cos\left(\frac{\pi}{4}-\frac{2\pi}{3}\right)\\ &= \cos\frac{\pi}{4}\cos\frac{2\pi}{3}+\sin\frac{\pi}{4}\sin\frac{2\pi}{3}\\ &= \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{2}\right)+\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)\\ &= \frac{\sqrt{3}-1}{2\sqrt{2}} \text{ or } \frac{\sqrt{6}-\sqrt{2}}{4}\end{aligned}$$

Cumulative Review, Chapters 4-6 Page 327 Question 22

a) $2\cos^2\frac{3\pi}{8}-1=\cos 2\left(\frac{3\pi}{8}\right)$

$=\cos\frac{3\pi}{4}$

$=-\frac{1}{\sqrt{2}}$

b) sin 10° cos 80° + cos 10° sin 80° = sin (10° + 80°)

= sin 90°

= 1

c) $\dfrac{\tan\frac{5\pi}{12}+\tan\frac{23\pi}{12}}{1-\tan\frac{5\pi}{12}\tan\frac{23\pi}{12}}=\tan\left(\frac{5\pi}{12}+\frac{23\pi}{12}\right)$

$=\tan\frac{28\pi}{12}$

$=\tan\frac{7\pi}{3}$

$=\sqrt{3}$

Cumulative Review, Chapters 4-6 Page 327 Question 23

a) Substitute A = 30°:

Left Side = $\sin^2 A + \cos^2 A + \tan^2 A$

$= \sin^2 30° + \cos^2 30° + \tan^2 30°$

$=\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2$

$=\frac{1}{4}+\frac{3}{4}+\frac{1}{3}$

$=\frac{4}{3}$

Right Side = $\sec^2 A$

$= \sec^2 30°$

$=\frac{1}{\cos^2 30°}$

$=\frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}$

$=\frac{4}{3}$

Left Side = Right Side

b) Left Side = $\sin^2 A + \cos^2 A + \tan^2 A$

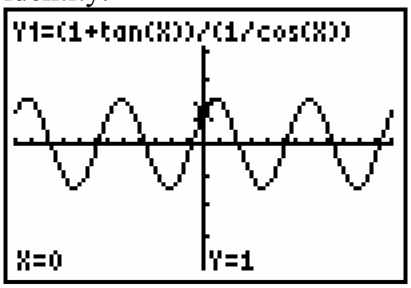
$= 1 + \tan^2 A$

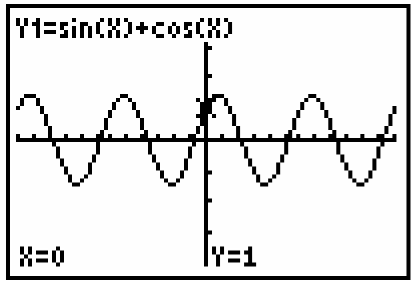
$=\sec^2 A$

= Right Side

Cumulative Review, Chapters 4-6 Page 327 Question 24

a) The graphs of each side of the equation look the same, so the equation may be an identity.





b)
$$\begin{aligned}\text{Left Side} &= \frac{1+\tan x}{\sec x}\\ &= \frac{1}{\sec x}+\frac{\tan x}{\sec x}\\ &= \cos x+\frac{\sin x}{\cos x}\div\frac{1}{\cos x}\\ &= \cos x+\sin x\\ &= \text{Right Side}\end{aligned}$$

Cumulative Review, Chapters 4-6 Page 327 Question 25

$$\text{Left Side} = \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$$

$$\begin{aligned}\text{Right Side} &= \frac{\cos 2\theta}{1+\sin 2\theta}\\ &= \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta}\\ &= \frac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{(\cos\theta+\sin\theta)(\cos\theta+\sin\theta)}\\ &= \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\end{aligned}$$

Left Side = Right Side

Cumulative Review, Chapters 4-6 Page 327 Question 26

a) $\sec^2 x = 4\tan^2 x$

$1 + \tan^2 x = 4\tan^2 x$

$1 = 3\tan^2 x$

$\tan^2 x = \dfrac{1}{3}$

$\tan x = \pm\dfrac{1}{\sqrt{3}}$

$x = \dfrac{\pi}{6} + \pi n,\ \dfrac{5\pi}{6} + \pi n,\ n \in \mathrm{I}$

b) $\sin 2x + \cos x = 0$

$2\sin x\cos x + \cos x = 0$

$\cos x(2\sin x + 1) = 0$

$\cos x = 0$ or $\sin x = -\dfrac{1}{2}$

$x = \dfrac{\pi}{2} + \pi n,\ n \in \mathrm{I}$ or

$x = \dfrac{7\pi}{6} + 2\pi n,\ \dfrac{11\pi}{6} + 2\pi n,\ n \in \mathrm{I}$

Cumulative Review, Chapters 4-6 Page 327 Question 27

a) $(\sin\theta + \cos\theta)^2 - \sin 2\theta = 1$

$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta - 2\sin\theta\cos\theta = 1$

$\sin^2\theta + \cos^2\theta = 1$

This is an identity, so the solution is all values of θ.

b) Yes, the equation is an identity because the left side simplifies to 1.

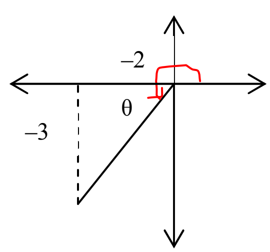
Unit 2 Test

Unit 2 Test Page 328 Question 1

If $\tan\theta = \dfrac{3}{2}$ and $\cos\theta < 0$, then θ is in quadrant III.

$r^2 = (-2)^2 + (-3)^2$

$r = \sqrt{13}$



Then, $\cos 2\theta = 2\cos^2\theta - 1$

$= 2\left(\dfrac{-2}{\sqrt{13}}\right)^2 - 1$

$= \dfrac{8}{13} - 1$

$= -\dfrac{5}{13}$

The best answer is **B**.

Unit 2 Test Page 328 Question 2

The point (3, –5) is is quadrant IV.

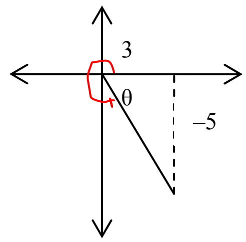
$r^2 = (3)^2 + (-5)^2$

$r = \sqrt{34}$

$\sin(\pi - \theta) = \sin\pi\cos\theta - \sin\theta\cos\pi$

$$= 0\left(\frac{3}{\sqrt{34}}\right) - \left(\frac{-5}{\sqrt{34}}\right)(-1)$$

$$= -\frac{5}{\sqrt{34}}$$



The best answer is **D**.

Unit 2 Test Page 328 Question 3

If the range is $-2 \leq y \leq 6$, the amplitude is 4 and the vertical displacement is 2 units up.
So, $a = 4$ and $d = 2$.
The best answer is **C**.

Unit 2 Test Page 328 Question 4

For the function $f(x) = 3\cos\left(4x + \frac{\pi}{2}\right)$ or $f(x) = 3\cos 4\left(x + \frac{\pi}{8}\right)$,

the period is $\frac{2\pi}{4}$, or $\frac{\pi}{2}$, and the phase shift is $\frac{\pi}{8}$ units to the left.

The best answer is **C**.

Unit 2 Test Page 328 Question 5

When the graph of $y = \cos x$ is translated to the right $\frac{\pi}{2}$ units it is the same as the graph of $y = \sin x$. So, $y = 3\cos\left(x - \frac{\pi}{2}\right)$ has a graph that is equivalent to $y = 3\sin x$.

The best answer is **B**.

Unit 2 Test Page 328 Question 6

$y = \tan x$ is not defined when $\cos x = 0$. This occurs when $x = 90° + 180°n$, $n \in I$.
The best answer is **D**.

Unit 2 Test Page 328 Question 7

$$\begin{aligned}\frac{\sin\theta+\tan\theta}{1+\cos\theta} &= \frac{\left(\sin\theta+\dfrac{\sin\theta}{\cos\theta}\right)}{1+\cos\theta}\\ &= \frac{\sin\theta\cos\theta+\sin\theta}{\cos\theta(1+\cos\theta)}\\ &= \frac{\sin\theta(1+\cos\theta)}{\cos\theta(1+\cos\theta)}\\ &= \tan\theta\end{aligned}$$

The best answer is **C**.

Unit 2 Test Page 328 Question 8

By inspection, the equation $\frac{\sec\theta\csc\theta}{\cot\theta} = \sec\theta$ cannot possibly be true.

Consider B: Left Side $= \tan^2\theta - \sin^2\theta$

$$\begin{aligned}&= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta\\ &= \frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}\\ &= \frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}\\ &= \tan^2\theta\sin^2\theta\\ &= \text{Right Side}\end{aligned}$$

Consider C: Left Side $= \frac{1-\cos 2\theta}{2}$

$$\begin{aligned}&= \frac{1-(1-2\sin^2\theta)}{2}\\ &= \sin^2\theta\\ &= \text{Right Side}\end{aligned}$$

Consider D: Left Side $= \frac{\tan^2\theta}{1+\tan^2\theta}$

$$\begin{aligned}&= \frac{\tan^2\theta}{\sec^2\theta}\\ &= \left(\frac{\sin^2\theta}{\cos^2\theta}\right) \div \left(\frac{1}{\cos^2\theta}\right)\\ &= \sin^2\theta\\ &= \text{Right Side}\end{aligned}$$

Equations B, C and D are identities.
The best answer is **A**.

Unit 2 Test Page 328 Question 9

$$\frac{17\pi}{3} = \frac{12\pi + 5\pi}{3}$$
$$= 4\pi + \frac{5\pi}{3}$$

So $\frac{17\pi}{3}$ is coterminal with $\frac{5\pi}{3}$ and is in quadrant IV with reference angle $\frac{\pi}{3}$.

$\sin\frac{17\pi}{3} = \frac{-\sqrt{3}}{2}$.

The exact value of $\sin\frac{17\pi}{3}$ is $\frac{-\sqrt{3}}{2}$.

Unit 2 Test Page 328 Question 10

If $P\left(x, \frac{\sqrt{5}}{3}\right)$ is on the unit circle then

$$x^2 + \left(\frac{\sqrt{5}}{3}\right)^2 = 1$$
$$x^2 + \frac{5}{9} = 1$$
$$x^2 = \frac{4}{9}$$
$$x = \pm\frac{2}{3}$$

The possible values of x are $\frac{2}{3}$ and $-\frac{2}{3}$.

Unit 2 Test Page 328 Question 11

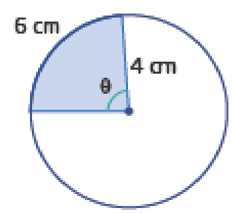
If $\cos\theta = \frac{-5}{13}$ and $\frac{\pi}{2} \le \theta \le \pi$, then θ is in quadrant II and $y = 12$.

$$\text{Then, } \sin\left(\theta + \frac{\pi}{4}\right) = \sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4}$$
$$= \left(\frac{12}{13}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{-5}{13}\right)\left(\frac{1}{\sqrt{2}}\right)$$
$$= \frac{7}{13\sqrt{2}} \text{ or } \frac{7\sqrt{2}}{26}$$

The exact value of $\sin\left(\theta + \frac{\pi}{4}\right)$ is $\frac{7}{13\sqrt{2}}$ or $\frac{7\sqrt{2}}{26}$.

Unit 2 Test Page 328 Question 12

$$\frac{\theta}{2\pi} = \frac{6}{2\pi(4)} \qquad \text{or} \qquad \frac{\theta}{360°} = \frac{6}{2\pi(4)}$$
$$\theta = \frac{3}{2} \qquad\qquad \theta = \frac{3(360°)}{4\pi}$$
$$\approx 85.9°$$



The measures of θ in radians and degrees, to the nearest tenth of a unit, are 1.5 and 85.9°.

Unit 2 Test Page 329 Question 13

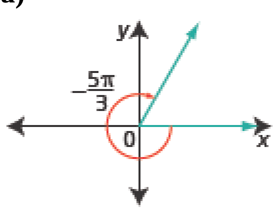
If $\sqrt{3}\sec\theta - 2 = 0$
$$\sec\theta = \frac{2}{\sqrt{3}}$$

So θ is in quadrant I or IV with reference angle $\frac{\pi}{6}$.

For $-2\pi \le \theta \le 2\pi$, $\theta = -\frac{11\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}$, and $\frac{11\pi}{6}$.

Unit 2 Test Page 329 Question 14

a)



b) $-\frac{5\pi}{3} = -\frac{5\pi(180°)}{3\pi}$
$$= -300°$$

c) All coterminal angles are given by $-\frac{5\pi}{3} \pm 2\pi n,\ n \in \mathrm{N}$.

d) No. $\frac{10\pi}{3} = 2\pi + \frac{4\pi}{3}$; its terminal arm is in quadrant III, and so this angle is never coterminal with $-\frac{5\pi}{3}$.

Unit 2 Test Page 329 Question 15

$$5\sin^2\theta + 3\sin\theta - 2 = 0$$
$$(5\sin\theta - 2)(\sin\theta + 1) = 0$$
$$5\sin\theta - 2 = 0 \text{ or } \sin\theta + 1 = 0$$
$$\sin\theta = 0.4 \text{ or } \sin\theta = -1$$

For $0 \le \theta \le 2\pi$, $\theta \approx 0.412, 2.730$, or 4.712.

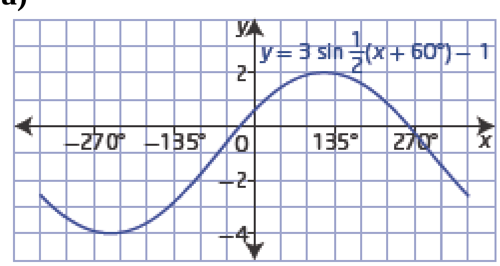
Unit 2 Test Page 329 Question 16

Sam is correct. In the first step, Pat has lost two solutions by forgetting the negative square root.
The correct solution is:

$$4\sin^2 x = 3$$
$$2\sin x = \pm\sqrt{3}$$
$$\sin x = \pm\frac{\sqrt{3}}{2}$$
$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

Unit 2 Test Page 329 Question 17

a)



b) The range is $-4 \le y \le 2$.

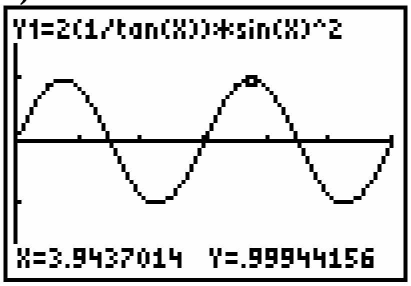
c) The amplitude is 3, the period is 720°, the phase shift is 60° to the left, and the vertical displacement is 1 unit down.

d) $3\sin\frac{1}{2}(x + 60°) - 1 = 0$

$$\sin\frac{1}{2}(x+60°) = \frac{1}{3}$$
$$\frac{1}{2}(x+60°) \approx 19.5° \text{ or } 160.5°$$
$$x \approx -21° \text{ or } 261°$$

Unit 2 Test Page 329 Question 18

a)



b) $g(\theta) = \sin 2\theta$

c)
$$\begin{aligned} f(\theta) &= 2 \cot \theta \sin^2 \theta \\ &= 2\left(\frac{\cos\theta}{\sin\theta}\right)\sin^2\theta \\ &= 2\cos\theta\sin\theta \\ &= \sin 2\theta \end{aligned}$$

Unit 2 Test Page 329 Question 19

a) For $x = \frac{2\pi}{3}$:

$$\begin{aligned} \text{Left Side} &= \tan\frac{2\pi}{3} + \frac{1}{\tan\frac{2\pi}{3}} \\ &= -\sqrt{3} + \frac{1}{-\sqrt{3}} \\ &= \frac{-3-1}{\sqrt{3}} \\ &= -\frac{4\sqrt{3}}{3} \end{aligned}$$

$$\begin{aligned} \text{Right Side} &= \frac{\sec\frac{2\pi}{3}}{\sin\frac{2\pi}{3}} \\ &= -2\left(\frac{2}{\sqrt{3}}\right) \\ &= -\frac{4\sqrt{3}}{3} \end{aligned}$$

Left Side = Right Side

b) Non-permissible values occur when $\sin x = 0$ and $\cos x = 0$. In general, the non-permissible values are $x \neq \frac{\pi n}{2}, n \in \mathrm{I}$.

c)
$$\begin{aligned} \text{Left Side} &= \tan x + \frac{1}{\tan x} \\ &= \frac{\tan^2 x + 1}{\tan x} \\ &= \frac{\sec^2 x}{\tan x} \\ &= \sec x\left(\frac{1}{\cos x}\right)\left(\frac{\cos x}{\sin x}\right) \\ &= \frac{\sec x}{\sin x} \\ &= \text{Right Side} \end{aligned}$$

Unit 2 Test Page 329 Question 20

a) For the function $h(t) = 2.962 \sin (0.508t - 0.107) + 3.876$,
the amplitude is 2.962 and the vertical displacement is 3.876 and the maximum height of the tide presumably occurs at the maximum of the curve which is 2.962 + 3.876. The maximum height of the tide is predicted to be 6.838 m.

b) The period is $\frac{2\pi}{0.508}$, or 12.368. The period of the function is approximately 12.37 h.

c) At noon $t = 12$.

$$h(12) = 2.962 \sin (0.508(12) - 0.107) + 3.876$$
$$\approx 3.017$$

The height of the tide at 12 noon was about 3.017 m.