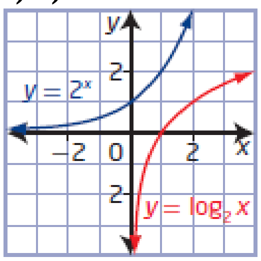
Chapter 8 Logarithmic Functions

Section 8.1 Understanding Logarithms

Section 8.1 Page 380 Question 1

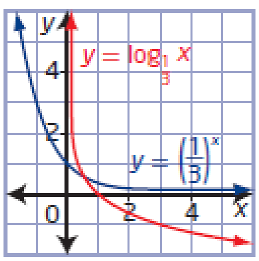
a) i)



ii) $y = \log_2 x$

iii) For $y = \log_2 x$,
- domain: $\{x \mid x > 0, x \in \mathrm{R}\}$ and range: $\{y \mid y \in \mathrm{R}\}$
- x-intercept: 1
- no y-intercept
- equation of the asymptote: $x = 0$

b) i)



ii) $y = \log_{\frac{1}{3}} x$

iii) For $y = \log_{\frac{1}{3}} x$,
- domain: $\{x \mid x > 0, x \in \mathrm{R}\}$ and range: $\{y \mid y \in \mathrm{R}\}$
- x-intercept: 1
- no y-intercept
- equation of the asymptote: $x = 0$

Section 8.1 Page 380 Question 2

a) In logarithmic form, $12^2 = 144$ is $\log_{12} 144 = 2$.

b) In logarithmic form, $8^{\frac{1}{3}} = 2$ is $\log_8 2 = \dfrac{1}{3}$.

c) In logarithmic form, $10^{-5} = 0.000\ 01$ is $\log_{10} 0.000\ 01 = -5$.

d) In logarithmic form, $7^{2x} = y + 3$ is $\log_7 (y + 3) = 2x$.

Section 8.1 Page 380 Question 3

a) In exponential form, $\log_5 25 = 2$ is $5^2 = 25$.

b) In exponential form, $\log_8 4 = \frac{2}{3}$ is $8^{\frac{2}{3}} = 4$.

c) In exponential form, $\log 1\,000\,000 = 6$ is $10^6 = 1\,000\,000$.

d) In exponential form, $\log_{11} (x + 3) = y$ is $11^y = x + 3$.

Section 8.1 Page 380 Question 4

a) Since $5^3 = 125$, the value of the logarithm is 3. Therefore, $\log_5 125 = 3$.

b) Since $10^0 = 1$, the value of the logarithm is 0. Therefore, $\log 1 = 0$.

c) Let $\log_4 \sqrt[3]{4} = x$. Express in exponential form.

$$4^x = \sqrt[3]{4}$$
$$4^x = 4^{\frac{1}{3}}$$
$$x = \frac{1}{3}$$

Therefore, $\log_4 \sqrt[3]{4} = \frac{1}{3}$.

d) Let $\log_{\frac{1}{3}} 27 = x$. Express in exponential form.

$$\left(\frac{1}{3}\right)^x = 27$$
$$\left(\frac{1}{3}\right)^x = \left(\frac{1}{3}\right)^{-3}$$
$$x = -3$$

Therefore, $\log_{\frac{1}{3}} 27 = -3$.

Section 8.1 Page 380 Question 5

Write $a < \log_2 28 < b$ in exponential form: $2^a < 28 < 2^b$.
Since $2^4 = 16$ and $2^5 = 32$, then $a = 4$ and $b = 5$.

Section 8.1 Page 380 Question 6

a) For $\log_3 x$ to be a positive number, $x > 1$.

b) For $\log_3 x$ to be a negative number, $0 < x < 1$.

c) For $\log_3 x$ to be zero, $x = 1$.

d) Example: For $\log_3 x$ to be a rational number, $x = \sqrt{3}$.

Section 8.1 Page 380 Question 7

a) The base of a logarithm cannot be 0 because $0^y = 0$, $y \neq 0$.

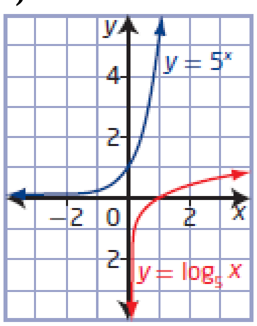
b) The base of a logarithm cannot be 1 because $1^y = 1$.

c) The base of a logarithm cannot be negative because exponential functions are only defined for $c > 0$.

Section 8.1 Page 380 Question 8

a) If $f(x) = 5^x$, then $f^{-1}(x) = \log_5 x$.

b)

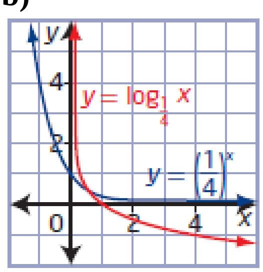


For $y = \log_5 x$,
- domain: $\{x \mid x > 0, x \in \mathrm{R}\}$ and range: $\{y \mid y \in \mathrm{R}\}$
- x-intercept: 1
- no y-intercept
- equation of the asymptote: $x = 0$

Section 8.1 Page 380 Question 9

a) If $g(x) = \log_{\frac{1}{4}} x$, then $g^{-1}(x) = \left(\frac{1}{4}\right)^x$.

b)



For $y = \left(\frac{1}{4}\right)^x$,

- domain: $\{x \mid x \in \mathrm{R}\}$ and range: $\{y \mid y > 0, y \in \mathrm{R}\}$
- no x-intercept
- y-intercept: 1
- equation of the asymptote: $y = 0$

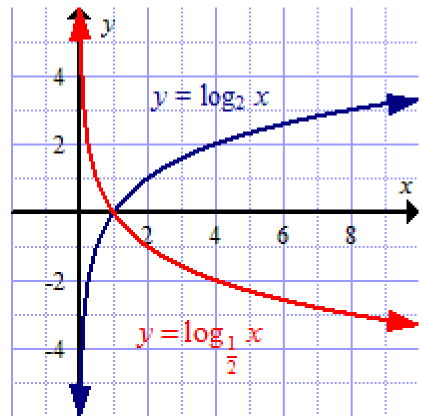
Section 8.1 Page 381 Question 10

The relationship between the characteristics of the functions $y = 7^x$ and $y = \log_7 x$ is that the graphs are reflections of each other in the line $y = x$. This means that the domain, range, y-intercept, and horizontal asymptote of the exponential function become the range, domain, x-intercept, and vertical asymptote of the logarithmic function.

Section 8.1 Page 381 Question 11

a) The graphs have the same domain, range, x-intercept, and vertical asymptote.

b) The graphs differ in that one is increasing and the other is decreasing.



Section 8.1 Page 381 Question 12

a)
$$\begin{aligned}\log_6 x &= 3\\ 6^3 &= x\\ x &= 216\end{aligned}$$

b)
$$\begin{aligned}\log_x 9 &= \frac{1}{2}\\ x^{\frac{1}{2}} &= 9\\ x &= 9^2\\ x &= 81\end{aligned}$$

c)
$$\begin{aligned}\log_{\frac{1}{4}} x &= -3\\ \left(\frac{1}{4}\right)^{-3} &= x\\ x &= 4^3\\ x &= 64\end{aligned}$$

d)
$$\begin{aligned}\log_x 16 &= \frac{4}{3}\\ x^{\frac{4}{3}} &= 16\\ x &= 16^{\frac{3}{4}}\\ x &= 8\end{aligned}$$

Section 8.1 Page 381 Question 13

a) Use the inverse property $c^{\log_c x} = x$. For $m = \log_5 7$,

$$\begin{aligned}5^m &= 5^{\log_5 7}\\ &= 7\end{aligned}$$

b) Use the inverse property $c^{\log_c x} = x$. For $n = \log_8 6$,
$8^n = 8^{\log_8 6}$
$= 6$

Section 8.1 Page 381 Question 14

a) $\log_2 (\log_3 (\log_4 64)) = \log_2 (\log_3 3)$
$= \log_2 1$
$= 0$

b) $\log_4 (\log_2 (\log 10^{16})) = \log_4 (\log_2 16)$
$= \log_4 4$
$= 1$

Section 8.1 Page 381 Question 15

Substitute $y = 0$.
$y = \log_7 (x + 2)$
$0 = \log_7 (x + 2)$
$x + 2 = 7^0$
$x = -1$
The x-intercept of $y = \log_7 (x + 2)$ is -1.

Section 8.1 Page 381 Question 16

Use the given point $\left(\frac{1}{8}, -3\right)$ on the graph of $f(x) = \log_c x$ to determine the value of c.
$f(x) = \log_c x$
$-3 = \log_c \frac{1}{8}$
$c^{-3} = \frac{1}{8}$
$c^{-3} = 2^{-3}$
$c = 2$
So, the inverse of $f(x) = \log_2 x$ is $f^{-1}(x) = 2^x$.
For the point $(4, k)$ on the graph of the inverse, substitute $x = 4$.
$f^{-1}(x) = 2^x$
$f^{-1}(4) = 2^4$
$f^{-1}(4) = 16$
Therefore, the value of k is 16.

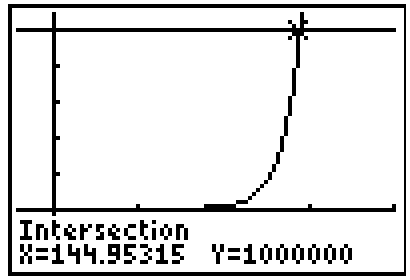
Section 8.1 Page 381 Question 17

a) Given the exponential function $N(t) = 1.1^t$, the equation of the inverse is $t = \log_{1.1} N$.

b) Substitute $N = 1\ 000\ 000$.

$$N(t) = 1.1^t$$
$$1\ 000\ 000 = 1.1^t$$

Use graphing technology to graph each side of the equation and determine the point of intersection.



It will take approximately 145 days for the number of users to exceed 1 000 000.

Section 8.1 Page 381 Question 18

Determine the relative risk for each asteroid from the Palermo scale.

Substitute $P = –1.66$.

$$P = \log R$$
$$–1.66 = \log R$$
$$R = 10^{–1.66}$$

Substitute $P = –4.83$.

$$P = \log R$$
$$–4.83 = \log R$$
$$R = 10^{–4.83}$$

Compare the relative risks.

$$\frac{10^{-1.66}}{10^{-4.83}} = 10^{3.17}$$
$$= 1479.108\ldots$$

The larger asteroid had a relative risk that is about 1479 times as dangerous as the smaller asteroid.

Section 8.1 Page 381 Question 19

Determine the amplitude of each earthquake.

Nahanni earthquake:

$$M = \log \frac{A}{A_0}$$
$$6.9 = \log \frac{A}{A_0}$$
$$10^{6.9} = \frac{A}{A_0}$$
$$A = 10^{6.9} A_0$$

Saskatchewan earthquake:

$$M = \log \frac{A}{A_0}$$
$$3.9 = \log \frac{A}{A_0}$$
$$10^{3.9} = \frac{A}{A_0}$$
$$A = 10^{3.9} A_0$$

Compare the amplitudes.

$$\frac{10^{6.9}}{10^{3.9}} = 10^3$$

The seismic shaking of the Nahanni earthquake was 1000 times that of the Saskatchewan earthquake.

Section 8.1 Page 381 Question 20

If $\log_5 x = 2$, then $x = 5^2$, or 25.

$$\begin{aligned}\log_5 125x &= \log_5 125(25)\\ &= \log_5 5^3(5^2)\\ &= \log_5 5^5\\ &= 5\end{aligned}$$

Section 8.1 Page 381 Question 21

$$\begin{aligned}\log_3 (m - n) &= 0\\ 3^0 &= m - n\\ 1 &= m - n \quad ①\end{aligned}$$

$$\begin{aligned}\log_3 (m + n) &= 3\\ 3^3 &= m + n\\ 27 &= m + n \quad ②\end{aligned}$$

Solve the system of equations.

$$\begin{aligned}1 &= m - n\\ \underline{27} &\underline{= m + n}\\ 28 &= 2m \qquad ① + ②\\ m &= 14\end{aligned}$$

Substitute $m = 14$ into ①.

$1 = m - n$

$1 = 14 - n$

$n = 13$

Section 8.1 Page 381 Question 22

If $\log_3 m = n$, then $3^n = m$.

$$\begin{aligned}\log_3 m^4 &= \log_3 (3^n)^4\\ &= \log_3 3^{4n}\\ &= 4n\end{aligned}$$

Section 8.1 Page 381 Question 23

If $y = \log_2 (\log_3 x)$, then the inverse is

$x = \log_2 (\log_3 y)$

$2^x = \log_3 y$

$y = 3^{2^x}$

Section 8.1 Page 381 Question 24

If $m = \log_2 n$, then $2^m = n$.

$2m + 1 = \log_2 16n$

$2m + 1 = \log_2 16(2^m)$

$2m + 1 = \log_2 2^4(2^m)$

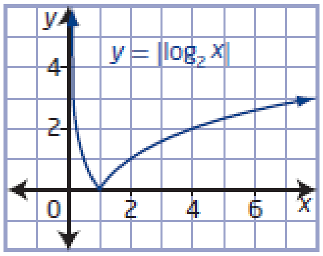
$2m + 1 = \log_2 2^{4 + m}$

$2m + 1 = 4 + m$

$m = 3$

Substitute $m = 3$ into $2^m = n$.
$2^m = n$
$2^3 = n$
$8 = n$

Section 8.1 Page 381 Question C1



The graph of $y = |\log_2 x|$ is the same as the graph of $y = \log_2 x$ for $x \geq 1$. The graph of $y = |\log_2 x|$ is the reflection in the x-axis of the graph of $y = \log_2 x$ for $0 < x < 1$.

Section 8.1 Page 381 Question C2

Answers may vary. Mind maps should include a graph showing how exponential functions and logarithmic functions are related, domain, range, intercept, and equation of the asymptote.

Section 8.1 Page 382 Question C3

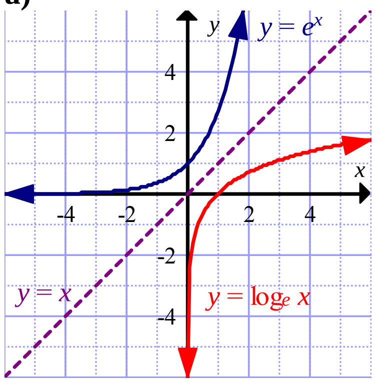
Step 1
a) $e = 2.718\ 281\ 828$

b) The minimum value of x needed to approximate e correctly to nine decimal places is 10^{10}.

x	$\left(1+\frac{1}{x}\right)^x$
10^1	2.593 742 460
10^2	2.704 813 829
10^3	2.716 923 932
10^4	2.718 145 927
10^5	2.718 268 237
10^6	2.718 280 469
10^7	2.718 281 693
10^8	2.718 281 815
10^9	2.718 281 827
10^{10}	2.718 281 828

Step 2

a)



For $y = \log_e x$,
- domain: $\{x \mid x > 0, x \in \mathrm{R}\}$ and range: $\{y \mid y \in \mathrm{R}\}$
- x-intercept: 1
- no y-intercept
- equation of the asymptote: $x = 0$

b) The inverse of $y = e^x$ is $y = \ln x$.

Step 3

a) Substitute $\theta = 2\pi$.

$$\begin{aligned} r &= e^{0.14\theta} \\ &= e^{0.14(2\pi)} \\ &= 2.410\ldots \end{aligned}$$

The distance, r, from point P to the origin after the point has rotated 2π is approximately 2.41.

b) i) $r = e^{0.14\theta}$

$$\begin{aligned} 0.14\theta &= \ln r \\ \theta &= \frac{\ln r}{0.14} \end{aligned}$$

The logarithmic form of $r = e^{0.14\theta}$ is $\theta = \dfrac{\ln r}{0.14}$.

ii) Substitute $r = 12$.

$$\begin{aligned} \theta &= \frac{\ln r}{0.14} \\ &= \frac{\ln 12}{0.14} \\ &= 17.749... \end{aligned}$$

The angle, θ, of rotation that corresponds to a value for r of 12 is approximately 17.75.

Section 8.2 Transformations of Logarithmic Functions

Section 8.2 Page 389 Question 1

Compare each function to the form $y = a \log_5 (b(x - h)) + k$.

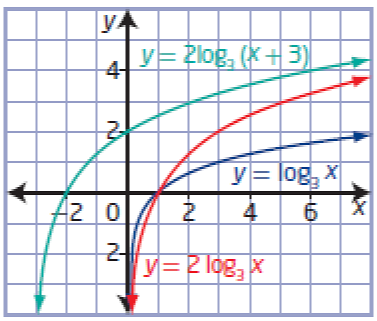
a) For $y = \log_5 (x - 1) + 6$, $h = 1$ and $k = 6$. This is a translation of 1 unit to the right and 6 units up of the graph of $y = \log_5 x$.

b) For $y = -4 \log_5 3x$, $a = -4$ and $b = 3$. This is a vertical stretch about the x-axis by a factor of 4, a reflection in the x-axis, and a horizontal stretch by a factor of $\frac{1}{3}$ of the graph of $y = \log_5 x$.

c) For $y = \frac{1}{2} \log_5 (-x) + 7$, $a = \frac{1}{2}$, $b = -1$, and $k = 7$. This is a vertical stretch about the x-axis by a factor of $\frac{1}{2}$, a reflection in the y-axis, and a translation of 7 units up of the graph of $y = \log_5 x$.

Section 8.2 Page 389 Question 2

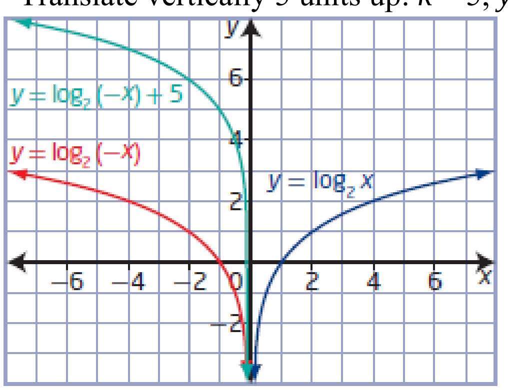
a) Given: $y = \log_3 x$
• Stretch vertically about the x-axis by a factor of 2: $a = 2$, $y = 2 \log_3 x$
• Translate 3 units to the left: $h = -3$, $y = 2 \log_3 (x + 3)$



b) $y = 2 \log_3 (x + 3)$

Section 8.2 Page 390 Question 3

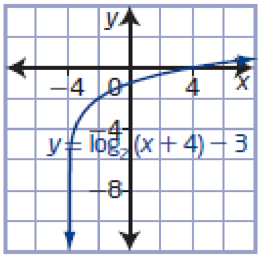
a) Given: $y = \log_2 x$
• Reflect in the y-axis: $b = -1$, $y = \log_2 (-x)$
• Translate vertically 5 units up: $k = 5$, $y = \log_2 (-x) + 5$



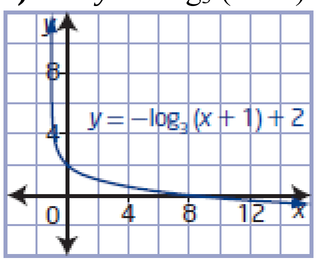
b) $y = \log_2 (-x) + 5$

Section 8.2 Page 390 Question 4

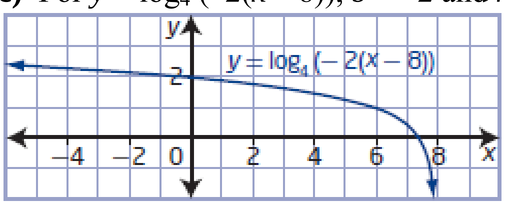
a) For $y = \log_2 (x + 4) - 3$, $h = -4$ and $k = -3$.



b) For $y = -\log_3 (x + 1) + 2$, $a = -1$, $h = -1$, and $k = 2$.



c) For $y = \log_4 (-2(x - 8))$, $b = -2$ and $h = 8$.



Section 8.2 Page 390 Question 5

a) $y = -5 \log_3 (x + 3)$

i) The equation of the vertical asymptote occurs when $x + 3 = 0$. Therefore, the equation of the vertical asymptote is $x = -3$.

ii) The domain is $\{x \mid x > -3, x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

iii) Substitute $x = 0$. Then, solve for y.

$y = -5 \log_3 (x + 3)$
$\quad = -5 \log_3 (0 + 3)$
$\quad = -5 \log_3 3$
$\quad = -5$

The y-intercept is -5.

iv) Substitute $y = 0$. Then, solve for x.

$y = -5 \log_3 (x + 3)$
$0 = -5 \log_3 (x + 3)$
$0 = \log_3 (x + 3)$

$3^0 = x + 3$
$1 = x + 3$
$x = -2$
The x-intercept is –2.

b) $y = \log_6 (4(x + 9))$
i) The equation of the vertical asymptote occurs when $4(x + 9) = 0$. Therefore, the equation of the vertical asymptote is $x = -9$.

ii) The domain is $\{x \mid x > -9, x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

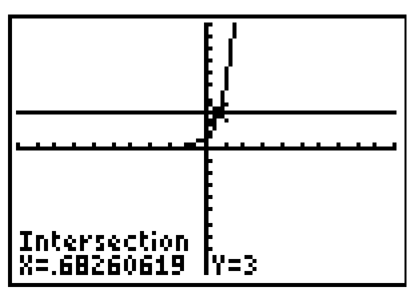
iii) Substitute $x = 0$. Then, solve for y.
$y = \log_6 (4(x + 9))$
$\quad = \log_6 (4(0 + 9))$
$\quad = \log_6 36$
$\quad = 2$
The y-intercept is 2.

iv) Substitute $y = 0$. Then, solve for x.
$y = \log_6 (4(x + 9))$
$0 = \log_6 (4(x + 9))$
$6^0 = 4(x + 9)$
$1 = 4x + 36$
$4x = -35$
$$x = -\frac{35}{4}$$
The x-intercept is $-\frac{35}{4}$, or –8.75.

c) $y = \log_5 (x + 3) - 2$
i) The equation of the vertical asymptote occurs when $x + 3 = 0$. Therefore, the equation of the vertical asymptote is $x = -3$.

ii) The domain is $\{x \mid x > -3, x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

iii) Substitute $x = 0$. Then, solve for y.
$y = \log_5 (x + 3) - 2$
$\quad = \log_5 (0 + 3) - 2$
$\quad = \log_5 3 - 2$
To obtain an approximate value for $\log_5 3$, graph $y = 5^x$ and $y = 3$ and find the point of intersection.
$\log_5 3 \approx 0.68$



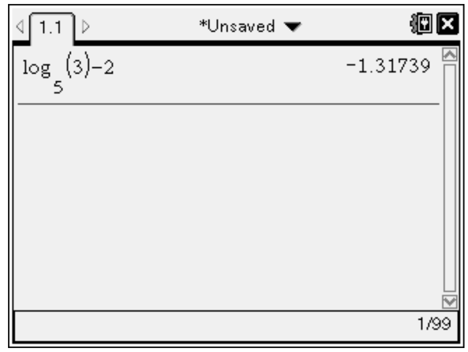
$y \approx 0.68 - 2$
$\quad \approx -1.3$
The y-intercept is about –1.3.

A possible alternative is to use a calculator that can evaluate a logarithm of any base.

$y = \log_5 3 - 2$

≈ -1.3

The y-intercept is about –1.3.



iv) Substitute $y = 0$. Then, solve for x.

$y = \log_5 (x + 3) - 2$

$0 = \log_5 (x + 3) - 2$

$2 = \log_5 (x + 3)$

$5^2 = x + 3$

$25 = x + 3$

$x = 22$

The x-intercept is 22.

d) $y = -3 \log_2 (x + 1) - 6$

i) The equation of the vertical asymptote occurs when $x + 1 = 0$. Therefore, the equation of the vertical asymptote is $x = -1$.

ii) The domain is $\{x \mid x > -1, x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

iii) Substitute $x = 0$. Then, solve for y.

$y = -3 \log_2 (x + 1) - 6$

$= -3 \log_2 (0 + 1) - 6$

$= -3 \log_2 1 - 6$

$= -3(0) - 6$

$= -6$

The y-intercept is –6.

iv) Substitute $y = 0$. Then, solve for x.

$y = -3 \log_2 (x + 1) - 6$

$0 = -3 \log_2 (x + 1) - 6$

$6 = -3 \log_2 (x + 1)$

$-2 = \log_2 (x + 1)$

$2^{-2} = x + 1$

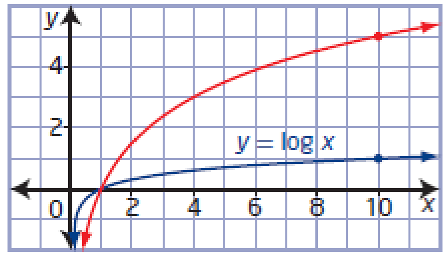
$\frac{1}{4} = x + 1$

$x = -\frac{3}{4}$

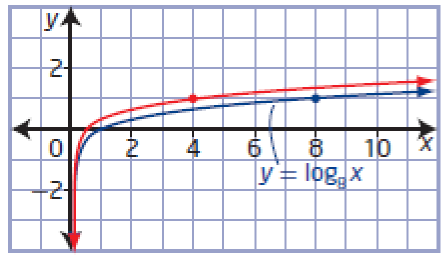
The x-intercept is $-\frac{3}{4}$, or –0.75.

Section 8.2 Page 390 Question 6

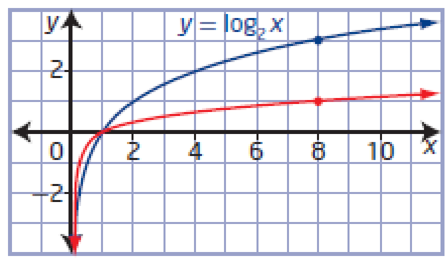
a) The key point (10, 1) on the graph of $y = \log x$ has become the image point (10, 5) on the red graph. Thus, the red graph can be generated by vertically stretching the graph of $y = \log x$ about the x-axis by a factor of 5. The red graph can be described by the equation $y = 5 \log x$.



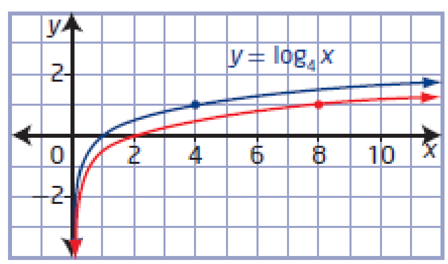
b) The key point (8, 1) on the graph of $y = \log_8 x$ has become the image point (4, 1) on the red graph. Thus, the red graph can be generated by horizontally stretching the graph of $y = \log_8 x$ about the y-axis by a factor of $\frac{1}{2}$. The red graph can be described by the equation $y = \log_8 2x$.



c) The key point (8, 3) on the graph of $y = \log_2 x$ has become the image point (8, 1) on the red graph. Thus, the red graph can be generated by vertically stretching the graph of $y = \log_2 x$ about the x-axis by a factor of $\frac{1}{3}$. The red graph can be described by the equation $y = \frac{1}{3} \log_2 x$.



d) The key point (4, 1) on the graph of $y = \log_4 x$ has become the image point (8, 1) on the red graph. Thus, the red graph can be generated by horizontally stretching the graph of $y = \log_4 x$ about the y-axis by a factor of 2. The red graph can be described by the equation $y = \log_4 \left(\frac{1}{2}x\right)$.



Section 8.2 Page 390 Question 7

a) For $y = \log_7 (4(x + 5)) + 6$, $b = 4$, $h = –5$, and $k = 6$. To obtain the graph of $y = \log_7 (4(x + 5)) + 6$, the graph of $y = \log_7 x$ must be horizontally stretched about the y-axis by a factor of $\frac{1}{4}$ and translated 5 units to the left and 6 units up.

b) For $y = 2\log_7\left(-\frac{1}{3}(x-1)\right)-4$, $a = 2$, $b = -\frac{1}{3}$, $h = 1$, and $k = –4$. To obtain the graph of $y = 2\log_7\left(-\frac{1}{3}(x-1)\right)-4$, the graph of $y = \log_7 x$ must be horizontally stretched about the y-axis by a factor of 3, reflected in the y-axis, vertically stretched about the x-axis by a factor of 2, and translated 1 unit to the right and 4 units down.

Section 8.2 Page 390 Question 8

a) For a reflection in the x-axis and a translation of 6 units left and 3 units up, $a = –1$, $h = –6$, and $k = 3$. The equation of the transformed function is $y = –\log_3 (x + 6) + 3$.

b) For a vertical stretch by a factor of 5 about the x-axis and a horizontal stretch about the y-axis by a factor of $\frac{1}{3}$, $a = 5$, $b = 3$, $h = 0$, and $k = 0$. The equation of the transformed function is $y = 5 \log_3 3x$.

c) For a vertical stretch about the x-axis by a factor of $\frac{3}{4}$, a horizontal stretch about the y-axis by a factor of 4, a reflection in the y-axis, and a translation of 2 units right and 5 units down, $a = \frac{3}{4}$, $b = -\frac{1}{4}$, $h = 2$, and $k = –5$. The equation of the transformed function is $y = \frac{3}{4}\log_3\left(-\frac{1}{4}(x-2\right)-5$.

Section 8.2 Page 390 Question 9

a) $y = 5 \log_3 (–4x + 12) – 2$
$y = 5 \log_3 (–4(x – 3)) – 2$
For $y = 5 \log_3 (–4(x – 3)) – 2$, $a = 5$, $b = –4$, $h = 3$, and $k = –2$. To obtain the graph of $y = 5 \log_3 (–4(x – 3)) – 2$, the graph of $y = \log_3 x$ must be horizontally stretched about the y-axis by a factor of $\frac{1}{4}$, reflected in the y-axis, vertically stretched about the x-axis by a factor of 5, and translated 3 units to the right and 2 units down.

b) $y = -\frac{1}{4} \log_3 (6 – x) + 1$
$y = -\frac{1}{4} \log_3 (–(x – 6)) + 1$
For $y = -\frac{1}{4} \log_3 (–(x – 6)) + 1$, $a = -\frac{1}{4}$, $b = –1$, $h = 6$, and $k = 1$. To obtain the graph of $y = -\frac{1}{4} \log_3 (–(x – 6)) + 1$, the graph of $y = \log_3 x$ must be reflected in the y-axis,

vertically stretched about the x-axis by a factor of $\frac{1}{4}$, reflected in the x-axis, and translated 6 units to the right and 1 unit up.

Section 8.2 Page 390 Question 10

a) For a vertical translation, compare the point on the graph of $y = \log_3 x$ with the same x-coordinate as the given point on the transformed function, (9, –4).
$(9, 2) \rightarrow (9, -4)$
So, $k = -6$ and the equation of the transformed image is $\log_3 x - 6$.

b) For a horizontal stretch, compare the point on the graph of $y = \log_2 x$ with the same y-coordinate as the given point on the transformed function, (8, 1).
$(2, 1) \rightarrow (8, 1)$
So, $b = \frac{1}{4}$ and the equation of the transformed image is $\log_2\left(\frac{1}{4}x\right)$.

Section 8.2 Page 391 Question 11

$$\frac{1}{3}(y + 2) = \log_6 (x - 4)$$
$$y + 2 = 3 \log_6 (x - 4)$$
$$y = 3 \log_6 (x - 4) - 2$$
For $y = 3 \log_6 (x - 4) - 2$, $a = 3$, $h = 4$, and $k = -2$. To obtain the graph of $y = 3 \log_6 (x - 4) - 2$, the graph of $y = \log_6 x$ must be vertically stretched about the x-axis by a factor of 3 and translated 4 units to the right and 2 units down.

Section 8.2 Page 391 Question 12

a) For $R = 0.67 \log 0.36E + 1.46$, $a = 0.67$, $b = 0.36$, and $k = 1.46$. The function is transformed from $R = \log E$ by a horizontal stretch about the y-axis by a factor of $\frac{1}{0.36}$ or $\frac{25}{9}$, vertically stretched about the x-axis by a factor of 0.67, and translated 1.46 units up.

b) Substitute $R = 7.0$.
$$R = 0.67 \log 0.36E + 1.46$$
$$7.0 = 0.67 \log 0.36E + 1.46$$
$$5.54 = 0.67 \log 0.36E$$
$$\frac{5.54}{0.67} = \log 0.36E$$

$$10^{\frac{5.54}{0.67}} = 0.36E$$

$$E = \frac{10^{\frac{5.54}{0.67}}}{0.36}$$

$$E = 515\,649\,042.5$$

The equivalent amount of energy released, to the nearest kilowatt-hour, is 515 649 043 kWh.

Section 8.2 Page 391 Question 13

a) Substitute $P = 110$.

$V = 0.23 + 0.35 \log (P - 56.1)$
$\quad = 0.23 + 0.35 \log (110 - 56.1)$
$\quad = 0.23 + 0.35 \log 53.9$
$\quad = 0.836\ldots$

To the nearest tenth of a microlitre, the vessel volume is 0.8 μL.

b) Substitute $V = 0.7$.

$V = 0.23 + 0.35 \log (P - 56.1)$
$0.7 = 0.23 + 0.35 \log (P - 56.1)$
$0.47 = 0.35 \log (P - 56.1)$

$$\frac{0.47}{0.35} = \log (P - 56.1)$$

$$10^{\frac{0.47}{0.35}} = P - 56.1$$

$$P = 10^{\frac{0.47}{0.35}} + 56.1$$

$$P = 78.122...$$

To the nearest millimetre of mercury, the arterial blood pressure is 78 mmHg.

Section 8.2 Page 391 Question 14

a) Substitute $m = 60$.

$\log m = 0.008h + 0.4$
$\log 60 = 0.008h + 0.4$
$\log 60 - 0.4 = 0.008h$

$$h = \frac{\log 60 - 0.4}{0.008}$$

$h = 172.268\ldots$

The height of the child, to the nearest centimetre, is 172 cm.

b) Substitute $h = 150$.

$\log m = 0.008h + 0.4$
$\log m = 0.008(150) + 0.4$
$\log m = 1.6$
$m = 10^{1.6}$
$m = 39.810\ldots$

The mass of the child, to the nearest kilogram, is 40 kg.

Section 8.2 Page 391 Question 15

For example, the point (8, 1) is on the graph of $f(x) = \log_8 a$.
Determine the value of a such that the point (8, 1) is on the graph of $g(x) = a \log_2 x$.

$$
\begin{aligned}
a \log_2 8 &= 1 \\
\log_2 8 &= \frac{1}{a} \\
2^{\frac{1}{a}} &= 8 \\
2^{\frac{1}{a}} &= 2^3 \\
\frac{1}{a} &= 3 \\
a &= \frac{1}{3}
\end{aligned}
$$

Section 8.2 Page 391 Question 16

a) The graph of $y = 2 \log_5 x - 7$ is reflected in the x-axis and translated 6 units up.
For a base function being transformed: $(x, y) \rightarrow (x, -y + 6)$
Using the given function as the base function:

$$
\begin{aligned}
(x, 2y - 7) &\rightarrow (x, -(2y - 7) + 6) \\
&\rightarrow (x, -2y + 13)
\end{aligned}
$$

The equation of the transformed image is $y = -2 \log_5 x + 13$.

b) The graph of $y = \log (6(x - 3))$ is stretched horizontally about the y-axis by a factor of 3 and translated 9 units left.
For a base function being transformed: $(x, y) \rightarrow (3x - 9, y)$
Using the given function as the base function:

$$
\begin{aligned}
\left(\frac{x}{6} + 3, y\right) &\rightarrow \left(3\left(\frac{x}{6} + 3\right) - 9, y\right) \\
&\rightarrow \left(\frac{x}{2}, y\right)
\end{aligned}
$$

The equation of the transformed image is $y = \log 2x$.

Section 8.2 Page 391 Question 17

The graph of $f(x) = \log_2 x$ has been transformed to $g(x) = a \log_2 x + k$: $(x, y) \rightarrow (x, ay + k)$.
Given points on the transformed image: $\left(\frac{1}{4}, -9\right)$ and $(16, -6)$.

Use the mapping to create a system of equations.

$\left(\frac{1}{4}, -2\right) \rightarrow \left(\frac{1}{4}, -9\right)$: $-2a + k = -9$ ①

$(16, 4) \rightarrow (16, -6)$: $4a + k = -6$ ②

$2 \times ①: -4a + 2k = -18$
$+ ②: \quad \underline{4a + k = -6}$
$3k = -24$
$k = -8$
Substitute $k = -8$ into ①.
$-2a + k = -9$
$-2a + -8 = -9$
$-2a = -1$
$a = \frac{1}{2}$

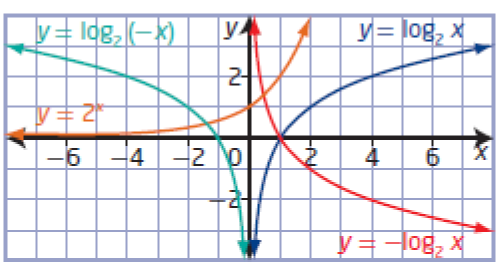
Section 8.2 Page 391 Question C1

The graph of $f(x) = 5^x$ is
• reflected in the line $y = x$: $g(x) = \log_5 x$
• vertically stretched about the x-axis by a factor of $\frac{1}{4}$: $a = \frac{1}{4}$
• horizontally stretched about the y-axis by a factor of 3: $b = \frac{1}{3}$
• translated 4 units right and 1 unit down: $h = 4$ and $k = -1$
The equation of transformed image is $g(x) = \frac{1}{4}\ \log_5 \frac{1}{3}(x - 4) - 1$.

Section 8.2 Page 391 Question C2

a) For $f(x) = \log_2 x$,
$y = -f(x) = -\log_2 x$
$y = f(-x) = \log_2 (-x)$
$y = f^{-1}(x) = 2^x$

b) The graph of $y = -\log_2 x$ is a reflection in the x-axis of the graph of $f(x) = \log_2 x$.
The graph of $y = \log_2 (-x)$ is a reflection in the y-axis of the graph of $f(x) = \log_2 x$.
The graph of $y = 2^x$ is a reflection in the line $y = x$ of the graph of $f(x) = \log_2 x$.



Section 8.2 Page 391 Question C3

a) The graph of $y = 3(7^{2x-1}) + 5$ is reflected in the line $y = x$. Determine the equation of the inverse of the function.

$$y = 3(7^{2x-1}) + 5$$
$$x = 3(7^{2y-1}) + 5$$
$$x - 5 = 3(7^{2y-1})$$
$$\frac{1}{3}(x-5) = 7^{2y-1}$$
$$2y - 1 = \log_7 \frac{1}{3}(x-5)$$
$$2y = \log_7 \frac{1}{3}(x-5) + 1$$
$$y = \frac{1}{2}\log_7 \frac{1}{3}(x-5) + \frac{1}{2}$$

b) Given $f(x) = 2\log3(x-1) + 8$, find $f^{-1}(x)$.

$$f(x) = 2\log3(x-1) + 8$$
$$y = 2\log3(x-1) + 8$$
$$x = 2\log3(y-1) + 8$$
$$x - 8 = 2\log3(y-1)$$
$$\frac{1}{2}(x-8) = \log3(y-1)$$
$$y - 1 = 3^{\frac{1}{2}(x-8)}$$
$$y = 3^{\frac{1}{2}(x-8)} + 1$$

Section 8.3 Laws of Logarithms

Section 8.3 Page 400 Question 1

a)
$$\begin{aligned}\log_7 xy^3\sqrt{z} &= \log_7 x + \log_7 y^3 + \log_7 \sqrt{z}\\ &= \log_7 x + 3\log_7 y + \frac{1}{2}\log_7 z\end{aligned}$$

b)
$$\begin{aligned}\log_5 (xyz)^8 &= 8\log_5 xyz\\ &= 8(\log_5 x + \log_5 y + \log_5 z)\end{aligned}$$

c)
$$\begin{aligned}\log\left(\frac{x^2}{y\sqrt[3]{z}}\right) &= \log x^2 - \log y\sqrt[3]{z}\\ &= 2\log x - (\log y + \log\sqrt[3]{z})\\ &= 2\log x - \log y - \frac{1}{3}\log z\end{aligned}$$

d)
$$\begin{aligned}\log_3 x\sqrt{\frac{y}{z}} &= \log_3 x + \log_3\sqrt{\frac{y}{z}}\\ &= \log_3 x + \frac{1}{2}\log_3 \frac{y}{z}\\ &= \log_3 x + \frac{1}{2}(\log_3 y - \log_3 z)\end{aligned}$$

Section 8.3 Page 400 Question 2

a) $\log_{12} 24 - \log_{12} 6 + \log_{12} 36$
$= \log_{12} \frac{24(36)}{6}$
$= \log_{12} 144$
$= 2$

b) $3\log_5 10 - \frac{1}{2}\log_5 64$
$= \log_5 10^3 - \log_5 \sqrt{64}$
$= \log_5 \frac{1000}{8}$
$= \log_5 125$
$= 3$

c) $\log_3 27\sqrt{3} = \log_3 27 + \log_3 \sqrt{3}$
$= \log_3 27 + \frac{1}{2}\log_3 3$
$= 3 + \frac{1}{2}(1)$
$= \frac{7}{2}$

d) $\log_2 72 - \frac{1}{2}(\log_2 3 + \log_2 27)$
$= \log_2 72 - \log_2 \sqrt{81}$
$= \log_2 72 - \log_2 9$
$= \log_2 \frac{72}{9}$
$= \log_2 8$
$= 3$

Section 8.3 Page 400 Question 3

a) $\log_9 x - \log_9 y + 4\log_9 z$
$= \log_9 \frac{x}{y} + \log_9 z^4$
$= \log_9 \frac{xz^4}{y}$

b) $\frac{\log_3 x}{2} - 2\log_3 y$
$= \log_3 \sqrt{x} - \log_3 y^2$
$= \log_3 \frac{\sqrt{x}}{y^2}$

c) $\log_6 x - \frac{1}{5}(\log_6 x + 2\log_6 y)$
$= \log_6 x - \frac{1}{5}\log_6 xy^2$
$= \log_6 x - \log_6 \sqrt[5]{xy^2}$
$= \log_6 \frac{x}{\sqrt[5]{xy^2}}$

d) $\frac{\log x}{3} + \frac{\log y}{3}$
$= \frac{1}{3}(\log x + \log y)$
$= \frac{1}{3}\log xy$
$= \log \sqrt[3]{xy}$

Section 8.3 Page 400 Question 4

Given: log 1.44 ≈ 0.158 36, log 1.2 ≈ 0.079 18, and log 1.728 ≈ 0.237 54.

a) For 1.44 × 1.2,

$$\begin{aligned}\log (1.44 \times 1.2) &= \log 1.44 + \log 1.2\\ &= 0.158\ 36 + 0.079\ 18\\ &= 0.237\ 54\\ &= \log 1.728\end{aligned}$$

So, 1.44 × 1.2 = 1.728.

b) For 1.728 ÷ 1.2,

$$\begin{aligned}\log (1.728 \div 1.2) &= \log 1.728 - \log 1.2\\ &= 0.237\ 54 - 0.079\ 18\\ &= 0.158\ 36\\ &= \log 1.44\end{aligned}$$

So, 1.728 ÷ 1.2 = 1.44.

b) For $\sqrt{1.44}$,

$$\begin{aligned}\log \sqrt{1.44} &= 0.5 \log 1.44\\ &= 0.5(0.158\ 36)\\ &= 0.079\ 18\\ &= \log 1.2\end{aligned}$$

So, $\sqrt{1.44} = 1.2$.

Section 8.3 Page 400 Question 5

a) Given: $k = \log_2 40 - \log_2 5$

$$\begin{aligned}3^k &= 3^{\log_2 40 - \log_2 5}\\ &= 3^{\log_2 \frac{40}{5}}\\ &= 3^{\log_2 8}\\ &= 3^3\\ &= 27\end{aligned}$$

b) Given: $n = 3 \log_8 4$

$$\begin{aligned}7^n &= 7^{3\log_8 4}\\ &= 7^{\log_8 4^3}\\ &= 7^{\log_8 64}\\ &= 7^2\\ &= 49\end{aligned}$$

Section 8.3 Page 400 Question 6

a) You need to apply a horizontal stretch about the y-axis by a factor of $\frac{1}{8}$ to the graph of $y = \log_2 x$ to result in the graph of $y = \log_2 8x$.

b) Using the product law of logarithms, the function $y = \log_2 8x$ can be written as $y = \log_2 8 + \log_2 x$, or $y = \log_2 x + 3$.
You need to apply a translation of 3 units up to the graph of $y = \log_2 x$ to result in the graph of $y = \log_2 8x$.

Section 8.3 Page 401 Question 7

a) The equation $\dfrac{\log_c x}{\log_c y} = \log_c x - \log_c y$ is false, as $\log_c \dfrac{x}{y} = \log_c x - \log_c y$.

b) The equation $\log_c (x + y) = \log_c x + \log_c y$ is false, as $\log_c xy = \log_c x + \log_c y$.

c) The equation $\log_c c^n = n$ is true, since

$$\begin{aligned}\log_c c^n &= n \log_c c \\ &= n(1) \\ &= n\end{aligned}$$

d) The equation $(\log_c x)^n = n \log_c x$ is false, as $\log_c x^n = n \log_c x$.

e) The equation $-\log_c \left(\dfrac{1}{x}\right) = \log_c x$ is true, since

$$\begin{aligned}-\log_c \left(\frac{1}{x}\right) &= \log_c \left(\frac{1}{x}\right)^{-1} \\ &= \log_c x\end{aligned}$$

Section 8.3 Page 401 Question 8

Given: $\log 3 = P$ and $\log 5 = Q$

a) $$\begin{aligned}\log \frac{3}{5} &= \log 3 - \log 5 \\ &= P - Q\end{aligned}$$

b) $$\begin{aligned}\log 15 &= \log 3(5) \\ &= \log 3 + \log 5 \\ &= P + Q\end{aligned}$$

c) $$\begin{aligned}\log 3\sqrt{5} &= \log 3 + \log \sqrt{5} \\ &= \log 3 + \frac{1}{2}\log 5 \\ &= P + \frac{1}{2}Q\end{aligned}$$

d) $$\begin{aligned}\log \frac{25}{9} &= \log \left(\frac{5}{3}\right)^2 \\ &= 2\log \frac{5}{3} \\ &= 2(\log 5 - \log 3) \\ &= 2(Q - P)\end{aligned}$$

Section 8.3 Page 401 Question 9

Given: $\log_2 7 = K$

a) $$\begin{aligned}\log_2 7^6 &= 6\log_2 7 \\ &= 6K\end{aligned}$$

b) $$\begin{aligned}\log_2 14 &= \log_2 7(2) \\ &= \log_2 7 + \log_2 2 \\ &= K + 1\end{aligned}$$

c) $\log_2(49 \times 4) = \log_2 49 + \log_2 4$
$$= \log_2 7^2 + \log_2 4$$
$$= 2\log_2 7 + \log_2 4$$
$$= 2K + 2$$

d) $\log_2 \dfrac{\sqrt[5]{7}}{8} = \log_2 \sqrt[5]{7} - \log_2 8$
$$= \frac{1}{5}\log_2 7 - \log_2 8$$
$$= \frac{1}{5}K - 3$$

Section 8.3 Page 401 Question 10

a) $\log_5 x + \log_5 \sqrt{x^3} - 2\log_5 x = \log_5 x\sqrt{x^3} - \log_5 x^2$
$$= \log_5 \frac{x\sqrt{x^3}}{x^2}$$
$$= \log_5 \frac{x^2\sqrt{x}}{x^2}$$
$$= \log_5 \sqrt{x}$$
$$= \frac{1}{2}\log_5 x,\ x > 0$$

b) $\log_{11} \dfrac{x}{\sqrt{x}} + \log_{11} \sqrt{x^5} - \dfrac{7}{3}\log_{11} x = \log_{11} \dfrac{x\sqrt{x^5}}{\sqrt{x}} - \log_{11} x^{\frac{7}{3}}$
$$= \log_{11} \frac{x^3\sqrt{x}}{\sqrt{x}} - \log_{11} x^2\sqrt[3]{x}$$
$$= \log_{11} \frac{x^3}{x^2\sqrt[3]{x}}$$
$$= \log_{11} x^{\frac{2}{3}}$$
$$= \frac{2}{3}\log_{11} x,\ x > 0$$

Section 8.3 Page 401 Question 11

a) $\log_2(x^2 - 25) - \log_2(3x - 15) = \log_2 \dfrac{x^2 - 25}{3x - 15}$
$$= \log_2 \frac{(x-5)(x+5)}{3(x-5)}$$
$$= \log_2 \frac{x+5}{3},\ x < -5 \text{ or } x > 5$$

b) $\log_7(x^2-16)-\log_7(x^2-2x-8)=\log_7\dfrac{x^2-16}{x^2-2x-8}$

$$=\log_7\frac{(x-4)(x+4)}{(x-4)(x+2)}$$

$$=\log_7\frac{x+4}{x+2}, x<-4 \text{ or } x>4$$

c) $2\log_8(x+3)-\log_8(x^2+x-6)=\log_8\dfrac{x+3}{x^2+x-6}$

$$=\log_8\frac{x+3}{(x+3)(x-2)}$$

$$=\log_8\frac{1}{x-2}, x>2$$

Section 8.3 Page 401 Question 12

a) Left Side
$\log_c 48 - (\log_c 3 + \log_c 2)$
$= \log_c 48 - \log_c 3(2)$
$= \log_c \dfrac{48}{6}$
$= \log_c 8$
Left Side = Right Side

Right Side
$\log_c 8$

b) Left Side
$7 \log_c 4$
$= \log_c 4^7$
$= \log_c (2^2)^7$
$= \log_c 2^{14}$
$= 14 \log_c 2$
Left Side = Right Side

Right Side
$14 \log_c 2$

c) Left Side
$\dfrac{1}{2}(\log_c 2+\log_c 6)$
$=\dfrac{1}{2}\log_c 12$
$=\log_c \sqrt{12}$
$=\log_c 2\sqrt{3}$
$=\log_c 2+\log_c \sqrt{3}$
Left Side = Right Side

Right Side
$\log_c 2+\log_c \sqrt{3}$

d) Left Side Right Side

$\log_c (5c)^2$ $2(\log_c 5 + 1)$

$= 2 \log_c 5c$

$= 2(\log_c 5 + \log_c c)$

$= 2(\log_c 5 + 1)$

Left Side = Right Side

Section 8.3 Page 401 Question 13

a) Substitute $I = 0.000\ 01$ and $I_0 = 10^{-12}$.

$$\begin{aligned}
\beta &= 10\log\left(\frac{I}{I_0}\right) \\
&= 10\log\left(\frac{0.000\ 01}{10^{-12}}\right) \\
&= 10\log\left(\frac{10^{-5}}{10^{-12}}\right) \\
&= 10\log 10^7 \\
&= 10(7) \\
&= 70
\end{aligned}$$

The decibel level of the a hairdryer is 70 dB.

b) Let the decibel levels of two sounds be $\beta_1 = 10 \log \frac{I_1}{I_0}$ and $\beta_2 = 10 \log \frac{I_2}{I_0}$.

From Example 4 on page 398, comparing the two intensities results in the equation $\beta_2 - \beta_1 = 10\left(\log \frac{I_2}{I_1}\right)$. Substitute $\beta_2 = 118$ and $\beta_1 = 85$.

$$\begin{aligned}
\beta_2 - \beta_1 &= 10\left(\log \frac{I_2}{I_1}\right) \\
118 - 85 &= 10\left(\log \frac{I_2}{I_1}\right) \\
33 &= 10\left(\log \frac{I_2}{I_1}\right) \\
3.3 &= \log \frac{I_2}{I_1} \\
10^{3.3} &= \frac{I_2}{I_1} \\
1995.262\ldots &= \frac{I_2}{I_1}
\end{aligned}$$

The fire truck siren is approximately 1995 times as loud as city traffic.

c) Substitute $\frac{I_2}{I_1} = 63$ and $\beta_1 = 80$ into $\beta_2 - \beta_1 = 10\left(\log\frac{I_2}{I_1}\right)$.

$$\beta_2 - \beta_1 = 10\left(\log\frac{I_2}{I_1}\right)$$
$$\beta_2 - 80 = 10 \log 63$$
$$\beta_2 = 10 \log 63 + 80$$
$$\beta_2 = 97.993\ldots$$

The decibel level of the farm tractor is approximately 98 dB.

Section 8.3 Page 401 Question 14

The decibel scale is logarithmic, not linear. So, a 20 dB sound is actually 10^1 times as loud as a 10 dB sound.

Section 8.3 Page 401 Question 15

Substitute $G = 24$ and $V_i = 0.2$.

$$G = 20\log\frac{V}{V_i}$$
$$24 = 20\log\frac{V}{0.2}$$
$$1.2 = \log\frac{V}{0.2}$$
$$10^{1.2} = \frac{V}{0.2}$$
$$V = 0.2(10^{1.2})$$
$$V = 3.169...$$

The voltage is 3.2 V, to the nearest tenth of a volt.

Section 8.3 Page 402 Question 16

a) Substitute pH = 7.0.

$$\text{pH} = -\log [\text{H}+]$$
$$7.0 = -\log [\text{H}+]$$
$$-7.0 = \log [\text{H}+]$$
$$[\text{H}+] = 10^{-7.0}$$

The hydrogen ion concentration is 10^{-7} mol/L.

b) Let the pH levels of two rains be $\text{pH}_1 = -\log [\text{H}_1+]$ and $\text{pH}_2 = -\log [\text{H}_2+]$.
Compare the two pH levels.

$$\text{pH}_2 - \text{pH}_1 = -\log [\text{H}_2+] - (-\log [\text{H}_1+])$$
$$\text{pH}_2 - \text{pH}_1 = \log [\text{H}_2+]^{-1} - \log [\text{H}_1+]^{-1}$$

$\text{pH}_2 - \text{pH}_1 = \log \dfrac{[\text{H}_1+]}{[\text{H}_2+]}$

Substitute $\text{pH}_2 = 5.6$ and $\text{pH}_1 = 4.5$.

$$5.6 - 4.5 = \log \frac{[\text{H}_1+]}{[\text{H}_2+]}$$

$$1.1 = \log \frac{[\text{H}_1+]}{[\text{H}_2+]}$$

$$10^{1.1} = \frac{[\text{H}_1+]}{[\text{H}_2+]}$$

$$12.589\ldots = \frac{[\text{H}_1+]}{[\text{H}_2+]}$$

Acid rain is approximately 12.6 times more acidic than normal rain.

c) Substitute $\dfrac{[\text{H}_1+]}{[\text{H}_2+]} = 500$ and $\text{pH}_2 = 6.1$ into $\text{pH}_2 - \text{pH}_1 = \log \dfrac{[\text{H}_1+]}{[\text{H}_2+]}$.

$\text{pH}_2 - \text{pH}_1 = \log \dfrac{[\text{H}_1+]}{[\text{H}_2+]}$

$6.1 - \text{pH}_1 = \log 500$

$-\text{pH}_1 = \log 500 - 6.1$

$\text{pH}_1 = -\log 500 + 6.1$

$\text{pH}_1 = 3.401\ldots$

The pH of the hair conditioner is approximately 3.4.

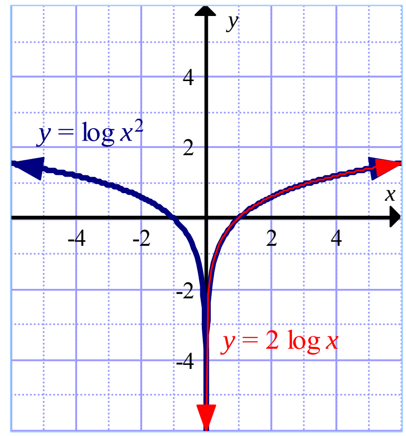
Section 8.3 Page 402 Question 17

Given: $\Delta v = \dfrac{3.1}{0.434}(\log m_0 - \log m_f)$ and $\dfrac{m_0}{m_f} = 1.06$

$$\begin{aligned}\Delta v &= \frac{3.1}{0.434}(\log m_0 - \log m_f) \\ &= \frac{3.1}{0.434}\log \frac{m_0}{m_f} \\ &= \frac{3.1}{0.434}\log 1.06 \\ &\quad 0.180...\end{aligned}$$

The change in velocity of the rocket is 0.18 km/s, to the nearest hundredth of a kilometre per second.

Section 8.3 Page 402 Question 18



a) The graphs are the same for $x > 0$. However, the graph of $y = \log x^2$ has a second branch for $x < 0$, which is the reflection in the y-axis of the branch for $x > 0$.

b) The graphs are not identical because the domains are different. For $y = \log x^2$, the domain is $\{x \mid x \neq 0, x \in \mathrm{R}\}$. The domain for $y = 2\log x$ is $\{x \mid x > 0, x \in \mathrm{R}\}$.

c) For $\log x^2 = 2\log x$, the restriction $x > 0$ is required.

Section 8.3 Page 402 Question 19

a)

$$\begin{aligned} y &= \log_c x \\ c^y &= x \\ \log_d c^y &= \log_d x \\ y\log_d c &= \log_d x \\ y &= \frac{\log_d x}{\log_d c} \end{aligned}$$

b) Use the change in base formula: $\log_c x = \dfrac{\log x}{\log c}$.

$$\begin{aligned} \log_2 9.5 &= \frac{\log 9.5}{\log 2} \\ &\approx 3.2479 \end{aligned}$$

c)

$$\begin{aligned} \varphi &= -\log_2 D \\ &= -\frac{\log D}{\log 2} \end{aligned}$$

d)

Use the formula $\varphi = -\log_2 D$. Let the φ-values be $\varphi_1 = -\log_2 D_1$ and $\varphi_2 = -\log_2 D_2$. Compare the two values. $\varphi_2 - \varphi_1 = -\log_2 D_2 - (-\log_2 D_1)$ $\varphi_2 - \varphi_1 = \log_2 D_2^{-1} - \log_2 D_1^{-1}$ $\varphi_2 - \varphi_1 = \log_2 \dfrac{D_1}{D_2}$ Substitute $\varphi_2 = 2$ and $\varphi_1 = -5.7$. $2 - (-5.7) = \log_2 \dfrac{D_1}{D_2}$ $7.7 = \log_2 \dfrac{D_1}{D_2}$ $2^{7.7} = \dfrac{D_1}{D_2}$ $207.936\ldots = \dfrac{D_1}{D_2}$	Use the formula $\varphi = -\dfrac{\log D}{\log 2}$. Let the φ-values be $\varphi_1 = -\dfrac{\log D_1}{\log 2}$ and $\varphi_2 = -\dfrac{\log D_2}{\log 2}$. Compare the two values. $\varphi_2 - \varphi_1 = -\dfrac{\log D_2}{\log 2} - \left(-\dfrac{\log D_1}{\log 2}\right)$ $\varphi_2 - \varphi_1 = -\dfrac{1}{\log 2}(\log D_2 - \log D_1)$ $\varphi_2 - \varphi_1 = -\dfrac{1}{\log 2}\log \dfrac{D_2}{D_1}$ Substitute $\varphi_2 = -5.7$ and $\varphi_1 = 2$. $-5.7 - 2 = -\dfrac{1}{\log 2}\log \dfrac{D_2}{D_1}$

	$7.7 \log 2 = \log \frac{D_2}{D_1}$ $10^{7.7 \log 2} = \frac{D_2}{D_1}$ $207.936\ldots = \frac{D_2}{D_1}$

Using either form of the formula, the diameter of the pebble is approximately 207.9 times that of the medium sand.

Section 8.3 Page 402 Question 20

a) Left Side

$$= \log_{q^3} p^3$$
$$= \frac{\log_q p^3}{\log_q q^3}$$
$$= \frac{3\log_q p}{3\log_q q}$$
$$= \frac{\log_q p}{1}$$
= Right Side

b) Left Side

$$= \frac{1}{\log_p 2} - \frac{1}{\log_q 2}$$
$$= \frac{1}{\frac{\log_2 2}{\log_2 p}} - \frac{1}{\frac{\log_2 2}{\log_2 q}}$$
$$= \frac{\log_2 p}{1} - \frac{\log_2 q}{1}$$
$$= \log_2 \frac{p}{q}$$
= Right Side

c) Left Side

$$= \frac{1}{\log_q p} + \frac{1}{\log_q p}$$
$$= \frac{1}{\frac{\log_{q^2} p}{\log_{q^2} q}} + \frac{1}{\frac{\log_{q^2} p}{\log_{q^2} q}}$$
$$= \frac{\log_{q^2} q}{\log_{q^2} p} + \frac{\log_{q^2} q}{\log_{q^2} p}$$
$$= \frac{1}{\log_{q^2} p}(\log_{q^2} q + \log_{q^2} q)$$
$$= \frac{1}{\log_{q^2} p}\log_{q^2} q^2$$
$$= \frac{1}{\log_{q^2} p}$$
= Right Side

d) Left Side

$$= \log_{\frac{1}{q}} p$$
$$= \frac{\log_q p}{\log_q \frac{1}{q}}$$
$$= \frac{\log_q p}{\log_q 1 - \log_q q}$$
$$= \frac{\log_q p}{0-1}$$
$$= -\log_q p$$
$$= \log_q p^{-1}$$
$$= \log_q \frac{1}{p}$$
= Right Side

Section 8.3 Page 403 Question C1

a) The function $y = \log x^3$ can be written as $y = 3 \log x$. You need to apply a vertical stretch about the x-axis by a factor of 3 to the graph of $y = \log x$ to result in the graph of $y = \log x^3$.

b) The function $y = \log (x + 2)^5$ can be written as $y = 5 \log (x + 2)$. You need to apply a vertical stretch about the x-axis by a factor of 5 and a translation of 2 units to the left to the graph of $y = \log x$ to result in the graph of $y = \log (x + 2)^5$.

c) The function $y = \log \frac{1}{x}$ can be written as $y = –\log x$. You need to apply a reflection in the x-axis to the graph of $y = \log x$ to result in the graph of $y = \log \frac{1}{x}$.

d) The function $y = \log \frac{1}{\sqrt{x-6}}$ can be written as $y = -\frac{1}{2} \log (x – 6)$. You need to apply a vertical stretch about the x-axis by a factor of $\frac{1}{2}$, a reflection in the x-axis, and a translation of 6 units to the right to the graph of $y = \log x$ to result in the graph of $y = \log \frac{1}{\sqrt{x-6}}$.

Section 8.3 Page 403 Question C2

$$\begin{aligned}
\log_2\left(\sin\frac{\pi}{4}\right)+\log_2\left(\sin\frac{3\pi}{4}\right) &= \log_2\frac{\sqrt{2}}{2}+\log_2\frac{\sqrt{2}}{2} \\
&= 2(\log_2\sqrt{2}-\log_2 2) \\
&= 2\left(\frac{1}{2}\log_2 2-1\right) \\
&= 2\left(\frac{1}{2}-1\right) \\
&= -1
\end{aligned}$$

Section 8.3 Page 403 Question C3

a) $$\begin{aligned} d &= \log 4 - \log 2 \\ &= \log \frac{4}{2} \\ &= \log 2 \end{aligned}$$

The common difference in the arithmetic series $\log 2 + \log 4 + \log 8 + \log 16 + \log 32$ is $\log 2$.

b) Use the formula for the sum of an arithmetic series: $S_n = \frac{n}{2}(t_1 + t_n)$. Substitute $n = 5$, $t_1 = \log 2$, and $t_n = \log 32$.

$$S_n = \frac{n}{2}(t_1 + t_n)$$
$$S_5 = \frac{5}{2}(\log 2 + \log 32)$$
$$S_5 = \frac{5}{2}(\log 2 + \log 2^5)$$
$$S_5 = \frac{5}{2}(\log 2 + 5\log 2)$$
$$S_5 = \frac{5}{2}(6\log 2)$$
$$S_5 = 15\log 2$$

Section 8.3 Page 403 Question C4

Example:

	Product Law	**Quotient Law**	**Power Law**
Algebraic Representation	$\log_c MN = \log_c M + \log_c N$	$\log_c \frac{M}{N} = \log_c M - \log_c N$	$\log_c M^P = P \log_c M$
Written Description	The logarithm of a product of numbers is the sum of the logarithms of the numbers.	The logarithm of a quotient of numbers is the difference of the logarithms of the dividend and divisor.	The logarithm of a power of a number is the exponent times the logarithm of the number.
Example	$\log_2 5x = \log_2 5 + \log_2 x$	$\log \frac{x}{5} = \log x - \log 5$	$\log_3 x^2 = 2 \log_3 x$
Common Error	$\log_2 5 + \log_2 x \neq \log_2 (5 + x)$	$\log x - \log 5 \neq \log_2 (x - 5)$	$2 \log_3 x \neq \log_3 2x$

Section 8.4 Logarithmic and Exponential Equations

Section 8.4 Page 412 Question 1

a) $15 = 12 + \log x$
$3 = \log x$
$10^3 = x$
$1000 = x$

b) $\log_5 (2x - 3) = 2$
$5^2 = 2x - 3$
$28 = 2x$
$x = 14$

c) $4 \log_3 x = \log_3 81$
$\log_3 x^4 = \log_3 81$
$x^4 = 81$
$x^4 = 3^4$
$x = 3$

d) $2 = \log (x - 8)$
$x - 8 = 10^2$
$x = 108$

Section 8.4 Page 412 Question 2

a)
$$\begin{aligned} 4(7^x) &= 92 \\ 7^x &= 23 \\ \log 7^x &= \log 23 \\ x \log 7 &= \log 23 \\ x &= \frac{\log 23}{\log 7} \\ x &\approx 1.61 \end{aligned}$$

b)
$$\begin{aligned} 2^{\frac{x}{3}} &= 11 \\ \log 2^{\frac{x}{3}} &= \log 11 \\ \frac{x}{3}\log 2 &= \log 11 \\ x &= \frac{3\log 11}{\log 2} \\ x &\approx 10.38 \end{aligned}$$

c)
$$\begin{aligned} 6^{x-1} &= 271 \\ \log 6^{x-1} &= \log 271 \\ (x-1)\log 6 &= \log 271 \\ x &= \frac{\log 271}{\log 6} + 1 \\ x &\approx 4.13 \end{aligned}$$

d)
$$\begin{aligned} 4^{2x+1} &= 54 \\ \log 4^{2x+1} &= \log 54 \\ (2x+1)\log 4 &= \log 54 \\ x &= \frac{1}{2}\left(\frac{\log 54}{\log 4} - 1\right) \\ x &\approx 0.94 \end{aligned}$$

Section 8.4 Page 412 Question 3

I disagree with Hamdi's check. Neither $\log_3 (x - 8)$ nor $\log_3 (x - 6)$ are defined for $x = 5$.

Section 8.4 Page 413 Question 4

a) The equation $\log_7 x + \log_7 (x - 1) = \log_7 4x$ is defined for $x > 1$. So, the possible root $x = 0$ is extraneous.

b) The equation $\log_6 (x^2 - 24) - \log_6 x = \log_6 5$ is defined for $x > \sqrt{24}$, or approximately $x > 4.9$. So, both possible roots, $x = 3$ and $x = -8$, are extraneous.

c) The equation $\log_3 (x + 3) - \log_3 (x + 5) = 1$ is defined for $x > -3$. So, the possible root $x = -6$ is extraneous.

d) The equation $\log_2 (x - 2) = 2 - \log_2 (x - 5)$ is defined for $x > 5$. So, the possible root $x = 1$ is extraneous.

Section 8.4 Page 413 Question 5

a) $2 \log_3 x = \log_3 32 + \log_3 2$

$\log_3 x^2 = \log_3 64$

$x^2 = 64$

$x = 8$

b) $\frac{3}{2}\log_7 x = \log_7 125$

$\log_7 x^{\frac{3}{2}} = \log_7 125$

$x^{\frac{3}{2}} = 125$

$x = 125^{\frac{2}{3}}$

$x = 25$

c) $\log_2 x - \log_2 3 = 5$

$\log_2 \frac{x}{3} = 5$

$\frac{x}{3} = 2^5$

$x = 32(3)$

$x = 96$

d) $\log_6 x = 2 - \log_6 4$

$\log_6 x + \log_6 4 = 2$

$\log_6 4x = 2$

$6^2 = 4x$

$\frac{36}{4} = x$

$9 = x$

Section 8.4 Page 413 Question 6

a) Rubina subtracted the contents of the logarithmic expressions on the left side of the equation when she should have divided them.
Correct solution:

$\log_6 (2x + 1) - \log_6 (x - 1) = \log_6 5$

$\log_6 \frac{2x+1}{x-1} = \log_6 5$

$\frac{2x+1}{x-1} = 5$

$2x+1 = 5(x-1)$

$2x+1 = 5x-5$

$-3x = -6x$

$x = 2$

b) Ahmed's work is correct. However, he incorrectly concluded that there was no solution. The equation $2 \log_5 (x + 3) = \log_5 9$ is defined for $x > -3$. So, the solution is $x = 0$.

c) Jennifer incorrectly eliminated the logarithmic expression in the third line. The right side should have been 2^3, not 3.
Correct solution:

$$\begin{aligned}
\log_2 x + \log_2 (x + 2) &= 3\\
\log_2 (x(x + 2)) &= 3\\
\log_2 (x^2 + 2x) &= 3\\
x^2 + 2x &= 2^3\\
x^2 + 2x - 8 &= 0\\
(x + 4)(x - 2) &= 0
\end{aligned}$$

$x = -4$ or $x = 2$

The solution is $x = 2$, since $x > 0$.

Section 8.4 Page 413 Question 7

a)

$$\begin{aligned}
7^{2x} &= 2^{x+3}\\
\log 7^{2x} &= \log 2^{x+3}\\
2x \log 7 &= (x + 3) \log 2\\
2x \log 7 &= x \log 2 + 3 \log 2\\
2x \log 7 - x \log 2 &= 3 \log 2\\
x(2 \log 7 - \log 2) &= 3 \log 2\\
x &= \frac{3\log 2}{2\log 7 - \log 2}\\
x &\approx 0.65
\end{aligned}$$

b)

$$\begin{aligned}
1.6^{x-4} &= 5^{3x}\\
\log 1.6^{x-4} &= \log 5^{3x}\\
(x - 4) \log 1.6 &= 3x \log 5\\
x \log 1.6 - 4 \log 1.6 &= 3x \log 5\\
x \log 1.6 - 3x \log 5 &= 4 \log 1.6\\
x(\log 1.6 - 3 \log 5) &= 4 \log 1.6\\
x &= \frac{4\log 1.6}{\log 1.6 - 3\log 5}\\
x &\approx -0.43
\end{aligned}$$

c)

$$\begin{aligned}
9^{2x-1} &= 71^{x+2}\\
\log 9^{2x-1} &= \log 71^{x+2}\\
(2x - 1) \log 9 &= (x + 2) \log 71\\
2x \log 9 - \log 9 &= x \log 71 + 2 \log 71\\
2x \log 9 - x \log 71 &= 2 \log 71 + \log 9\\
x(2 \log 9 - \log 71) &= 2 \log 71 + \log 9\\
x &= \frac{2\log 71 + \log 9}{2\log 9 - \log 71}\\
x &\approx 81.37
\end{aligned}$$

d)

$$\begin{aligned}
4(7^{x+2}) &= 9^{2x-3}\\
\log 4(7^{x+2}) &= \log 9^{2x-3}\\
\log 4 + \log 7^{x+2} &= \log 9^{2x-3}\\
\log 4 + (x + 2) \log 7 &= (2x - 3) \log 9\\
\log 4 + x \log 7 + 2 \log 7 &= 2x \log 9 - 3 \log 9\\
x \log 7 - 2x \log 9 &= -3 \log 9 - \log 4 - 2 \log 7\\
x(\log 7 - 2 \log 9) &= -3 \log 9 - \log 4 - 2 \log 7\\
x &= \frac{-3\log 9 - \log 4 - 2\log 7}{\log 7 - 2\log 9}\\
x &\approx 4.85
\end{aligned}$$

Section 8.4 Page 413 Question 8

a) $\log_5(x-18)-\log_5 x=\log_5 7$

$$\log_5\frac{x-18}{x}=\log_5 7$$
$$\frac{x-18}{x}=7$$
$$x-18=7x$$
$$-6x=18$$
$$x=-3$$

Since the equation is defined for $x > 18$, there is no solution.

b) $\log_2(x-6)+\log_2(x-8)=3$

$$\log_2((x-6)(x-8))=3$$
$$(x-6)(x-8)=2^3$$
$$x^2-14x+48=8$$
$$x^2-14x+40=0$$
$$(x-10)(x-4)=0$$

$x = 10$ or $x = 4$

Since the equation is defined for $x > 8$, the solution is $x = 10$.

c)

$$2\log_4(x+4)-\log_4(x+12)=1$$
$$\log_4(x+4)^2-\log_4(x+12)=1$$
$$\log_4\frac{(x+4)^2}{x+12}=1$$
$$\frac{(x+4)^2}{x+12}=4^1$$
$$(x+4)^2=4(x+12)$$
$$x^2+8x+16=4x+48$$
$$x^2+4x-32=0$$
$$(x+8)(x-4)=0$$

$x = –8$ or $x = 4$

Since the equation is defined for $x > –4$, the solution is $x = 4$.

d) $$\log_3(2x-1) = 2 - \log_3(x+1)$$
$$\log_3(2x-1) + \log_3(x+1) = 2$$
$$\log_3((2x-1)(x+1)) = 2$$
$$(2x-1)(x+1) = 3^2$$
$$2x^2 + x - 1 = 9$$
$$2x^2 + x - 10 = 0$$
$$(2x+5)(x-2) = 0$$
$x = -\dfrac{5}{2}$ or $x = 2$

Since the equation is defined for $x > \dfrac{1}{2}$, the solution is $x = 2$.

e) $$\log_2 \sqrt{x^2 + 4x} = \frac{5}{2}$$
$$\frac{1}{2}\log_2(x^2 + 4x) = \frac{5}{2}$$
$$\log_2(x^2 + 4x) = 5$$
$$x^2 + 4x = 2^5$$
$$x^2 + 4x - 32 = 0$$
$$(x+8)(x-4) = 0$$
$x = –8$ or $x = 4$

Since the equation is defined for $x < –4$ or $x > 0$, the solutions are $x = –8$ and $x = 4$.

Section 8.4 Page 413 Question 9

a) Substitute $m = –1.44$ and $M = 1.45$ into $m – M = 5 \log d – 5$.
$$m - M = 5 \log d - 5$$
$$-1.44 - 1.45 = 5 \log d - 5$$
$$-2.89 = 5 \log d - 5$$
$$2.11 = 5 \log d$$
$$\frac{2.11}{5} = \log d$$
$$d = 10^{\frac{2.11}{5}}$$
$$d = 2.642\ldots$$
Sirius is approximately 2.64 parsecs from Earth.

b) The distance 2.64 pc is equivalent to 2.64(3.26), or about 8.61 light years.

Section 8.4 Page 413 Question 10

Substitute $E = 24$ into $\log E = \log 10.61 + 0.1964 \log m$.

$\log E = \log 10.61 + 0.1964 \log m$
$\log 24 = \log 10.61 + 0.1964 \log m$
$\log 24 - \log 10.61 = 0.1964 \log m$

$$\log \frac{24}{10.61} = 0.1964 \log m$$
$$\frac{1}{0.1964} \log \frac{24}{10.61} = \log m$$
$$m = 10^{\frac{1}{0.1964} \log \frac{24}{10.61}}$$
$$m = 63.821...$$

The mass of the mountain goat is 64 kg, to the nearest kilogram.

Section 8.4 Page 414 Question 11

a) Substitute $t = 0$.

$P = 10\,000(1.035)^t$
$= 10\,000(1.035)^0$
$= 10\,000$

When the lake was stocked, 10 000 northern pike were put in the lake.

b) Since the base is 1.035, or 1 + 0.035, the annual growth rate as a percent is 3.5%.

c) Substitute $P = 20\,000$.

$P = 10\,000(1.035)^t$
$20\,000 = 10\,000(1.035)^t$
$2 = (1.035)^t$
$\log 2 = \log (1.035)^t$
$\log 2 = t \log 1.035$

$$\frac{\log 2}{\log 1.035} = t$$

$20.148\ldots = t$

It will take approximately 20.1 years for the number of northern pike in the lake to double.

Section 8.4 Page 414 Question 12

a) Substitute $d = 5906$ into $\log T = \frac{3}{2}\log d - 3.263$.

$\log T = \frac{3}{2}\log d - 3.263$

$\log T = \frac{3}{2}\log 5906 - 3.263$

$T = 10^{\frac{3}{2}\log 5906 - 3.263}$

$T = 247.708\ldots$

To the nearest Earth year, it takes Pluto 248 Earth years to revolve around the sun.

b) Substitute $T = 1.88$ into $\log T = \frac{3}{2}\log d - 3.263$.

$\log T = \frac{3}{2}\log d - 3.263$

$\log 1.88 = \frac{3}{2}\log d - 3.263$

$\log 1.88 + 3.263 = \frac{3}{2}\log d$

$\frac{2}{3}(\log 1.88 + 3.263) = \log d$

$d = 10^{\frac{2}{3}(\log 1.88 + 3.263)}$

$d = 228.089\ldots$

Mars is 228 million kilometres from the sun, to the nearest million kilometres.

Section 8.4 Page 414 Question 13

a) Substitute $P = 10\ 000$, $i = \frac{0.06}{2}$, or 0.03, and $A = 11\ 000$.

$A = P(1 + i)^n$

$11\ 000 = 10\ 000(1 + 0.03)^n$

$1.1 = 1.03^n$

$\log 1.1 = \log 1.03^n$

$\log 1.1 = n \log 1.03$

$\frac{\log 1.1}{\log 1.03} = n$

$3.224\ldots = n$

Since $n = 3$ results in \$10 927.27, and interest is compounded at the end of each 6 months, it will take 2 years for the GIC to be worth \$11 000.

b) Substitute $P = 1200$, $i = \dfrac{0.28}{365}$, and $A = 1241.18$.

$$A = P(1 + i)^n$$
$$1241.18 = 1200\left(1 + \frac{0.28}{365}\right)^n$$
$$\frac{1241.18}{1200} = \left(1 + \frac{0.28}{365}\right)^n$$
$$\log\frac{1241.18}{1200} = \log\left(1 + \frac{0.28}{365}\right)^n$$
$$\log\frac{1241.18}{1200} = n\log\left(1 + \frac{0.28}{365}\right)$$
$$n = \frac{\log\dfrac{1241.18}{1200}}{\log\left(1 + \dfrac{0.28}{365}\right)}$$
$$n = 44.000...$$

Linda's payment is 44 days overdue.

c) Substitute $A = 3P$ and $i = \dfrac{0.055}{2}$, or 0.0275.

$$A = P(1 + i)^n$$
$$3P = P(1 + 0.0275)^n$$
$$3 = 1.0275^n$$
$$\log 3 = \log 1.0275^n$$
$$\log 3 = n \log 1.0275$$
$$\frac{\log 3}{\log 1.0275} = n$$
$$40.496\ldots = n$$

Since $n = 40$ results in $A = \$2.96$ for $P = \$1$, and compound interest is added at the end of each 6 months, it will take $41 \div 2$, or 20.5 years for the money to triple in value.

Section 8.4 Page 414 Question 14

Substitute $PV = 250\ 000$, $i = \dfrac{0.074}{2}$, or 0.037, and $R = 10\ 429.01$.

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

$$250\,000 = \frac{10\,429.01\,[1-(1+0.037)^{-n}]}{0.037}$$

$$\frac{9250}{10\,429.01} = 1 - 1.037^{-n}$$

$$\frac{9250}{10\,429.01} - 1 = -1.037^{-n}$$

$$\frac{1179.01}{10\,429.01} = 1.037^{-n}$$

$$\log \frac{1179.01}{10\,429.01} = \log 1.037^{-n}$$

$$\log \frac{1179.01}{10\,429.01} = -n \log 1.037$$

$$n = -\frac{\log \frac{1179.01}{10\,429.01}}{\log 1.037}$$

$$n = 60.000...$$

The mortgage will be completely paid off after 60 ÷ 2, or 30 years.

Section 8.4 Page 414 Question 15

Substitute $m(t) = 0.315m_0$ into $m(t) = m_0\left(\frac{1}{2}\right)^{\frac{t}{5730}}$.

$$m(t) = m_0\left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

$$0.315m_0 = m_0\left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

$$0.315 = 0.5^{\frac{t}{5730}}$$

$$\log 0.315 = \log 0.5^{\frac{t}{5730}}$$

$$\log 0.315 = \frac{t}{5730} \log 0.5$$

$$t = \frac{5730 \log 0.315}{\log 0.5}$$

$$t = 9549.482...$$

The tree was almost 9550 years old when it was discovered.

Section 8.4 Page 415 Question 16

Substitute $m(t) = 274$, $m_0 = 280$, and $t = 6$ into $m(t) = m_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$, where $m(t)$ and m_0 are measured in megabecquerels, t is time, in hours, and h is the half-life of I-131, in hours.

$$m(t) = m_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$274 = 280\left(\frac{1}{2}\right)^{\frac{6}{h}}$$

$$\frac{274}{280} = 0.5^{\frac{6}{h}}$$

$$\log\frac{274}{280} = \log 0.5^{\frac{6}{h}}$$

$$\log\frac{274}{280} = \frac{6}{h}\log 0.5$$

$$h = \frac{6\log 0.5}{\log\frac{274}{280}}$$

$$h = 191.994...$$

The half-life of I-131 is 192 ÷ 24, or 8 days, to the nearest day.

Section 8.4 Page 415 Question 17

Let the light intensity, $l(d)$, below the water's surface be represented by $l(d) = l_0(0.96)^d$, where l_0 is the intensity at the surface and d is the depth, in metres.

Substitute $l(d) = 0.25l_0$.

$$l(d) = l_0(0.96)^d$$

$$0.25l_0 = l_0(0.96)^d$$

$$0.25 = 0.96^d$$

$$\log 0.25 = \log 0.96^d$$

$$\log 0.25 = d \log 0.96$$

$$d = \frac{\log 0.25}{\log 0.96}$$

$$d = 33.959\ldots$$

To the nearest tenth of a metre, at 34.0 m the light intensity is 25% of the intensity at the surface.

Section 8.4 Page 415 Question 18

Solve the system of equations, $\log_3 81 = x - y$ and $\log_2 32 = x + y$, by elimination.

$\log_3 81 = x - y$
$\underline{\log_2 32 = x + y}$
$\log_3 81 + \log_2 32 = 2x$

$$\begin{aligned} \log_3 3^4 + \log_2 2^5 &= 2x \\ 4 + 5 &= 2x \\ x &= \frac{9}{2} \end{aligned}$$

Substitute $x = \frac{9}{2}$ into $\log_3 81 = x - y$.

$$\begin{aligned} \log_3 81 &= x - y \\ \log_3 81 &= \frac{9}{2} - y \\ 4 &= \frac{9}{2} - y \\ y &= \frac{1}{2} \end{aligned}$$

Section 8.4 Page 415 Question 19

a) The first line, $\log 0.1 < 3 \log 0.1$, is not true.

b) Since $\log x < 0$, for $0 < x < 1$, the inequality symbol in the last line should be reversed. In other words, from line 4 to line 5 you are dividing by a negative quantity.

Section 8.4 Page 415 Question 20

a)
$$\begin{aligned} x^{\frac{2}{\log x}} &= x \\ \frac{2}{\log x} &= 1 \\ 2 &= \log x \\ x &= 10^2 \\ x &= 100 \end{aligned}$$

b)
$$\begin{aligned} \log x^{\log x} &= 4 \\ \log x(\log x) &= 4 \\ (\log x)^2 &= 4 \\ \log x &= \pm 2 \end{aligned}$$

$x = 10^{-2}$ or $x = 10^2$

$= \frac{1}{100}$ $\quad = 100$

c)
$$\begin{aligned} (\log x)^2 &= \log x^2 \\ (\log x)^2 &= 2 \log x \\ (\log x)^2 - 2 \log x &= 0 \\ \log x(\log x - 2) &= 0 \end{aligned}$$

$\log x = 0$ or $\log x - 2 = 0$

$x = 1$ $\quad \log x = 2$

$\quad x = 100$

Section 8.4 Page 415 Question 21

a)

$$\log_4 x + \log_2 x = 6$$
$$\frac{\log_2 x}{\log_2 4} + \log_2 x = 6$$
$$\frac{\log_2 x}{2} + \log_2 x = 6$$
$$\frac{3}{2}\log_2 x = 6$$
$$\log_2 x = 4$$
$$x = 2^4$$
$$x = 16$$

b)

$$\log_3 x - \log_{27} x = \frac{4}{3}$$
$$\log_3 x - \frac{\log_3 x}{\log_3 27} = \frac{4}{3}$$
$$\log_3 x - \frac{\log_3 x}{3} = \frac{4}{3}$$
$$\frac{2}{3}\log_3 x = \frac{4}{3}$$
$$\log_3 x = 2$$
$$x = 3^2$$
$$x = 9$$

Section 8.4 Page 415 Question 22

$$(x^2 + 3x - 9)^{2x-8} = 1$$
$$\log (x^2 + 3x - 9)^{2x-8} = \log 1$$
$$(2x - 8)\log (x^2 + 3x - 9) = 0$$

$2x - 8 = 0$ or $\log (x^2 + 3x - 9) = 0$

$2x = 8$ $\quad$ $x^2 + 3x - 9 = 1$

$x = 4$ $\quad$ $x^2 + 3x - 10 = 0$

$(x + 5)(x - 2) = 0$

$x = -5$ or $x = 2$

Section 8.4 Page 415 Question C1

a)

$$8(2^x) = 512$$
$$\log 8(2^x) = \log 512$$
$$\log 8 + \log 2^x = \log 512$$
$$\log 8 + x \log 2 = \log 512$$
$$x \log 2 = \log 512 - \log 8$$
$$x = \frac{\log 64}{\log 2}$$
$$x = 6$$

b) Example: Fatima could have divided both sides of the equation by 8 to avoid taking the logarithm of each side.

$8(2^x) = 512$

$2^x = 64$

$2^x = 2^6$

$x = 6$

c) Example: I prefer the approach in part b). It is much shorter.

Section 8.4 Page 415 Question C2

For the sequence 4, 12, 36, …, 708 588, $t_1 = 4$ and $r = 3$.
Substitute $t_n = 708\ 588$, $t_1 = 4$, and $r = 3$ into $t_n = t_1r^{n-1}$.

$$\begin{aligned}
t_n &= t_1r^{n-1} \\
708\ 588 &= 4(3)^{n-1} \\
177\ 147 &= 3^{n-1} \\
\log 177\ 147 &= \log 3^{n-1} \\
\log 177\ 147 &= (n-1)\log 3 \\
\frac{\log 177\ 147}{\log 3} &= n-1 \\
\frac{\log 177\ 147}{\log 3} + 1 &= n \\
12 &= n
\end{aligned}$$

Section 8.4 Page 415 Question C3

For the series 8192 + 4096 + 2048 + …, $t_1 = 8192$ and $r = 0.5$.

Substitute $S_n = 16\ 383$, $t_1 = 8192$, and $r = 0.5$ into $S_n = \dfrac{t_1(r^n - 1)}{r - 1}$.

$$\begin{aligned}
S_n &= \frac{t_1(r^n - 1)}{r - 1} \\
16\ 383 &= \frac{8192(0.5^n - 1)}{0.5 - 1} \\
-\frac{8191.5}{8192} &= 0.5^n - 1 \\
1 - \frac{8191.5}{8192} &= 0.5^n \\
\frac{0.5}{8192} &= 0.5^n \\
\log\frac{0.5}{8192} &= \log 0.5^n \\
\log\frac{0.5}{8192} &= n\log 0.5 \\
n &= \frac{\log\frac{0.5}{8192}}{\log 0.5} \\
n &= 14
\end{aligned}$$

Section 8.4 Page 415 Question C4

a) $2 \log_2 (\cos x) + 1 = 0$

$$\log_2 (\cos x) = -\frac{1}{2}$$

$$2^{-\frac{1}{2}} = \cos x$$

$$\frac{1}{\sqrt{2}} = \cos x$$

$$x = \frac{\pi}{4} \text{ or } x = \frac{7\pi}{4}$$

b) $\log (\sin x) + \log (2 \sin x - 1) = 0$

$$\log ((\sin x)(2 \sin x - 1)) = 0$$

$$(\sin x)(2 \sin x - 1) = 1$$

$$2 \sin^2 x - \sin x - 1 = 0$$

$$(2 \sin x + 1)(\sin x - 1) = 0$$

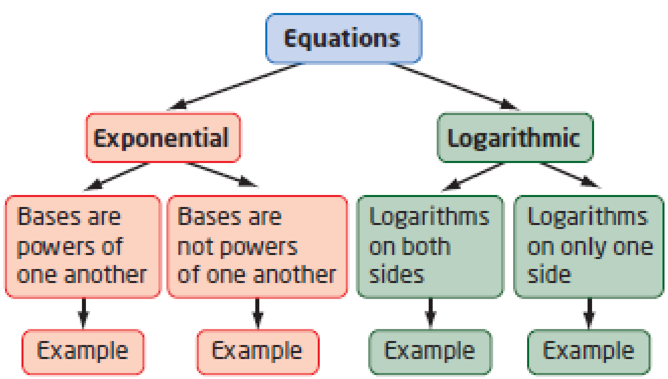
$2 \sin x + 1 = 0$ or $\sin x - 1 = 0$

$\sin x = -\frac{1}{2}$ $\sin x = 1$

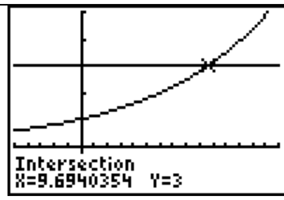
$x = \frac{4\pi}{3}, \frac{5\pi}{3}$ $x = \frac{\pi}{2}$

Since the equation is defined for $\sin x > \frac{1}{2}$, the solution is $x = \frac{\pi}{2}$.

Section 8.4 Page 415 Question C5



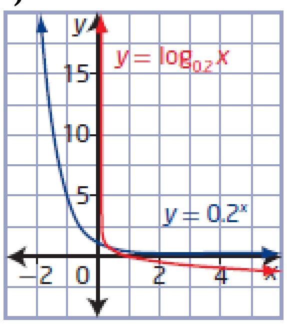
Exponential Equations		Logarithmic Equations	
Example	**Example**	**Example**	**Example**
$3^{5x} = 27^{x-1}$ $3^{5x} = (3^3)^{x-1}$ $3^{5x} = 3^{3x-3}$ Equate the exponents.	$3 = 1.12^x$ Graph $y = 3$ and $y = 1.12^x$ and find the point of intersection.	$4 \log_3 x = \log_3 81$ $\log_3 x^4 = \log_3 81$ $x^4 = 81$ $x^4 = 3^4$ $x = 3$	$\log_5 (2x - 3) = 2$ $5^2 = 2x - 3$ $28 = 2x$ $x = 14$

$5x = 3x - 3$ $2x = -3$ $x = -\frac{3}{2}$	 The solution is $x \approx 9.7$.		

Chapter 8 Review

Chapter 8 Review Page 416 Question 1

a)



b) The graph of $y = \log_{0.2} x$ has the following characteristics
i) domain: $\{x \mid x > 0, x \in \mathrm{R}\}$ and range: $\{y \mid y \in \mathrm{R}\}$
ii) x-intercept: 1
iii) no y-intercept
iv) equation of the asymptote: $x = 0$

c) Since $f(x) = 0.2^x$, the equation of inverse is $f^{-1}(x) = \log_{0.2} x$.

Chapter 8 Review Page 416 Question 2

Use the given point (2, 16) on the graph of the inverse of $y = \log_c x$, or $y = c^x$ to determine the value of c.

$y = c^x$
$16 = c^2$
$4^2 = c^2$
$c = 4$

Chapter 8 Review Page 416 Question 3

Write $a < \log_2 24 < b$ in exponential form: $2^a < 24 < 2^b$.
Since $2^4 = 16$ and $2^5 = 32$, then $a = 4$ and $b = 5$.
So, the value of $\log_2 24$ must be between 4 and 5.

Chapter 8 Review Page 366 Question 4

a) $\log_{125} x = \frac{2}{3}$

$$125^{\frac{2}{3}} = x$$
$$x = 25$$

b) $\log_9 \frac{1}{81} = x$

$$9^x = \frac{1}{81}$$
$$9^x = 9^{-2}$$
$$x = -2$$

c) $\log_3 27\sqrt{3} = x$

$$3^x = 27\sqrt{3}$$
$$3^x = 3^{\frac{7}{2}}$$
$$x = \frac{7}{2}$$

d) $\log_x 8 = \frac{3}{4}$

$$x^{\frac{3}{4}} = 8$$
$$x = 8^{\frac{4}{3}}$$
$$x = 16$$

e) $6^{\log x} = \frac{1}{36}$

$$6^{\log x} = 6^{-2}$$
$$\log x = -2$$
$$10^{-2} = x$$
$$x = \frac{1}{100}$$

Chapter 8 Review Page 416 Question 5

Determine the amplitude of each earthquake.

Japan earthquake:

$$M = \log \frac{A}{A_0}$$
$$9.0 = \log \frac{A}{A_0}$$
$$10^{9.0} = \frac{A}{A_0}$$
$$A = 10^{9.0} A_0$$

Japan aftershock:

$$M = \log \frac{A}{A_0}$$
$$7.4 = \log \frac{A}{A_0}$$
$$10^{7.4} = \frac{A}{A_0}$$
$$A = 10^{7.4} A_0$$

Compare the amplitudes.

$$\frac{10^{9.0}}{10^{7.4}} = 10^{1.6}$$
$$= 39.810\ldots$$

The seismic shaking of the Japan earthquake was approximately 40 times that of the aftershock.

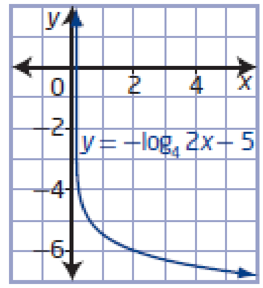
Chapter 8 Review Page 416 Question 6

a) Given: $y = \log_4 x$

• Stretch horizontally about the y-axis by a factor of $\frac{1}{2}$: $b = 2$, $y = \log_4 2x$

• Reflect in the x-axis: $a = -1$, $y = -\log_4 2x$

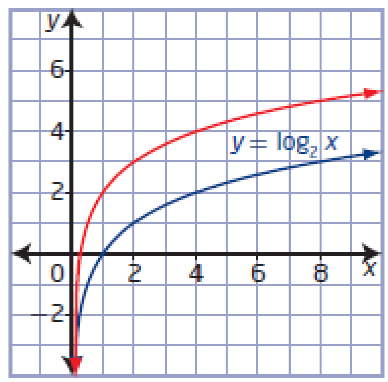
• Translate 5 units down: $k = -5$, $y = -\log_4 2x - 5$



b) The equation of the transformed image in the form $y = a \log_c (b(x - h)) + k$ is $y = -\log_4 2x - 5$. So, $a = -1$, $b = 2$, $c = 4$, $h = 0$, and $k = -5$.

Chapter 8 Review Page 416 Question 7

Choose a key points on the blue graph, say (4, 2) and (8, 3).
The key points (4, 2) and (8, 3) on the graph of $y = \log_2 x$ have become the image points (1, 2) and (2, 3) on the red graph. Thus, the red graph can be generated by horizontally stretching the graph of $y = \log_2 x$ about the y-axis by a factor of $\frac{1}{4}$. The red graph can be described by the equation $y = \log_2 4x$.



Chapter 8 Review Page 417 Question 8

a) For $y = -\log_5 (3(x - 12)) + 2$, $a = -1$, $b = 3$, $h = 12$, and $k = 2$. To obtain the graph of $y = -\log_5 (3(x - 12)) + 2$, the graph of $y = \log_5 x$ must be reflected in the x-axis, horizontally stretched about the y-axis by a factor of $\frac{1}{3}$, and translated 12 units to the right and 2 units up.

b) For $y + 7 = \frac{\log_5(6 - x)}{4}$, or $y = \frac{1}{4} \log_5 (-(x - 6)) - 7$, $a = \frac{1}{4}$, $b = -1$, $h = 6$, and $k = -7$. To obtain the graph of $y + 7 = \frac{\log_5(6 - x)}{4}$, the graph of $y = \log_5 x$ must be vertically stretched about the x-axis by a factor of $\frac{1}{4}$, reflected in the y-axis, and translated 6 units to the right and 7 units down.

Chapter 8 Review Page 417 Question 9

Given: $y = 3 \log_2 (x + 8) + 6$

a) The equation of the vertical asymptote occurs when $x + 8 = 0$. Therefore, the equation of the vertical asymptote is $x = -8$.

b) The domain is $\{x \mid x > -8, x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

c) Substitute $x = 0$. Then, solve for y.

$$\begin{aligned} y &= 3 \log_2 (x + 8) + 6 \\ &= 3 \log_2 (0 + 8) + 6 \\ &= 3 \log_2 8 + 6 \\ &= 3(3) + 6 \\ &= 15 \end{aligned}$$

The y-intercept is 15.

d) Substitute $y = 0$. Then, solve for x.

$$\begin{aligned} y &= 3 \log_2 (x + 8) + 6 \\ 0 &= 3 \log_2 (x + 8) + 6 \\ -6 &= 3 \log_2 (x + 8) \\ -2 &= \log_2 (x + 8) \\ 2^{-2} &= x + 8 \\ \frac{1}{4} &= x + 8 \\ x &= -\frac{31}{4} \end{aligned}$$

The x-intercept is $-\frac{31}{4}$, or -7.75.

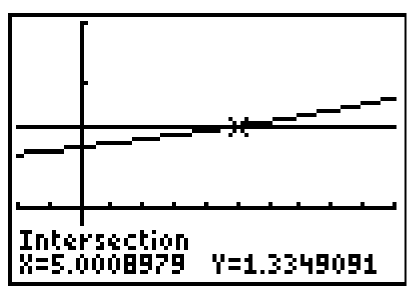
Chapter 8 Review Page 417 Question 10

a) For $n = 12 \log_2 \frac{f}{440}$, $a = 12$ and $b = \frac{1}{440}$. The function is transformed from $n = \log_2 f$ by a horizontal stretch about the y-axis by a factor of 440 and vertically stretched about the x-axis by a factor of 12.

b) Substitute $f = 587.36$.

$$\begin{aligned} n &= 12 \log_2 \frac{f}{440} \\ n &= 12 \log_2 \frac{587.36}{440} \\ \frac{n}{12} &= \log_2 \frac{587.36}{440} \\ 2^{\frac{n}{12}} &= \frac{587.36}{440} \end{aligned}$$

Graph $y = 2^{\frac{x}{12}}$ and $y = \frac{587.36}{440}$ and determine the point of intersection.



The note D is 5 notes above A.

c) Substitute $n = 8$.

$$n = 12\log_2 \frac{f}{440}$$

$$8 = 12\log_2 \frac{f}{440}$$

$$\frac{8}{12} = \log_2 \frac{f}{440}$$

$$\frac{2}{3} = \log_2 \frac{f}{440}$$

$$2^{\frac{2}{3}} = \frac{f}{440}$$

$$f = 440\left(2^{\frac{2}{3}}\right)$$

$$f = 698.456\ldots$$

The frequency of F is 698.46 Hz, to the nearest hundredth of a hertz.

Chapter 8 Review Page 417 Question 11

a) $\log_5\left(\frac{x^5}{y\sqrt[3]{z}}\right) = \log_5 x^5 - \log_5 y\sqrt[3]{z}$

$$= 5\log_5 x - (\log_5 y + \log_5 \sqrt[3]{z})$$

$$= 5\log_5 x - \log_5 y - \frac{1}{3}\log_5 z$$

b) $\log\sqrt{\frac{xy^2}{z}} = \frac{1}{2}\log\frac{xy^2}{z}$

$$= \frac{1}{2}\left(\log xy^2 - \log z\right)$$

$$= \frac{1}{2}\left(\log x + \log y^2 - \log z\right)$$

$$= \frac{1}{2}\left(\log x + 2\log y - \log z\right)$$

Chapter 8 Review Page 417 Question 12

a) $\log x - 3\log y + \frac{2}{3}\log z$

$= \log x - \log y^3 + \log \sqrt[3]{z^2}$

$= \log \frac{x\sqrt[3]{z^2}}{y^3}$

b) $\log x - \frac{1}{2}(\log y + 3\log z)$

$= \log x - \frac{1}{2}(\log y + \log z^3)$

$= \log x - \frac{1}{2}\log yz^3$

$= \log x - \log \sqrt{yz^3}$

$= \log \frac{x}{\sqrt{yz^3}}$

Chapter 8 Review Page 417 Question 13

a) $2\log x + 3\log \sqrt{x} - \log x^3 = \log x^2 + \log \sqrt{x^3} - \log x^3$

$= \log \frac{x^2\sqrt{x^3}}{x^3}$

$= \log \frac{x^3\sqrt{x}}{x^3}$

$= \log \sqrt{x}$

$= \frac{1}{2}\log x,\ x > 0$

b) $\log(x^2 - 25) - 2\log(x+5) = \log(x^2 - 25) - \log(x+5)^2$

$= \log \frac{x^2 - 25}{(x+5)^2}$

$= \log \frac{(x-5)(x+5)}{(x+5)(x+5)}$

$= \log \frac{x-5}{x+5},\ x < -5 \text{ or } x > 5$

Chapter 8 Review Page 417 Question 14

a) $\log_6 18 - \log_6 2 + \log_6 4 = \log_6 \frac{18(4)}{2}$

$= \log_6 36$

$= 2$

b) $\log_4 \sqrt{12} + \log_4 \sqrt{9} - \log_4 \sqrt{27} = \log_4 \dfrac{\sqrt{12}\sqrt{9}}{\sqrt{27}}$

$$= \log_4 \frac{2\sqrt{3}(3)}{3\sqrt{3}}$$
$$= \log_4 2$$
$$= 0.5$$

Chapter 8 Review Page 417 Question 15

Let the pH levels of two berries be $pH_1 = -\log [H_1+]$ and $pH_2 = -\log [H_2+]$.
Compare the two pH levels.
$pH_2 - pH_1 = -\log [H_2+] - (-\log [H_1+])$
$pH_2 - pH_1 = \log [H_2+]^{-1} - \log [H_1+]^{-1}$
$pH_2 - pH_1 = \log \dfrac{[H_1+]}{[H_2+]}$
Substitute $pH_2 = 4.0$ and $pH_1 = 3.2$.

$$4.0 - 3.2 = \log \frac{[H_1+]}{[H_2+]}$$
$$0.8 = \log \frac{[H_1+]}{[H_2+]}$$
$$10^{0.8} = \frac{[H_1+]}{[H_2+]}$$
$$6.309\ldots = \frac{[H_1+]}{[H_2+]}$$

The blueberry is approximately 6.3 times more acidic than the Saskatoon berry.

Chapter 8 Review Page 417 Question 16

Substitute $m_2 = -26.74$ and $m_1 = -12.74$ in $m_2 - m_1 = -2.5\log\left(\dfrac{F_2}{F_1}\right)$.

$$m_2 - m_1 = -2.5\log\left(\frac{F_2}{F_1}\right)$$
$$-26.74 - (-12.74) = -2.5\log\left(\frac{F_2}{F_1}\right)$$
$$-14 = -2.5\log\left(\frac{F_2}{F_1}\right)$$
$$5.6 = \log\left(\frac{F_2}{F_1}\right)$$

$$10^{5.6} = \frac{F_2}{F_1}$$

$$398\ 107.170\ldots = \frac{I_2}{I_1}$$

The sun appears to be approximately 398 107 times brighter than the moon.

Chapter 8 Review   Page 418   Question 17

Substitute $\frac{I_2}{I_1} = 20$ and $\beta_1 = 80$ into $\beta_2 - \beta_1 = 10\left(\log\frac{I_2}{I_1}\right)$.

$\beta_2 - \beta_1 = 10\left(\log\frac{I_2}{I_1}\right)$

$\beta_2 - 80 = 10 \log 20$

$\beta_2 = 10 \log 20 + 80$

$\beta_2 = 93.010\ldots$

The decibel level at which the police can issue a fine to a motorcycle operator is approximately 93 dB.

Chapter 8 Review   Page 418   Question 18

a)

$$\begin{aligned}
3^{2x+1} &= 75\\
\log 3^{2x+1} &= \log 75\\
(2x+1)\log 3 &= \log 75\\
2x\log 3 + \log 3 &= \log 75\\
2x\log 3 &= \log 75 - \log 3\\
x &= \frac{\log 25}{2\log 3}\\
x &\approx 1.46
\end{aligned}$$

b)

$$\begin{aligned}
7^{x+1} &= 4^{2x-1}\\
\log 7^{x+1} &= \log 4^{2x-1}\\
(x+1)\log 7 &= (2x-1)\log 4\\
x\log 7 + \log 7 &= 2x\log 4 - \log 4\\
x\log 7 - 2x\log 4 &= -(\log 4 + \log 7)\\
x(\log 7 - 2\log 4) &= -(\log 28)\\
x &= -\frac{\log 28}{\log 7 - 2\log 4}\\
x &\approx 4.03
\end{aligned}$$

Chapter 8 Review   Page 418   Question 19

a)

$$\begin{aligned}
2\log_5(x-3) &= \log_5 4\\
\log_5(x-3)^2 &= \log_5 4\\
(x-3)^2 &= 4\\
x^2 - 6x + 9 &= 4\\
x^2 - 6x + 5 &= 0\\
(x-5)(x-1) &= 0
\end{aligned}$$

$x = 5$ or $x = 1$

Since the equation is defined for $x > 3$, the solution is $x = 5$.

b)

$$\begin{aligned}
\log_4(x+2) - \log_4(x-4) &= \frac{1}{2}\\
\log_4\frac{x+2}{x-4} &= \frac{1}{2}\\
\frac{x+2}{x-4} &= 4^{\frac{1}{2}}\\
x+2 &= 2(x-4)\\
x &= 10
\end{aligned}$$

Since the equation is defined for $x > 4$, the solution is $x = 10$.

c)

$$\log_2(3x+1) = 2 - \log_2(x-1)$$
$$\log_2(3x+1) + \log_2(x-1) = 2$$
$$\log_2((3x+1)(x-1)) = 2$$
$$(3x+1)(x-1) = 2^2$$
$$3x^2 - 2x - 1 = 4$$
$$3x^2 - 2x - 5 = 0$$
$$(3x-5)(x+1) = 0$$

$x = \frac{5}{3}$ 10 or $x = -1$

Since the equation is defined for $x > 1$, the solution is $x = \frac{5}{3}$.

d) $\log\sqrt{x^2 - 21x} = 1$

$$\frac{1}{2}\log(x^2 - 21x) = 1$$
$$\log(x^2 - 21x) = 2$$
$$x^2 - 21x = 10^2$$
$$x^2 - 21x - 100 = 0$$
$$(x-25)(x+4) = 0$$

$x = 25$ or $x = -4$

Since the equation is defined for $x < 0$ or $x > 21$, the solutions are $x = 25$ and $x = -4$.

Chapter 8 Review Page 418 Question 20

Let the value of the computer, $v(t)$, be represented by $v(t) = v_0(0.68)^t$, where v_0 is the initial value of the computer and t is the time, in years.
Substitute $v(t) = 100$ and $v_0 = 1200$.

$$v(t) = v_0(0.68)^t$$
$$100 = 1200(0.68)^t$$
$$\frac{100}{1200} = 0.68^t$$
$$\log\frac{1}{12} = \log 0.68^t$$
$$\log\frac{1}{12} = t\log 0.68$$
$$t = \frac{\log\frac{1}{12}}{\log 0.68}$$
$$t = 6.443\ldots$$

Since $t = 6.4$ results in a value of \$101.68, the computer will be worth less than \$100 in approximately 6.5 years.

Chapter 8 Review Page 418 Question 21

Substitute $R = 1050$ into $\log R = \log 73.3 + 0.75 \log m$.

$$\log R = \log 73.3 + 0.75\log m$$
$$\log 1050 = \log 73.3 + 0.75\log m$$
$$\log 1050 - \log 73.3 = 0.75\log m$$
$$\frac{1}{0.75}\log\frac{1050}{73.3} = \log m$$

$$10^{\frac{1}{0.75}\log\frac{1050}{73.3}} = m$$
$$34.789\ldots = m$$
The mass of the wolf is 35 kg, to the nearest kilogram.

Chapter 8 Review Page 418 Question 22

Substitute $m(t) = 600$, $m_0 = 800$, and $h = 6$ into $m(t) = m_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$, where $m(t)$ and m_0 are measured in megabecquerels, t is time, in hours, and h is the half-life of Tc-99m, in hours.

$$m(t) = m_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$
$$600 = 800\left(\frac{1}{2}\right)^{\frac{t}{6}}$$
$$0.75 = 0.5^{\frac{t}{6}}$$
$$\log 0.75 = \log 0.5^{\frac{t}{6}}$$
$$\log 0.75 = \frac{t}{6}\log 0.5$$
$$h = \frac{6\log 0.75}{\log 0.5}$$
$$h = 2.490...$$
The radioactivity of the Tc-99m in the patient's body will be 600 MBq in 2.5 h, to the nearest tenth of an hour.

Chapter 8 Review Page 418 Question 23

a) Substitute $P = 500$, $i = \frac{0.05}{4}$, or 0.0125, and $A = 1000$.
$$A = P(1 + i)^n$$
$$1000 = 500(1 + 0.0125)^n$$
$$2 = 1.0125^n$$
$$\log 2 = \log 1.0125^n$$
$$\log 2 = n \log 1.0125$$
$$\frac{\log 2}{\log 1.0125} = n$$
$$55.797\ldots = n$$
It will take approximately 56 ÷ 4, or 14 years for the GIC to be worth \$11 000.

b) Substitute $FV = 100\ 000$, $i = \dfrac{0.048}{2}$, or 0.012, and $R = 500$.

$$\begin{aligned}
FV &= \frac{R[(1+i)^n - 1]}{i} \\
100\ 000 &= \frac{500[(1+0.012)^n - 1]}{0.012} \\
\frac{0.012(100\ 000)}{500} &= 1.012^n - 1 \\
2.4 &= 1.012^n - 1 \\
3.4 &= 1.012^n \\
\log 3.4 &= \log 1.012^n \\
\log 3.4 &= n \log 1.012 \\
n &= \frac{\log 3.4}{\log 1.012} \\
n &= 102.591...
\end{aligned}$$

It will take about 103 ÷ 4, or 25.75 years for Mahal's investment to be worth \$100 000.

Chapter 8 Practice Test

Chapter 8 Practice Test Page 419 Question 1

The inverse of $y = \left(\frac{1}{4}\right)^x$ is $y = \log_{\frac{1}{4}} x$, which is represented by the graph in choice **D**.

Chapter 8 Practice Test Page 419 Question 2

The exponential form of $k = -\log_h 5$, or $k = \log_h 5^{-1}$, is $h^k = \dfrac{1}{5}$: choice **A**.

Chapter 8 Practice Test Page 419 Question 3

The function $y = \log_3 \sqrt{x+7}$ can be written as $y = \dfrac{1}{2} \log_3 (x + 7)$. Then, the graph of $y = \log_3 x$ must be vertically stretched about the x-axis by a factor of $\dfrac{1}{2}$ and translated 7 units to the left to obtain the graph of $y = \log_3 \sqrt{x+7}$: choice **B**.

Chapter 8 Practice Test Page 419 Question 4

$$\log_3 \frac{x^p}{x^q} = \log_3 x^p - \log_3 x^q$$
$$= p\log_3 x - q\log_3 x$$
$$= (p-q)\log_3 x$$

Choice **A**.

Chapter 8 Practice Test Page 419 Question 5

Given $x = \log_2 3$

$$\log_2 8\sqrt{3} = \log_2 8 + \log_2 \sqrt{3}$$
$$= \log_2 8 + \frac{1}{2}\log_2 3$$
$$= 3 + \frac{1}{2}x$$

Choice **C**.

Chapter 8 Practice Test Page 419 Question 6

Let the pH levels of two acids be $pH_1 = -\log [H_1+]$ and $pH_2 = -\log [H_2+]$.
Compare the two pH levels.

$pH_2 - pH_1 = -\log [H_2+] - (-\log [H_1+])$
$pH_2 - pH_1 = \log [H_2+]^{-1} - \log [H_1+]^{-1}$
$pH_2 - pH_1 = \log \dfrac{[H_1+]}{[H_2+]}$

Substitute $\dfrac{[H_1+]}{[H_2+]} = 4$ and $pH_2 = 2.9$.

$2.9 - pH_1 = \log 4$
$pH_1 = 2.9 - \log 4$
$pH_1 = 2.297\ldots$

The pH of formic acid is approximately 2.3: choice **B**.

Chapter 8 Practice Test Page 420 Question 7

a) $\log_9 x = -2$
$9^{-2} = x$
$\dfrac{1}{81} = x$

b) $\log_x 125 = \dfrac{3}{2}$
$x^{\frac{3}{2}} = 125$
$x = 25$

c) $\log_3 (\log_x 125) = 1$
$\log_x 125 = 3$
$x^3 = 125$
$x = 5$

d) $7^{\log_7 3} = x$
$3 = x$

e) $\log_2 8^{x-3} = 4$

$(x-3)\log_2 8 = 4$

$(x-3)3 = 4$

$3x - 9 = 4$

$3x = 13$

$x = \dfrac{13}{3}$

Chapter 8 Practice Test Page 420 Question 8

Given: $5^{m+n} = 125$ and $\log_{m-n} 8 = 3$

$5^{m+n} = 125$

$5^{m+n} = 5^3$

$m + n = 3$ ①

$\log_{m-n} 8 = 3$

$(m-n)^3 = 8$

$(m-n)^3 = 2^3$

$m - n = 2$ ②

Solve the system of equations.

$m + n = 3$

$\underline{m - n = 2}$

$2m = 5$ ① + ②

$m = \dfrac{5}{2}$

Substitute $m = \dfrac{5}{2}$ into ①.

$m + n = 3$

$\dfrac{5}{2} + n = 3$

$n = \dfrac{1}{2}$

Chapter 8 Practice Test Page 420 Question 9

For $y = -5 \log_2 (8(x-1))$, $a = -5$, $b = 8$, and $h = 1$.

Examples:

To obtain the graph of $y = -5 \log_2 (8(x-1))$, the graph of $y = \log_2 x$ must be reflected in the x-axis, vertically stretched about the x-axis by a factor of 5, horizontally stretched about the y-axis by a factor of $\dfrac{1}{8}$, and translated 1 unit to the right.

OR

To obtain the graph of $y = -5 \log_2 (8(x-1))$, the graph of $y = \log_2 x$ must be horizontally stretched about the y-axis by a factor of $\dfrac{1}{8}$, vertically stretched about the x-axis by a factor of 5, reflected in the x-axis, and translated 1 unit to the right.

Chapter 8 Practice Test Page 420 Question 10

Given: $y = 2 \log_5 (x + 5) + 6$

a) The equation of the vertical asymptote occurs when $x + 5 = 0$. Therefore, the equation of the vertical asymptote is $x = -5$.

b) The domain is $\{x \mid x > -5, x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

c) Substitute $x = 0$. Then, solve for y.

$$\begin{aligned} y &= 2 \log_5 (x + 5) + 6 \\ &= 2 \log_5 (0 + 5) + 6 \\ &= 2 \log_5 5 + 6 \\ &= 2(1) + 6 \\ &= 8 \end{aligned}$$

The y-intercept is 8.

d) Substitute $y = 0$. Then, solve for x.

$$\begin{aligned} y &= 2 \log_5 (x + 5) + 6 \\ 0 &= 2 \log_5 (x + 5) + 6 \\ -6 &= 2 \log_5 (x + 5) \\ -3 &= \log_5 (x + 5) \\ 5^{-3} &= x + 5 \\ \frac{1}{125} &= x + 5 \\ x &= -\frac{624}{125} \end{aligned}$$

The x-intercept is $-\frac{624}{125}$, or -4.992.

Chapter 8 Practice Test Page 420 Question 11

a) $\log_2 (x-4) - \log_2 (x+2) = 4$

$$\begin{aligned} \log_2 \frac{x-4}{x+2} &= 4 \\ \frac{x-4}{x+2} &= 2^4 \\ x-4 &= 16(x+2) \\ -15x &= 36 \\ x &= -\frac{36}{15} \\ x &= -\frac{12}{5} \end{aligned}$$

Since the equation is defined for $x > 4$, there is no solution.

b) $\log_2(x-4) = 4 - \log_2(x+2)$

$$\log_2(x-4) + \log_2(x+2) = 4$$
$$\log_2((x-4)(x+2)) = 4$$
$$(x-4)(x+2) = 2^4$$
$$x^2 - 2x - 8 = 16$$
$$x^2 - 2x - 24 = 0$$
$$(x-6)(x+4) = 0$$
$$x = 6 \quad \text{or} \quad x = -4$$

Since the equation is defined for $x > 4$, the solution is $x = 6$.

c) $\log_2(x^2 - 2x)^7 = 21$

$$7\log_2(x^2 - 2x) = 21$$
$$\log_2(x^2 - 2x) = 3$$
$$x^2 - 2x = 2^3$$
$$x^2 - 2x - 8 = 0$$
$$(x-4)(x+2) = 0$$

$x = 4$ or $x = -2$

Since the equation is defined for $x < 0$ or $x > 2$, the solutions are $x = 4$ and $x = -2$.

Chapter 8 Practice Test Page 420 Question 12

a) $3^{2x+1} = 75$

$$\log 3^{2x+1} = \log 75$$
$$(2x + 1) \log 3 = \log 75$$
$$2x \log 3 + \log 3 = \log 75$$
$$2x \log 3 = \log 75 - \log 3$$
$$x = \frac{\log 25}{2\log 3}$$
$$x \approx 1.46$$

b) $12^{x-2} = 3^{2x+1}$

$$\log 12^{x-2} = \log 3^{2x+1}$$
$$(x - 2) \log 12 = (2x + 1) \log 3$$
$$x \log 12 - 2 \log 12 = 2x \log 3 + \log 3$$
$$x \log 12 - 2x \log 3 = 2 \log 12 + \log 3$$
$$x(\log 12 - 2 \log 3) = 2 \log 12 + \log 3$$
$$x = \frac{2\log 12 + \log 3}{\log 12 - 2\log 3}$$
$$x \approx 21.09$$

Chapter 8 Practice Test Page 420 Question 13

Substitute $PV = 1\ 000\ 000$, $i = \dfrac{0.06}{2}$, or 0.03, and $R = 35\ 000$.

$$\begin{aligned}
PV &= \frac{R[1-(1+i)^{-n}]}{i}\\
1\ 000\ 000 &= \frac{35\ 000[1-(1+0.03)^{-n}]}{0.03}\\
\frac{0.03(1\ 000\ 000)}{35\ 000} &= 1-1.03^{-n}\\
\frac{6}{7} &= 1-1.03^{-n}\\
-\frac{1}{7} &= -1.03^{-n}\\
\frac{1}{7} &= 1.03^{-n}\\
\log\frac{1}{7} &= \log 1.03^{-n}\\
\log\frac{1}{7} &= -n\log 1.03\\
n &= -\frac{\log\frac{1}{7}}{\log 1.03}\\
n &= 65.831...
\end{aligned}$$

Holly can make semi-annual withdrawals for about 66 ÷ 2, or 33 years.

Chapter 8 Practice Test Page 420 Question 14

Substitute $\Delta G = 4200$ into $\Delta G = 1427.6(\log C_2 - \log C_1)$.

$$\begin{aligned}
\Delta G &= 1427.6(\log C_2 - \log C_1)\\
4200 &= 1427.6(\log C_2 - \log C_1)\\
\frac{4200}{1427.6} &= \log\frac{C_2}{C_1}\\
\frac{C_2}{C_1} &= 10^{\frac{4200}{1427.6}}\\
\frac{C_2}{C_1} &= 874.984...
\end{aligned}$$

The glucose concentration outside the cell is approximately 875 times as great as inside the cell.

Chapter 8 Practice Test Page 420 Question 15

Substitute $\frac{I_2}{I_1}$ = 2 and β_1 = 45 into $\beta_2 - \beta_1 = 10\left(\log\frac{I_2}{I_1}\right)$.

$$\beta_2 - \beta_1 = 10\left(\log\frac{I_2}{I_1}\right)$$
$$\beta_2 - 45 = 10 \log 2$$
$$\beta_2 = 10 \log 2 + 45$$
$$\beta_2 = 48.010\ldots$$

Since the decibel level with two refrigerators running is about 48 dB, the owner should not be worried. For comparison, this decibel level is between quiet and normal conversation on the decibel scale.

Chapter 8 Practice Test Page 420 Question 16

Substitute $c(t) = 12.8$, $c_0 = 4.0$, and $t = 8$ into $c(t) = c_0(2)^{\frac{t}{d}}$, where $c(t)$ and c_0 are measured in grams per litre, t is time, in hours, and d is the doubling time of the yeast cells, in hours.

$$c(t) = c_0(2)^{\frac{t}{d}}$$
$$12.8 = 4.0(2)^{\frac{8}{d}}$$
$$3.2 = 2^{\frac{8}{d}}$$
$$\log 3.2 = \log 2^{\frac{8}{d}}$$
$$\log 3.2 = \frac{8}{d}\log 2$$
$$d = \frac{8\log 2}{\log 3.2}$$
$$h = 4.767...$$

The doubling time of the yeast cells is 4.8 h, to the nearest tenth of an hour.

Chapter 8 Practice Test Page 420 Question 17

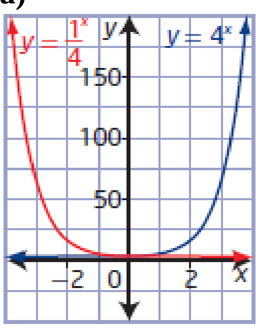
Let the CPI, $C(t)$, be represented by $C(t) = C_0(2)^{\frac{t}{d}}$, where C_0 is the CPI in 1992, t is the number of years since 1992, and d is the doubling time, in years.
Substitute $C_0 = 1$, $t = 14$, and $l(d) = 1.299$.

$$C(t) = C_0(2)^{\frac{t}{d}}$$
$$1.299 = 1(2)^{\frac{14}{d}}$$
$$1.299 = 2^{\frac{14}{d}}$$
$$\log 1.299 = \log 2^{\frac{14}{d}}$$
$$\log 1.299 = \frac{14}{d}\log 2$$
$$d = \frac{14\log 2}{\log 1.299}$$
$$d = 37.095...$$
If the CPI continues to grow at the same rate, in the year 2029 the price of the basket will be twice the 1992 price.

Cumulative Review, Chapters 7-8

Cumulative Review, Chapters 7-8 Page 422 Question 1

a)



b) The two functions have the same domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 0, y \in \mathrm{R}\}$, y-intercept 1, and equation of the asymptote $y = 0$.

c) The function $y = 4^x$ is increasing, since $c > 1$. The function $y = \left(\frac{1}{4}\right)^x$ is decreasing, since $0 < c < 1$.

Cumulative Review, Chapters 7-8 Page 422 Question 2

a) For $y = 5(2^x) + 1$, $c > 1$ so the graph is increasing. The graph will pass through the point (0, 6). Graph **B**.

b) For $y = \left(\frac{1}{2}\right)^{x+5}$, $c < 1$ so the graph is decreasing. The graph will pass through the point (0, 0.031 25). Graph **D**.

c) For $y + 1 = 2^{5-x}$ or $y = \left(\frac{1}{2}\right)^{x-5}$, $c < 1$ so the graph is decreasing. The graph will pass through the point (0, 31). Graph **A**.

d) For $y = 5\left(\frac{1}{2}\right)^{-x}$ or $y = 5(2^x)$, $c > 1$ so the graph is increasing. The graph will pass through the approximate point (0, 5). Graph **C**.

Cumulative Review, Chapters 7-8 Page 422 Question 3

a) Substitute $t = 0$.

$$B(t) = 1000\left(2^{\frac{t}{3}}\right)$$

$$B(0) = 1000\left(2^{\frac{0}{3}}\right)$$

$$B(0) = 1000$$

There were 1000 bacteria initially.

b) The doubling period is 3 h.

c) Substitute $t = 24$.

$$B(t) = 1000\left(2^{\frac{t}{3}}\right)$$

$$B(24) = 1000\left(2^{\frac{24}{3}}\right)$$

$$B(24) = 256\ 000$$

There were 256 000 bacteria after 24 h.

d) Substitute $B(t) = 128\ 000$.

$$B(t) = 1000\left(2^{\frac{t}{3}}\right)$$

$$128\ 000 = 1000\left(2^{\frac{t}{3}}\right)$$

$$128 = 2^{\frac{t}{3}}$$

$$2^7 = 2^{\frac{t}{3}}$$

$$7 = \frac{t}{3}$$

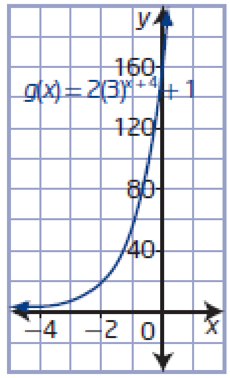
$$t = 21$$

There will be 128 000 bacteria in 21 h.

Cumulative Review, Chapters 7-8 Page 422 Question 4

a) For $g(x) = 2(3^{x+4}) + 1$, $a = 2$, $h = -4$, and $k = 1$. The graph of $f(x) = 3^x$ must be vertically stretch by a factor of 2 and translated 4 units to the left and 1 unit up.

b)



c) The domain remains the same. The range changes from $\{y \mid y > 0, y \in R\}$ to $\{y \mid y > 1, y \in R\}$ because of the vertical translation. The equation of the asymptote changes from $y = 0$ to $y = 1$ also because of the vertical translation. The y-intercept changes from 1 to 163 because of the vertical stretch and vertical translation.

Cumulative Review, Chapters 7-8 Page 422 Question 5

a) 2^{3x+6} and

$$\begin{aligned} 8^{x-5} &= (2^3)^{x-5} \\ &= 2^{3x-15} \end{aligned}$$

b) 27^{4-x} and $\left(\frac{1}{9}\right)^{2x} = (3^{-2})^{2x}$

$$\begin{aligned} &= (3^3)^{4-x} \\ &= 3^{12-3x} \end{aligned}$$

$$= 3^{-4x}$$

Cumulative Review, Chapters 7-8 Page 422 Question 6

a)

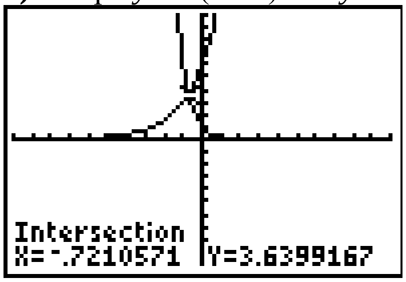
$$\begin{aligned} 5 &= 2^{x+4} - 3 \\ 8 &= 2^{x+4} \\ 2^3 &= 2^{x+4} \\ 3 &= x + 4 \\ x &= -1 \end{aligned}$$

b)

$$\begin{aligned} \frac{25^{x+3}}{625^{x-4}} &= 125^{2x+7} \\ \frac{(5^2)^{x+3}}{(5^4)^{x-4}} &= (5^3)^{2x+7} \\ \frac{5^{2x+6}}{5^{4x-16}} &= 5^{6x+21} \\ 5^{-2x+22} &= 5^{6x+21} \\ -2x + 22 &= 6x + 21 \\ -8x &= -1 \\ x &= \frac{1}{8} \end{aligned}$$

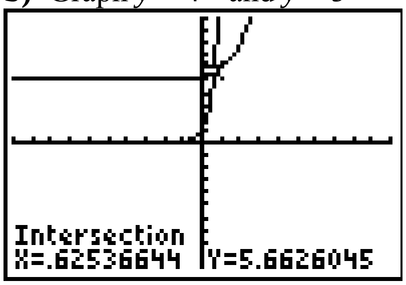
Cumulative Review, Chapters 7-8 Page 422 Question 7

a) Graph $y = 3(2^{x+1})$ and $y = 6^{-x}$ and identify the point of intersection.



The solution is $x = -0.72$, to two decimal places.

b) Graph $y = 4^{2x}$ and $y = 3^{x-1} + 5$ and identify the point of intersection.



The solution is $x = 0.63$, to two decimal places.

Cumulative Review, Chapters 7-8 Page 422 Question 8

a) Substitute $t = 5$.

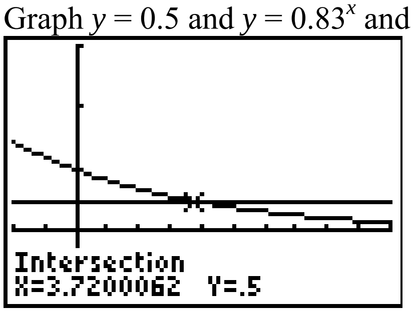
$p = 100(0.83^t)$
$= 100(0.83^5)$
$= 39.390\ldots$

The percent air pressure in the tank is approximately 39%.

b) Substitute $p = 50$.

$p = 100(0.83^t)$
$50 = 100(0.83^t)$
$0.5 = 0.83^t$

Graph $y = 0.5$ and $y = 0.83^x$ and identify the point of intersection.



The air pressure will be 50% of the starting pressure in approximately 3.7 s.

Cumulative Review, Chapters 7-8 Page 422 Question 9

a) In logarithmic form, $y = 3^x$ is $x = \log_3 y$.

b) In logarithmic form, $m = 2^{a+1}$ is $a + 1 = \log_2 m$.

Cumulative Review, Chapters 7-8 Page 422 Question 10

a) In exponential form, $\log_x 3 = 4$ is $x^4 = 3$.

b) In exponential form, $\log_a (x + 5) = b$ is $a^b = x + 5$.

Cumulative Review, Chapters 7-8 Page 423 Question 11

a) $$\begin{aligned}\log_3 \frac{1}{81} &= \log_3 3^{-4} \\ &= -4\end{aligned}$$

b) $$\begin{aligned}\log_2 \sqrt{8} + \frac{1}{3}\log_2 512 &= \frac{1}{2}\log_2 8 + \frac{1}{3}\log_2 512 \\ &= \frac{1}{2}(3) + \frac{1}{3}(9) \\ &= \frac{3}{2} + 3 \\ &= \frac{9}{2}\end{aligned}$$

c) $$\begin{aligned}\log_2(\log_5 \sqrt{5}) &= \log_2\left(\frac{1}{2}\log_5 5\right) \\ &= \log_2\left(\frac{1}{2}\right) \\ &= -1\end{aligned}$$

d) Use the inverse property $c^{\log_c x} = x$. For $k = \log_7 49$,
$$\begin{aligned}7^k &= 7^{\log_7 49} \\ &= 49\end{aligned}$$

Cumulative Review, Chapters 7-8 Page 423 Question 12

a) $$\begin{aligned}\log_x 16 &= 4 \\ x^4 &= 16 \\ x^4 &= 2^4 \\ x &= 2\end{aligned}$$

b) $$\begin{aligned}\log_2 x &= 5 \\ 2^5 &= x \\ 32 &= x\end{aligned}$$

c) $$\begin{aligned}5^{\log_5 x} &= \frac{1}{125} \\ x &= \frac{1}{125}\end{aligned}$$

d) $\log_x(\log_3 \sqrt{27}) = \frac{1}{5}$

$$\log_x\left(\frac{1}{2}\log_3 27\right) = \frac{1}{5}$$

$$\log_x\left(\frac{1}{2}(3)\right) = \frac{1}{5}$$

$$x^{\frac{1}{5}} = \frac{3}{2}$$

$$x = \frac{243}{32}$$

Cumulative Review, Chapters 7-8 Page 423 Question 13

For $y = \frac{\log_6(2x-8)}{3} + 5$ or $y = \frac{1}{3}\log_6(2(x-4)) + 5$, $a = \frac{1}{3}$, $b = 2$, $h = 4$, and $k = 4$. The graph of $y = \log_6 x$ must be transformed by a horizontal stretch about the y-axis by a factor of $\frac{1}{2}$, a vertical stretch about the x-axis by a factor of $\frac{1}{3}$, and translated by 4 units to the right and 5 units up.

Cumulative Review, Chapters 7-8 Page 423 Question 14

a) For a vertical stretch about the x-axis by a factor of 3 and a horizontal translation of 5 units left, $a = 3$ and $h = 5$. The equation of the transformed function is $y = 3\log(x + 5)$.

b) For a horizontal stretch about the y-axis by a factor of $\frac{1}{2}$, a reflection in the x-axis, and a vertical translation of 2 units down, $a = -1$, $b = 2$, and $k = -2$. The equation of the transformed function is $y = -\log 2x - 2$.

Cumulative Review, Chapters 7-8 Page 423 Question 15

a) Substitute pH = 6.2.

$\text{pH} = -\log[\text{H+}]$
$6.2 = -\log[\text{H+}]$
$-6.2 = \log[\text{H+}]$
$[\text{H+}] = 10^{-6}$
$[\text{H+}] \approx 6.3 \times 10^{-7}$

Substitute pH = 7.8.

$\text{pH} = -\log[\text{H+}]$
$7.8 = -\log[\text{H+}]$
$-7.8 = \log[\text{H+}]$
$[\text{H+}] = 10^{-7.8}$
$[\text{H+}] \approx 1.6 \times 10^{-8}$

The range of the concentration of hydrogen ions that is best for alfalfa is 1.6×10^{-8} mol/L to 6.3×10^{-7} mol/L.

b) Substitute [H+] = 3.0×10^{-6}.

pH = –log [H+]

= –log (3.0×10^{-6})

= 5.522…

Since the pH level is above 5.5, nitrogen is available to plants.

Cumulative Review, Chapters 7-8 Page 423 Question 16

a)
$$\begin{aligned} 2\log m - (\log\sqrt{n} + 3\log p) &= \log m^2 - (\log\sqrt{n} + \log p^3) \\ &= \log m^2 - \log\left(p^3\sqrt{n}\right) \\ &= \log\frac{m^2}{p^3\sqrt{n}},\ m > 0,\ n > 0,\ p > 0 \end{aligned}$$

b)
$$\begin{aligned} \frac{1}{3}(\log_a x - \log_a\sqrt{x}) + \log_a 3x^2 &= \frac{1}{3}\log_a\frac{x}{\sqrt{x}} + \log_a 3x^2 \\ &= \frac{1}{3}\log_a\sqrt{x} + \log_a 3x^2 \\ &= \log_a\sqrt[6]{x} + \log_a 3x^2 \\ &= \log_a\sqrt[6]{x}(3x^2) \\ &= \log_a 3x^{\frac{13}{6}},\ x > 0 \end{aligned}$$

c)
$$\begin{aligned} 2\log(x+1) + \log(x-1) - \log(x^2-1) &= \log(x+1)^2 + \log(x-1) - \log(x^2-1) \\ &= \log\frac{(x+1)^2(x-1)}{x^2-1} \\ &= \log\frac{(x+1)(x+1)(x-1)}{(x+1)(x-1)} \\ &= \log(x+1),\ x > 1 \end{aligned}$$

d)
$$\begin{aligned} \log_2 27^x - \log_2 3^x &= \log_2\frac{27^x}{3^x} \\ &= \log_2 9^x,\ x \in \mathrm{R} \text{ or} \\ &= \log_2 3^{2x},\ x \in \mathrm{R} \end{aligned}$$

Cumulative Review, Chapters 7-8 Page 423 Question 17

Zack incorrectly factored $x^2 - 8x - 65$ as $(x + 13)(x - 5)$. The correct factored form is $(x - 13)(x + 5)$. So, the solutions are $x = 13$ and $x = -5$.

Cumulative Review, Chapters 7-8 Page 423 Question 18

a)

$$\begin{aligned}
4^{2x+1} &= 9(4^{1-x}) \\
\frac{4^{2x+1}}{4^{1-x}} &= 9 \\
4^{3x} &= 9 \\
\log 4^{3x} &= \log 9 \\
3x\log 4 &= \log 9 \\
x &= \frac{\log 9}{3\log 4} \\
x &\approx 0.53
\end{aligned}$$

b)

$$\begin{aligned}
\log_3 x + 3\log_3 x^2 &= 14 \\
\log_3 x + \log_3 (x^2)^3 &= 14 \\
\log_3 x(x^6) &= 14 \\
\log_3 x^7 &= 14 \\
x^7 &= 3^{14} \\
x &= 3^2 \\
x &= 9
\end{aligned}$$

c)

$$\begin{aligned}
\log(2x-3) &= \log(4x-3) - \log x \\
\log(2x-3) - \log(4x-3) + \log x &= 0 \\
\log\frac{(2x-3)x}{4x-3} &= 0 \\
10^0 &= \frac{(2x-3)x}{4x-3} \\
1 &= \frac{2x^2-3x}{4x-3} \\
4x-3 &= 2x^2-3x \\
0 &= 2x^2-7x+3 \\
0 &= (2x-1)(x-3) \\
x = \frac{1}{2} \quad &\text{or} \quad x = 3
\end{aligned}$$

Since the equation is defined for $x > \frac{3}{2}$, the solution is $x = 3$.

d)

$$\begin{aligned}
\log_2 x + \log_2(x+6) &= 4 \\
\log_2(x(x+6)) &= 4 \\
x(x+6) &= 2^4 \\
x^2+6x &= 16 \\
x^2+6x-16 &= 0 \\
(x+8)(x-2) &= 0 \\
x = -8 \quad \text{or} \quad x &= 2
\end{aligned}$$

Since the equation is defined for $x > 0$, the solution is $x = 2$.

Cumulative Review, Chapters 7-8 Page 423 Question 19

a) Substitute $M = 4$.
$\log E = 4.4 + 1.4M$
$\log E = 4.4 + 1.4(4)$
$\log E = 10$
$E = 10^{10}$

Substitute $M = 5$.
$\log E = 4.4 + 1.4M$
$\log E = 4.4 + 1.4(5)$
$\log E = 11.4$
$E = 10^{11.4}$

The energy of earthquakes with magnitudes 4 and 5 are 10^{10} J and $10^{11.4}$ J, respectively.

b) For each increase in M of 1, E changes by a factor of $10^{1.4}$, or about 25.1 times.

Cumulative Review, Chapters 7-8 Page 423 Question 20

Substitute $FV = 1\ 000\ 000$, $i = \frac{0.06}{2}$, or 0.015, and $R = 625$.

$$\begin{aligned}
FV &= \frac{R[(1+i)^n - 1]}{i} \\
1\ 000\ 000 &= \frac{625[(1+0.015)^n - 1]}{0.015} \\
\frac{0.015(1\ 000\ 000)}{625} &= 1.015^n - 1 \\
24 &= 1.015^n - 1 \\
25 &= 1.015^n \\
\log 25 &= \log 1.015^n \\
\log 25 &= n \log 1.015 \\
n &= \frac{\log 25}{\log 1.015} \\
n &= 216.197...
\end{aligned}$$

Since $n = 216$ results in only \$996 946.64, it will take about $217 \div 4$, or 54.25 years for Aaron’s investment to be worth \$1 000 000.

Unit 2 Test

Unit 2 Test Page 424 Question 1

Use the given points, (3, –6) and (6, –12), to determine the value of a on the graph of $y = a(2^{bx})$.
For (3, –6),
$y = a(2^{bx})$
$-6 = a(2^{b3})$
$a = -\frac{6}{2^{3b}}$

Then, use (6, –12) and $a = -\dfrac{6}{2^{3b}}$,

$$
\begin{aligned}
y &= a(2^{bx}) \\
-12 &= -\frac{6}{2^{3b}}(2^{b6}) \\
2 &= 2^{3b} \\
1 &= 3b \\
b &= \frac{1}{3}
\end{aligned}
$$

Substitute $b = \dfrac{1}{3}$ into $a = -\dfrac{6}{2^{3b}}$.

$$
\begin{aligned}
a &= -\frac{6}{2^{3b}} \\
&= -\frac{6}{2^{3\left(\frac{1}{3}\right)}} \\
&= -3
\end{aligned}
$$

Choice **D**.

Unit 2 Test Page 424 Question 2

For $y = 3(b^{x+1}) - 2$, $a = 3$, $h = -1$, and $k = -2$. The graph of $y = b^x$ must be vertically stretched by a factor of 2 and translated 1 unit to the left and 2 units down to obtain the graph of $y = 3(b^{x+1}) - 2$. The domain stays the same, $\{x \mid x \in \mathrm{R}\}$, but the range changes from $\{y \mid y > 0, y \in \mathrm{R}\}$ to from $\{y \mid y > -2, y \in \mathrm{R}\}$. The x-intercept changes from none to one. The y-intercept changes from 1 to $3b - 2$.
Choice **B**.

Unit 2 Test Page 424 Question 3

The mass, m, of C-14 remaining at time t can be found using the relationship $m(t) = m_0\left(\dfrac{1}{2}\right)^{\frac{t}{5730}}$. If a bone has lost 40% of its carbon-14, then 60% remains. An equation that can be used to determine its age is $60 = 100\left(\dfrac{1}{2}\right)^{\frac{t}{5730}}$: choice **A**.

Unit 2 Test Page 424 Question 4

$$
\begin{aligned}
2x &= \log_3(y - 1) \\
y - 1 &= 3^{2x} \\
y &= 3^{2x} + 1 \\
y &= 9^x + 1
\end{aligned}
$$

An equivalent form for $2x = \log_3(y - 1)$ is $y = 9^x + 1$: choice **C**.

**Unit 2 Test	Page 424	Question 5**

The function $f(x) = –\log_2 (x + 3)$ is defined for $x + 3 > 0$, or $x > –3$. So, the domain is $\{x \mid x > –3, x \in R\}$: choice **A**.

**Unit 2 Test	Page 424	Question 6**

Given: $\log_2 5 = x$

$$\begin{aligned}\log_2 \sqrt[4]{25^3} &= \frac{3}{4}\log_2 25\\ &= \frac{3}{4}\log_2 5^2\\ &= \frac{3}{2}\log_2 5\\ &= \frac{3}{2}x\end{aligned}$$

Choice **A**.

**Unit 2 Test	Page 424	Question 7**

Given: $\log_4 16 = x + 2y$ and $\log 0.0001 = x – y$

$\log_4 16 = x + 2y$
$2 = x + 2y$ ①

$\log 0.0001 = x – y$
$–4 = x – y$ ②

Solve the system of equations.

$$\begin{aligned} 2 &= x + 2y\\ -4 &= x - y\\ \hline 6 &= 3y \quad ① - ②\\ 2 &= y\end{aligned}$$

Choice **D**.

**Unit 2 Test	Page 424	Question 8**

For a vertical stretch about the x-axis by a factor of 2, a reflection about the x-axis, and a horizontal translation of 3 units right, $a = –2$ and $h = 3$.

The graph of the function $f(x) = \left(\frac{1}{4}\right)^x$ is transformed by a vertical stretch about the x-axis by a factor of 2, a reflection about the x-axis, and a horizontal translation of 3 units right. The equation of the transformed function is $g(x) = -2\left(\frac{1}{4}\right)^{x-3}$.

Unit 2 Test Page 424 Question 9

$$\begin{aligned}\frac{9^{\frac{1}{2}}}{27^{\frac{2}{3}}} &= \frac{(3^2)^{\frac{1}{2}}}{(3^3)^{\frac{2}{3}}}\\ &= \frac{3}{3^2}\\ &= 3^{-1}\end{aligned}$$

The quotient $\frac{9^{\frac{1}{2}}}{27^{\frac{2}{3}}}$ expressed as a single power of 3 is 3^{-1}.

Unit 2 Test Page 425 Question 10

For a function that is reflected in the *x*-axis and translated 1 unit down, the mapping is $(x, y) \rightarrow (x, -y - 1)$.

The point P(2, 1) is on the graph of the logarithmic function $y = \log_2 x$. When the function is reflected in the *x*-axis and translated 1 unit down, the coordinates of the image of P are (2, –2).

Unit 2 Test Page 425 Question 11

$$\begin{aligned}\log 10^x &= 0.001\\ 10^{0.001} &= 10^x\\ x &= 0.001\end{aligned}$$

The solution to the equation $\log 10^x = 0.001$ is $x = 0.001$.

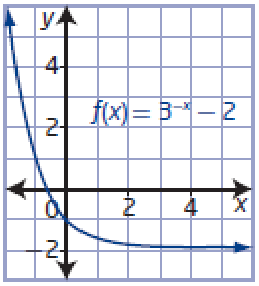
Unit 2 Test Page 425 Question 12

$$\begin{aligned}\log_5 40 - 3\log_5 10 &= \log_5 40 - \log_5 10^3\\ &= \log_5 \frac{40}{1000}\\ &= \log_5 \frac{1}{25}\\ &= -2\end{aligned}$$

Evaluating $\log_5 40 - 3\log_5 10$ results in –2.

Unit 2 Test Page 425 Question 13

a)



b) The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y > -2, y \in \mathrm{R}\}$.

c) Solve $f(x) = 0$.

$$\begin{aligned}
f(x) &= 3^{-x} - 2 \\
0 &= 3^{-x} - 2 \\
2 &= 3^{-x} \\
\log 2 &= \log 3^{-x} \\
\log 2 &= -x \log 3 \\
x &= -\frac{\log 2}{\log 3} \\
x &\approx -0.6
\end{aligned}$$

Unit 2 Test Page 425 Question 14

a)

$$\begin{aligned}
9^{\frac{1}{4}}\left(\frac{1}{3}\right)^{\frac{x}{2}} &= \sqrt[3]{27^4} \\
(3^2)^{\frac{1}{4}}(3^{-1})^{\frac{x}{2}} &= (3^3)^{\frac{4}{3}} \\
3^{\frac{1}{2}-\frac{x}{2}} &= 3^4 \\
\frac{1}{2}-\frac{x}{2} &= 4 \\
1-x &= 8 \\
x &= -7
\end{aligned}$$

b)

$$\begin{aligned}
5(2^{x-1}) &= 10^{2x-3} \\
\log 5(2^{x-1}) &= \log 10^{2x-3} \\
\log 5 + \log 2^{x-1} &= (2x-3)\log 10 \\
\log 5 + (x-1)\log 2 &= (2x-3)(1) \\
\log 5 + x\log 2 - \log 2 &= 2x-3 \\
x\log 2 - 2x &= -3 - \log 5 + \log 2 \\
x(\log 2 - 2) &= -3 - \log 5 + \log 2 \\
x &= \frac{-3-\log 5+\log 2}{\log 2 - 2} \\
x &= 2
\end{aligned}$$

Unit 2 Test Page 425 Question 15

a) For the function $f(x) = 1 - \log(x - 2)$, or $f(x) = -\log(x - 2) + 1$, $a = -1$, $h = 2$, and $k = 1$. The function is defined for $x - 2 > 0$ or $x > 2$. So, the domain is $\{x \mid x > 2, x \in \mathrm{R}\}$, the range is $\{y \mid y \in \mathrm{R}\}$, and the equation of the asymptote is $x = 2$.

b) $f(x) = 1 - \log (x - 2)$

$y = 1 - \log (x - 2)$

$x = 1 - \log (y - 2)$

$x - 1 = -\log (y - 2)$

$-(x - 1) = \log (y - 2)$

$y - 2 = 10^{-(x-1)}$

$y = 10^{-(x-1)} + 2$

$f^{-1}(x) = 10^{-(x-1)} + 2$

c) Substitute $x = 0$.

$f^{-1}(x) = 10^{-(x-1)} + 2$

$f^{-1}(0) = 10^{-(0-1)} + 2$

$f^{-1}(0) = 10 + 2$

$f^{-1}(0) = 12$

Unit 2 Test Page 425 Question 16

a) $\log 4 = \log x + \log (13 - 3x)$

$\log 4 = \log (x(13 - 3x))$

$\log 4 = \log (13x - 3x^2)$

$4 = 13x - 3x^2$

$0 = -3x^2 + 13x - 4$

$0 = 3x^2 - 13x + 4$

$0 = (3x - 1)(x - 4)$

$x = \frac{1}{3}$ or $x = 4$

Since the equation is defined for $0 < x < \frac{13}{3}$, the solutions are $x = \frac{1}{3}$ and $x = 4$.

b) $\log_3(3x+6) - \log_3(x-4) = 2$

$$\log_3 \frac{3x+6}{x-4} = 2$$

$$\frac{3x+6}{x-4} = 3^2$$

$$3x+6 = 9(x-4)$$

$$3x+6 = 9x-36$$

$$-6x = -42$$

$$x = 7$$

Since the equation is defined for $x > 4$, the solution is $x = 7$.

Unit 2 Test Page 425 Question 17

Giovanni's first error occurs in line 2. He multiplied the base by 2 when he should have divided both sides by 2. His next error occurs in line six, where he incorrectly applied the quotient law of logarithms: $\frac{\log 8}{\log 6} \neq \log 8 - \log 6$. The correct solution is

$2(3^x) = 8$

$3^x = 4$

$\log 3^x = \log 4$

$$x \log 3 = \log 4$$
$$x = \frac{\log 4}{\log 3}$$
$$x \approx 1.26$$

Unit 2 Test Page 425 Question 18

Determine the amplitude of the Tofino earthquake. Substitute $M = 5.6$.

$$M = \log \frac{A}{A_0}$$
$$5.6 = \log \frac{A}{A_0}$$
$$10^{5.6} = \frac{A}{A_0}$$
$$A = 10^{5.6} A_0$$

Then, the amplitude of the aftershock is $\frac{1}{4}A$, or $\frac{1}{4}10^{5.6}A_0$.

$$M = \log \frac{A}{A_0}$$
$$= \log \frac{\frac{1}{4}10^{5.6} A_0}{A_0}$$
$$= \log \frac{1}{4}10^{5.6}$$
$$= 4.997...$$

The magnitude of the aftershock is 5.0, to the nearest tenth.

Unit 2 Test Page 425 Question 19

a) Let the world population, $P(t)$, in billions, be represented by $P(t) = 6(1.03)^t$, where t is the number of years since 2000.

b) Substitute $P(t) = 10$.

$$P(t) = 6(1.013)^t$$
$$10 = 6(1.013)^t$$
$$\frac{5}{3} = 1.013^t$$
$$\log \frac{5}{3} = \log 1.013^t$$
$$\log \frac{5}{3} = t \log 1.013$$

$$t = \frac{\log \frac{5}{3}}{\log 1.013}$$
$$t = 39.549\ldots$$
The population will reach at least 10 billion by 2040.

Unit 2 Test **Page 425** **Question 20**

Substitute $FV = 150\ 000$, $i = \frac{0.05}{2}$, or 0.025, and $R = 11\ 500$.

$$FV = \frac{R[(1+i)^n - 1]}{i}$$
$$150\,000 = \frac{11\,500[(1+0.025)^n - 1]}{0.025}$$
$$\frac{0.025(150\,000)}{11\,500} = 1.025^n - 1$$
$$\frac{15}{46} = 1.025^n - 1$$
$$\frac{61}{46} = 1.025^n$$
$$\log \frac{61}{46} = \log 1.025^n$$
$$\log \frac{61}{46} = n \log 1.025$$
$$n = \frac{\log \frac{61}{46}}{\log 1.025}$$
$$n = 11.429...$$
Since $n = 11$ results in only \$143 559.86, it will take 12 deposits for the account to contain at least \$150 000.