AB Calculus Jan 6/2009 Name: ______________________________

1. Find the derivative of $f(x)=4x^2+7x-5$

A. $4x+7$ B. $4x+7$ C. $8x+7$ D. $8x^2+7x$ E. *none of these*

2. Given $f(x)=3x^2-4x+5$ find $f'(x)$

A. $5x+1$ B. $6x+1$ C. $5x-4$ D. $6x-4$ E. *none of these*

3. If $y=3x^3-4x^2+5$ find $\dfrac{dy}{dx}$

A. $3x^2-4x$ B. $3x^2-4x+5$ C. $9x^2-8x$ D. $9x^2-8x+5$ E. *none of these*

4. Find $\dfrac{dy}{dx}$ if $y=-3x^2+6x$

A. $-6x+6$ B. $9x+6$ C. $-x+7$ D. $-6x$ E. *none of these*

5. Find $f'(x)$ if $f(x)=3$

A. 0 B. 1 C. 3 D. $\dfrac{3}{x}$ E. *none of these*

6. Given that r is any real number, determine $\dfrac{d}{dx}(x^r)$

A. rx^{r-1} B. rx^{r+1} C. $(r-1)x^r$ D. $(r+1)x^r$ E. *none of these*

7. If $f(x)=\dfrac{3}{x}$ then $f'(x)=$

A. $-\dfrac{3}{x^2}$ B. $\dfrac{3}{x}$ C. $-3x$ D. $3x$ E. *none of these*

8. Find $\dfrac{dy}{dx}$ if $y=2\sqrt{x}$

A. $\dfrac{1}{\sqrt{x}}$ B. $-\sqrt{x}$ C. $\dfrac{1}{2\sqrt{x}}$ D. $-\dfrac{1}{2}x$ E. *none of these*

9. If $f(x)=\sqrt{x}$ determine the value of $f'(x)$ at $(16,\ 4)$

A. $-\dfrac{1}{4}$ B. $-\dfrac{1}{8}$ C. $\dfrac{1}{8}$ D. $\dfrac{1}{4}$ E. *none of these*

10. If $f(x)=k\sqrt{x}$ determine the value of the constant k so that $f'(4)=6$

A. $k=3$ B. $k=6$ C. $k=12$ D. $k=24$ E. *none of these*

11. For the curve $y=x^k$ $(k\neq 0)$, the slope of the tangent is equal to $16k$ when $x=2$ Determine the value of k

A. 3 B. 4 C. 5 D. 8 E. *none of these*

12. Given $f(x)=\frac{5}{x^2}$ determine $f'(x)$

A. $-\frac{10}{x}$ B. $-\frac{10}{x^3}$ C. $\frac{3}{x^3}$ D. $\frac{5}{2x}$ E. ***none of these***

13. Given $y=\frac{1}{x^3}$ determine $\frac{dy}{dx}$

A. $-\frac{3}{x^2}$ B. $-\frac{3}{x^4}$ C. $\frac{1}{3x^2}$ D. $\frac{1}{3x^4}$ E. ***none of these***

14. Find y' if $y=x^{\frac{3}{2}}$

A. $\frac{2}{3}x^{\frac{1}{2}}$ B. $\frac{3}{2}x^{\frac{1}{2}}$ C. $\frac{2}{3}x^{\frac{5}{2}}$ D. $\frac{3}{2}x^{\frac{5}{2}}$ E. ***none of these***

15. Which of the following represents the slope of the tangent to $f(x)$ at $x=2$

A. $f'(2)$ B. $f(2)$ C. $f'(x)=0$ D. $f'(x)=2$ E. ***none of these***

16. Given $f(x)=\frac{1}{x}$ determine $f'(x)$

A. $-\frac{1}{x^2}$ B. $\frac{1}{x^2}$ C. $-\frac{1}{x}$ D. $\frac{1}{x}$ E. ***none of these***

17. If $y=7$ determine $\frac{dy}{dx}$

A. 0 B. 1 C. 7 D. $\frac{7}{x}$ E. ***none of these***

18. Evaluate the derivative of the function $f(x)=3x^2-2x-1$ at the point where $x=0$

A. -2 B. -1 C. $\frac{1}{3}$ D. 1 E. ***none of these***

19. Evaluate the derivative of $f(x)=2x^2-3x+2$ at the point where $x=2$

A. $\frac{3}{4}$ B. $\frac{5}{4}$ C. 4 D. 5 E. ***none of these***

20. Given $f(x)=(2x-3)^2$ then $f'(x)=$

A. $4x$ B. $8x$ C. $4x-6$ D. $8x-12$ E. ***none of these***

21. Given the function $f(x)=\sqrt{2}$ determine $f'(x)$

A. 0 B. $\sqrt{2}$ C. $\frac{1}{2\sqrt{2}}$ D. $\frac{1}{\sqrt{2}}$ E. ***none of these***

22. If $f(x)=6g(x)$ then $f'(x)$ equals

A. $6g'(x)$ B. $g'(x)$ C. $g'(6)$ D. 6 E. ***none of these***

23. For what condition is $f(x)$ increasing ?

A. $f(x) > 0$ B. $f(x) < 0$ C. $f'(x) > 0$ D. $f'(x) < 0$ E. ***none of these***

24. Find k such that the function $f(x) = kx^2 + 12x - 4$ has a critical point at $x = 4$

A. $k = -6$ B. $k = -\frac{3}{2}$ C. $k = \frac{3}{2}$ D. $k = 6$ E. ***none of these***

25. Determine all values of x such that the function $f(x) = x^3 - 3x^2 + 5$ is decreasing.

A. $x < 2$ B. $x > 2$ C. $0 < x < 2$ D. $x < 0$ ***or*** $x > 2$

26. Find the x-value of the point on the graph of $y = x^2 - x$ where the slope of the tangent is 2

A. 0.5 B. 1.5 C. 2 D. 3 E. ***none of these***

27. Find all values of x such that the function $f(x) = 2x^3 - 3x^2$ is increasing

A. $x < 1$ B. $x > 0$ C. $0 < x < 1$ D. $x < 0$ ***or*** $x > 1$

28. Give all values of x where the function $f(x) = x^3 - 3x + 4$ is increasing

A. $x > 1$ B. $x < -1$ C. $-1 < x < 1$ D. $x < -1$ ***or*** $x > 1$

29. At which of the following values of x is the function $g(x) = x^3 - 4x^2$ decreasing ?

A. $x = -3$ B. $x = -1$ C. $x = 2$ D. $x = 4$ E. ***none of these***

30. If $f'(x) = -6x$ determine all values of x such that $f(x)$ is decreasing

A. $x > 0$ B. $x < 0$ C. $-6 < x < 0$ D. ***all real numbers***

31. Determine the x-values of the critical points for the function $f(x) = x^3 + 3x^2 - 24x$

A. $x = -4,\ x = 2$ B. $x = 4,\ x = -2$ C. $x = 0,\ x = 3.62,\ x = -6.62$

D. $x = 0,\ x = 3.62,\ x = 6.62$ E. ***none of these***

32. Determine all values of x such that the function $f(x) = x^4 - 18x^2 + 8$ is ***decreasing***.

33. Determine all values of x such that the function $f(x) = x^4 - 8x^2 - 9$ is increasing.

34. ***a***) Determine the x values of the critical points of $f(x) = x^4 - 8x^2$

b) For what values of x is $f(x) = x^4 - 8x^2$ **decreasing**

35. Given the function $f(x) = 2x^3 - 3x^2 - 12x + 4$

a) determine the coordinates of the critical points of $f(x)$

b) determine where $f(x)$ is increasing

36. For the function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-6x$, find the x-coordinate of the critical point where the local minimum point occurs.

A. -3 B. -2 C. 2 D. 3 E. *none of these*

37. Find the minimum value of the function $f(x)=2x^2-12x+6$

A. -24 B. -12 C. 3 D. 6 E. *none of these*

38. Determine the minimum value of the function $f(x)=3x^2-12x+13$

A. 0 B. 1 C. 2 D. 13 E. *none of these*

39. Determine the minimum value of the function $g(x)=2x^2-12x+25$

A. 0 B. 3 C. 7 D. 25 E. *none of these*

40. Determine the minimum value of the function $y=3x^2-24x-7$

A. -79 B. -55 C. 4 D. *no minimum value*

41. Find the maximum value of the function $y=-13-6x-x^2$

A. -40 B. -13 C. -4 D. -3 E. *none of these*

42. If $y=2ax+bx^2$ and a and b are positive constants, determine the minimum value of y

A. $-\frac{a}{b}$ B. $\frac{a}{b}$ C. $-\frac{a^2}{b}$ D. $-\frac{3a^2}{b}$ E. *none of these*

43. Determine the maximum value of the function $f(x)=-2x^2-x+6$

A. -0.25 B. 0 C. 5.625 D. 6.125 E. *none of these*

44. Find the minimum value of the function $f(x)=2x^2-12x+25$

A. 0 B. 3 C. 7 D. 25 E. *none of these*

45. If $f(x)=x^4+kx^2$ has a minimum at $x=1$, then determine the value of the constant k

A. -2 B. $-\frac{1}{2}$ C. $\frac{1}{2}$ D. 2 E. *none of these*

46. Determine the maximum value of the function $f(x)=2-18x-3x^2$

A. -79 B. -3 C. 3 D. 29 E. *none of these*

47. What is the maximum value of the function $f(x)=4+8x-x^2$

A. -4 B. 4 C. 12 D. 20 E. *none of these*

48. Find the slope of the line tangent to the graph of $f(x)=x^2+3$ at the point where $x=-1$

A. -2 B. 1 C. 2 D. 4 E. ***none of these***

49. Find the slope of the tangent to $y=x^3-2x^2+6$ at $(2, 6)$

A. 4 B. 6 C. 10 D. 20 E. ***none of these***

50. Find the slope of the line tangent to the graph of $y=x^3-4x^2+2$ at the point where $x=2$

A. -10 B. -6 C. -4 D. -2 E. ***none of these***

51. If $y=-3x+1$ is tangent to the curve $f(x)$ at $x=a$ which must be true ?

A. $f(a)=-3$ B. $f(a)=1$ C. $f'(a)=-3$ D. $f'(a)=1$ E. ***none of these***

52. Given the function $f(x)=3x^2-4x+3$ for what value(s) of x is the slope of the tangent line equal to 2

A. $\frac{2}{3}$ B. 1 C. 8 D. 1 *and* $\frac{1}{3}$ E. ***none of these***

53. Determine the slope of the line tangent to $y=\frac{6}{x}$ at $(2, 3)$

A. -3 B. -2 C. $-\frac{3}{2}$ D. $-\frac{2}{3}$ E. ***none of these***

54. Find the point on $y=2x^2+6x-1$ where the slope of the tangent line is 2

A. $(-1,-5)$ B. $(-1, 2)$ C. $(1, 7)$ D. $(2, 19)$ E. ***none of these***

55. Determine the slope of the line tangent to the graph of $y=\frac{1}{x}$ at $x=4$

A. -16 B. $-\frac{1}{16}$ C. $\frac{1}{16}$ D. 16 E. ***none of these***

56. Find an equation of the line tangent to the graph of $y=x^3-3x^2+3x+2$ at $(0, 2)$

A. $y=-3x+2$ B. $y=-2x+2$ C. $y=2x+2$ D. $y=3x+2$ E. ***none of these***

57. At what point on the curve $y=x^2-4$ is the tangent parallel to the line $6x+y=4$

A. $(-3, 22)$ B. $(-3, 5)$ C. $(3,-14)$ D. $(3, 5)$ E. ***none of these***

58. Determine the slope of the line tangent to the graph of $f(x)=\sqrt{x}$ at $x=9$

A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. $\frac{3}{2}$ D. 3 E. ***none of these***

59. The line $y=-4x+18$ is tangent to the parabola $y=ax^2+bx$ at the point where $x=3$
If the parabola has a critical point at $x=2$ determine the value of a

A. -4 B. -2 C. -1 D. 2 E. ***none of these***

60. What are the coordinates of the point on the graph of $y = \sqrt{x}$ where the slope of the tangent is $\frac{1}{8}$

A. $\left(\frac{1}{16}, \frac{1}{4}\right)$ B. $\left(\frac{1}{16}, 4\right)$ C. $\left(16, \frac{1}{8}\right)$ D. $(16, 4)$ E. ***none of these***

61. Determine the slope of the line tangent to the graph of $y = x^3 - x^2$ at the point where $x = 2$

A. 2 B. 4 C. 8 D. 10 E. ***none of these***

62. Determine the slope of the tangent line to $f(x) = -\frac{2}{x}$ at the point where $x = 2$

A. -2 B. $-\frac{1}{2}$ C. $\frac{1}{2}$ D. 2 E. ***none of these***

63. What is the slope of the tangent line to the graph of $y = -x^2 + 2x - 3$ at the point $(2, -3)$

A. -18 B. -3 C. -2 D. 8 E. ***none of these***

64. What is the slope of the tangent line to the function $y = 3 - x$

A. -1 B. 0 C. 2 D. 3 E. ***none of these***

65. The equation of the normal line to the curve $y = x^4 + 3x^3 + 2$ at the point where $x = 0$ is

A. $y = x$ B. $y = 13x$ C. $y = 0$ D. $y = x + 2$ E. $x = 0$

66. The line **L** is perpendicular to the parabola $y = kx^2$ at the point $(1, 5)$ What is the equation of **L**

A. $y = 10x - 5$ B. $x + y = 6$ C. $x + 10y = 51$ D. $10x + 3y = 25$ E. $y = 5x$

67. If $x + 7y = 29$ is an equation of the line ***normal*** to the graph of f at the point $(1, 4)$, then $f'(1) =$

A. 7 B. $\frac{1}{7}$ C. $-\frac{1}{7}$ D. $-\frac{7}{29}$ E. -7

68. The line perpendicular to the tangent of the curve represented by the equation $y = x^2 + 6x + 4$ at the point $(-2, -4)$ also intersects the curve at $x =$

A. -6 B. $-\frac{9}{2}$ C. $-\frac{7}{2}$ D. -3 E. $-\frac{1}{2}$

69. An equation of the line normal to the graph of $y = x^4 - 3x^2 + 1$ at the point where $x = 1$ is

A. $2x - y + 3 = 0$ B. $x - 2y + 3 = 0$ C. $2x - y - 3 = 0$ D. $x - 2y - 3 = 0$ E. $x - 2y = 0$

70. An equation of the line normal to the graph of $y = 7x^4 + 2x^3 + x^2 + 2x + 5$ at the point where $x = 0$ is

A. $x + 2y = 10$ B. $2x + y = 10$ C. $5x + 5y = 2$ D. $2x - y = -5$ E. $2x + y = -10$

71. Find the equation of the line normal to $y = 4x^2 + 2x + 9$ at the point where $x = 1$

A. $10x + y = -151$ B. $x + 10y = 151$ C. $x - y = 9$
D. $10x - y = -5$ E. $x - 10y = 151$

72. The coordinates of the point where the normal to the curve $y = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x$ at $x = 1$ intersects the y-axis are

A. $\left(0,\ \frac{3}{2}\right)$ B. $\left(\frac{3}{2},\ 0\right)$ C. $\left(0,\ \frac{13}{6}\right)$ D. $\left(\frac{13}{6},\ 0\right)$ E. $\left(0,\ \frac{5}{3}\right)$

73. The line normal to the curve $y = x^2$ at $(\ 2,\ 4)$ intersects the curve at $x =$

A. -3 B. $-\frac{5}{2}$ C. $-\frac{9}{4}$ D. -2 E. $-\frac{3}{2}$

74. Find the value of x at which the normal to the curve $y = x^2 + 1$ at $x = 3$ intersects the curve again.

75. The line normal to the function $f(x) = 4 - x^2$ at $x = -1$ intersects the curve again. Find the value of the function at that point.

76.

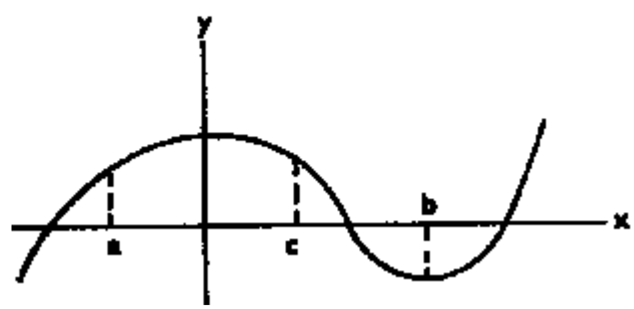
x	-4	-3	-2	-1	0	1	2	3	4
$g'(x)$	2	3	0	-3	-2	-1	0	3	2

The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals ?

A. $-2 \le x \le 2$ only B. $-1 \le x \le 1$ only C. $x \ge -2$
D. $x \ge 2$ only E. $x \le -2$ or $x \ge 2$

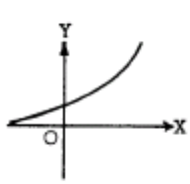
77. Given the function shown on the right, ***how many*** of the following statements are true ?

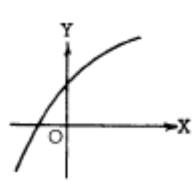
I. $f'(b) = 0$
II. $f''(a) < 0$
III. $f''(c) < 0$
IV. $f''(b) > 0$

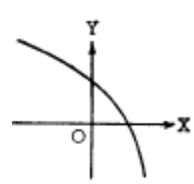


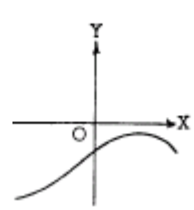
A. 0 B. 1 C. 2 D. 3 E. 4

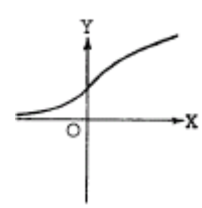
78. If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $f(x)$

A.


B.


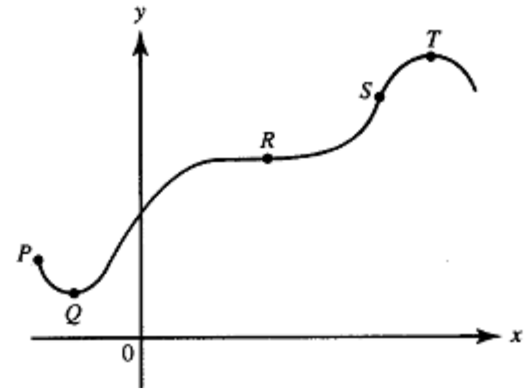
C.


D.


E.


79.

Use the graph on the right for this and the ***next two*** questions.



At which labelled point do both $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ equal zero ?

A. P B. Q C. R D. S E. T

80. At which labelled point is $\frac{dy}{dx}$ positive and $\frac{d^2y}{dx^2}$ equal to zero ?

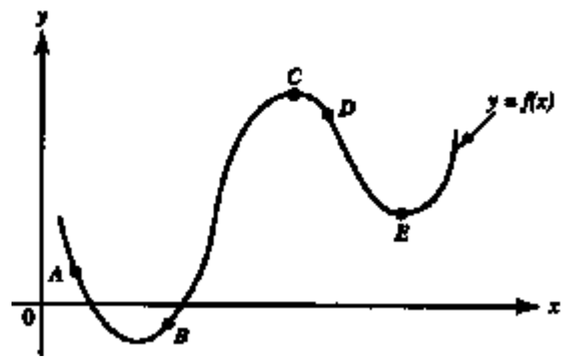
A. P B. Q C. R D. S E. T

81. At which labelled point is $\frac{dy}{dx}$ equal to zero and $\frac{d^2y}{dx^2}$ negative ?

A. P B. Q C. R D. S E. T

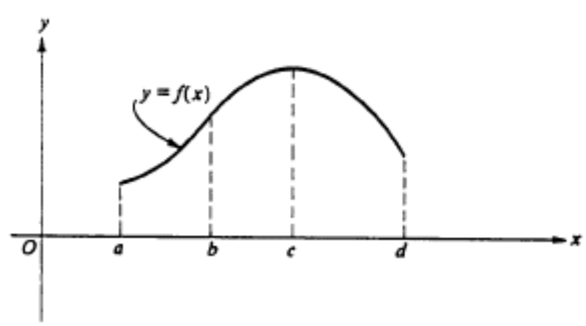
82.

At which point on the graph of $y = f(x)$ is $f'(x) < 0$ and $f''(x) > 0$



A. A B. B C. C D. D E. E

83. The graph of $y = f(x)$ is shown in the diagram. On which of the following intervals are $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$

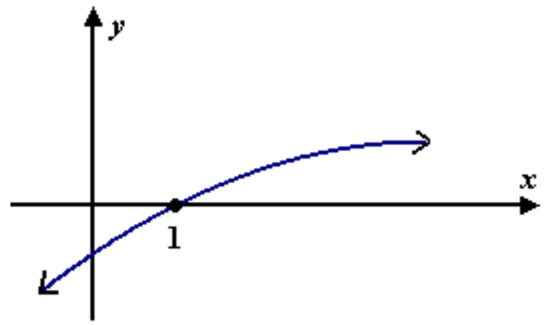


I. $a < x < b$ **II.** $b < x < c$ **III.** $c < x < d$

A. **I** ***only*** B. **II** ***only*** C. **III** ***only*** D. **I** ***and*** **II** E. **II** ***and*** **III**

84.

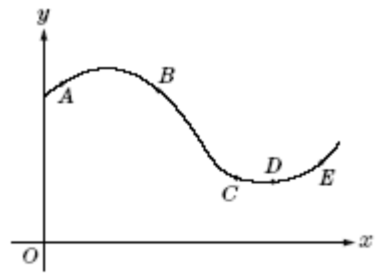
The graph of a twice-differentiable function f is shown in the figure on the right. Which of the following is true ?



A. $f(1) < f'(1) < f''(1)$ B. $f(1) < f''(1) < f'(1)$ C. $f'(1) < f(1) < f''(1)$
D. $f''(1) < f(1) < f'(1)$ E. $f''(1) < f'(1) < f(1)$

85.

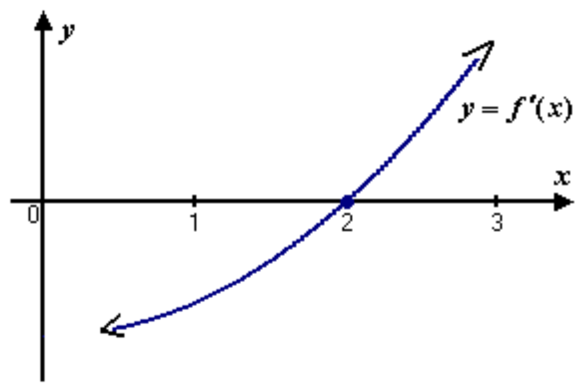
At which of the five points on the graph in the figure at the right are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both negative ?



A. **A** B. **B** C. **C** D. **D** E. **E**

86.

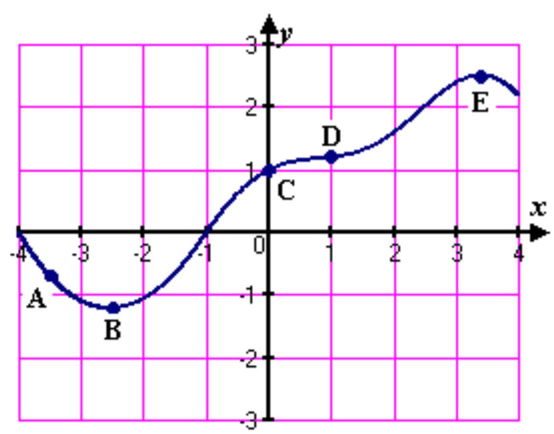
The graph of the derivative of a twice differentiable function f is shown in the graph. If $f(1) = -2$ which of the following is true ?



A. $f(2) < f'(2) < f''(2)$ B. $f''(2) < f'(2) < f(2)$ C. $f'(2) < f(2) < f''(2)$
D. $f(2) < f''(2) < f'(2)$ E. $f'(2) < f''(2) < f(2)$

87.

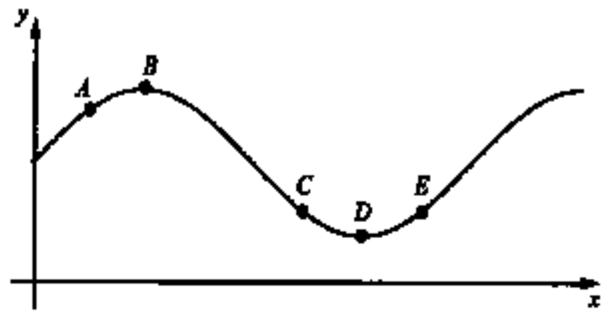
At which point on the graph of $y = g(x)$ on the right is $g'(x) = 0$ and $g''(x) < 0$



A. **A** B. **B** C. **C** D. **D** E. **E**

88.

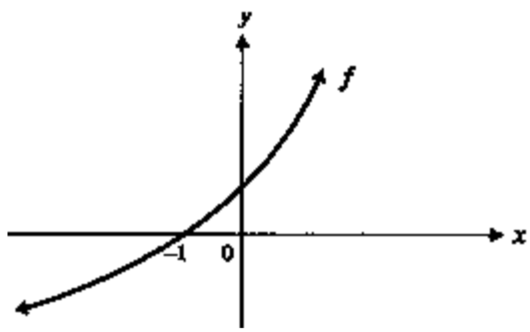
The graph of a function f is shown. At which of the marked points are both f' and f'' positive ?



A. **A** B. **B** C. **C** D. **D** E. **E**

89. The graph of f is shown in the diagram and f is twice differentiable. Which of the following has the smallest value ?

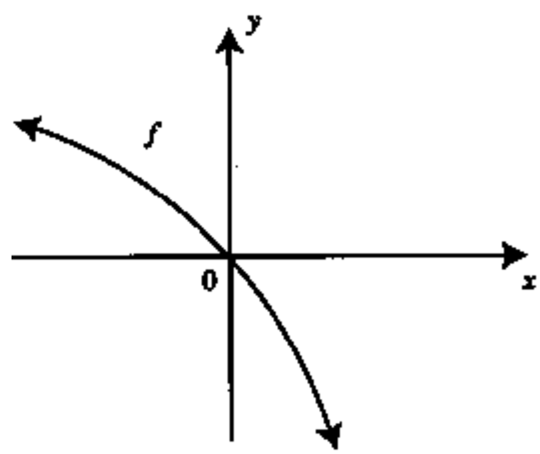
I. $f(-1)$
II. $f'(-1)$
III. $f''(-1)$



A. **I *only*** B. **II *only*** C. **III *only*** D. **I *and* II** E. **II *and* III**

90.

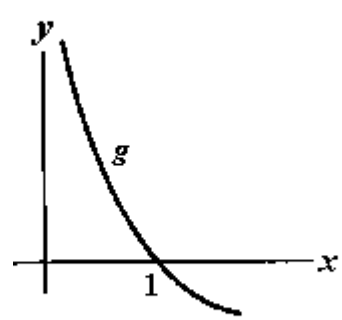
The graph of f is shown on the right and f is twice differentiable. Which of the following has the largest value $f(0)$, $f'(0)$ or $f''(0)$



A. $f(0)$ B. $f'(0)$ C. $f''(0)$
D. $f(0)$ and $f'(0)$ E. $f'(0)$ and $f''(0)$

91.

The graph of g, a twice-differentiable function is shown in the diagram. Choose the correct order for the values of $g(1)$, $g'(1)$ and $g''(1)$



A. $g(1) < g'(1) < g''(1)$ B. $g'(1) < g''(1) < g(1)$ C. $g''(1) < g(1) < g'(1)$
D. $g'(1) < g(1) < g''(1)$ E. cannot be determined

92.

Derivatives of	$y = e^u$	and	$y = \ln u$
	$y' = e^u \dfrac{du}{dx}$		$y' = \dfrac{1}{u}\dfrac{du}{dx} = \dfrac{u'}{u}$

93. If $f(x) = \ln x^3$ then $f''(3) =$

A. $-\dfrac{1}{3}$ B. -1 C. -3 D. 1 E. *none of these*

94. If $y = e^x(x-1)$ then $y''(0) =$

A. -2 B. -1 C. 0 D. 1 E. *none of these*

95. The domain of the function defined by $f(x) = \ln(x^2 - x - 6)$ is the set of all real numbers x such that

A. $x > 0$ B. $-2 \le x \le 3$ C. $-2 \le x$ *or* $x \ge 3$
D. $-2 < x < 3$ E. $-2 > x$ *or* $x > 3$

96. Find y' given $y = \ln\left(x\sqrt{x^2+1}\right)$

A. $1 + \dfrac{x}{x^2+1}$ B. $\dfrac{1}{x\sqrt{x^2+1}}$ C. $\dfrac{2x^2+1}{x\sqrt{x^2+1}}$ D. $\dfrac{2x^2+1}{x(x^2+1)}$ E. *none of these*

97. $\log_{\frac{1}{b}} x =$

A. $-\log_b x$ B. $\log_x b$ C. $-\log_x b$ D. b^x E. *none of these*

98. If $f(x) = 2e^x + e^{2x}$ then $f'''(0) =$

A. 10 B. 8 C. 6 D. 4 E. 3

99. If $e^{g(x)} = 2x + 1$ then $g'(x) =$

A. $\dfrac{1}{2x+1}$ B. $\dfrac{2}{2x+1}$ C. $2(2x+1)$ D. e^{2x+1} E. $\ln(2x+1)$

100. If $f(x) = (x+1)^{\frac{3}{2}} - e^{x^2-9}$ then $f'(3) =$

A. -5 B. -3 C. 0 D. 1 E. 3

101. Simplify: $\ln 2 + \ln 5 - \ln 8 - \ln 15 =$

A. $-\ln 12$ B. $\ln 12$ C. $\ln 7 - \ln 23$ D. $-\ln\left(\dfrac{1}{12}\right)$ E. $\ln(-16)$

102. Let $f(x) = \ln(x^2 - x - 6)$

a) the domain of $f(x)$ is

b) find $f(5)$

c) find $f'(-3)$

103. If $y = f(x) = x^3 + \ln x$ then $y' =$

A. $3x^2 \ln x + x^2$ B. $\frac{1}{4}x^4 + \frac{1}{x}$ C. $3x^2 + \frac{1}{x}$ D. $3x^2 + x\ln x$ E. *none of these*

104. Solve: $\log_9 x^2 = 9$

A. 1 B. 3^3 C. 3^9 D. $\pm 3^9$ E. 3^{18}

105. If $f(x) = x\ln x$, then $f'''(e) =$

A. $\frac{1}{e}$ B. 0 C. $-\frac{1}{e^2}$ D. $\frac{1}{e^2}$ E. $\frac{2}{e^3}$

106. If $e^{g(x)} = \frac{x^x}{x^2 - 1}$ then $g(x) =$

A. $x\ln x - 2x$ B. $\frac{\ln x}{2}$ C. $(x-2)\ln x$

D. $\frac{x\ln x}{\ln(x^2-1)}$ E. $x\ln x - \ln(x^2-1)$

107. If $\ln x - \ln\left(\frac{1}{x}\right) = 2$, then $x =$

A. $\frac{1}{e^2}$ B. $\frac{1}{e}$ C. e D. $2e$ E. e^2

108. If $y = x^2 e^x$ then $\frac{dy}{dx} =$

A. $2xe^x$ B. $x(x + 2e^x)$ C. $xe^x(x+2)$ D. $2x + e^x$ E. $2x + e$

109. If $y = \ln\left[(x+1)(x+2)\right]$, then $\frac{dy}{dx} =$

A. $\frac{1}{x+1} + (x+2)$ B. $\frac{1}{(x+2)} + (x+1)$ C. $\frac{1}{(x+1)(x+2)}$

D. $\frac{x+1}{x+2}$ E. $\frac{1}{x+1} + \frac{1}{x+2}$

110. Solve: $2x = 7^{1+\log_7 4}$

A. 3 B. 6 C. 7 D. 10 E. 14

111. What is x when $6 = e^{5x}$

A. $\frac{e^6}{5}$ B. $6 - \ln 5$ C. $5\ln 6$ D. $\frac{\ln 6}{5}$ E. $\frac{6}{e^5}$

112. $\ln_e 10 =$

A. $\ln_{10} e$ B. $\dfrac{1}{\ln_{10} e}$ C. e^{10} D. $\sqrt[10]{e}$ E. $10(\ln e)$

113. The tangent to the curve of $y = xe^{-x}$ is horizontal when x is equal to

A. 0 B. 1 C. -1 D. $\dfrac{1}{e}$ E. *none of these*

114. Find $\dfrac{dy}{dx}$ for $y = \ln\sqrt{x^2+4}$

A. $\dfrac{x}{\sqrt{x^2+4}}$ B. $\dfrac{2x}{\sqrt{x^2+4}}$ C. $\dfrac{x}{x^2+4}$ D. $\dfrac{1}{x}$ E. *none of these*

115. Find an equation for the tangent line to the graph of $f(x) = \ln(x^2-1)$ at the point where $x = 2$

A. $4x - 3y = 8$ B. $4x - y = 8 - \ln 3$ C. $4x - 3y = -1$

D. $4x - 3y = 8 - \ln 27$ E. *none of these*

116. If $f(x) = e^{-2x}$, then $f^{(4)}(x) =$

A. $16e^{-x}$ B. $16e^{-2x}$ C. $-8e^{-2x}$ D. $8e^{-2x}$ E. $-16e^{-2x}$

117. If $\log_b(3^b) = \dfrac{b}{2}$, then $b =$

A. $\dfrac{1}{9}$ B. $\dfrac{1}{3}$ C. $\dfrac{1}{2}$ D. 3 E. 9

118. Find $\dfrac{dy}{dx}$, if $y = x\ln^3 x$

A. $\dfrac{3\ln^2 x}{x}$ B. $3\ln^2 x$ C. $3x\ln^2 x + \ln^3 x$

D. $3(\ln x + 1)$ E. *none of these*

119. If $y = \dfrac{e^{\ln u}}{u}$, then $\dfrac{dy}{du} =$

A. $\dfrac{e^{\ln u}}{u^2}$ B. $e^{\ln u}$ C. $\dfrac{2e^{\ln u}}{u^2}$ D. 1 E. 0

120. What is the slope of the tangent line to the curve $y = \ln\dfrac{x^2}{\sqrt{x^2+1}}$ at the point where $x = 2$

A. 1 B. $-\dfrac{2}{3}$ C. $\dfrac{4}{7}$ D. $\dfrac{3}{5}$ E. $-\dfrac{2}{9}$

121. What is the slope of the tangent line to the curve $y = \ln(x^2+1)$ when $x = 3$

A. $\frac{1}{10}$ B. $\frac{3}{10}$ C. $\frac{1}{5}$ D. $\frac{1}{15}$ E. $\frac{3}{5}$

122. The derivative of $f(x) = \ln(x^2+2x+1)$ is

A. $\frac{2x}{x^2+2x+1}$ B. $\frac{2}{x+1}$ C. $\frac{1}{x^2+2x+1}$ D. $\frac{1}{x+1}$ E. $\frac{2x+3}{x^2+2x+1}$

123. If $f(x) = \ln(x+4+e^{-3x})$ then $f'(0) =$

A. $-\frac{2}{5}$ B. $\frac{1}{5}$ C. $\frac{1}{4}$ D. $\frac{2}{5}$ E. *nonexistent*

124. If $6y = 3e^{2x}$ then $y' =$

A. $\frac{1}{2}e^{2x}$ B. $3e^{x}$ C. $3e^{2x}$ D. $6e^{x}$ E. e^{2x}

125. If $f(x) = x^2 \ln x^3$ then $f'(x) =$

A. $3x + \ln x^3$ B. $3x(1+\ln x^2)$ C. $\frac{1}{x}$ D. $2x$ E. $2x\ln 3x^2$

126. If $y = e^{\frac{1}{2}\ln(x^2-4x+7)}$ then $\frac{dy}{dx} =$

A. $e^{\frac{1}{2}\ln(x^2-4x+7)}$ B. $x-2$ C. $\frac{1}{\sqrt{x^2-4x+7}}$

D. $\frac{x-2}{\sqrt{x^2-4x+7}}$ E. $(2x-4)e^{\frac{1}{2}\ln(x^2-4x+7)}$

127. Given the equation $y = 3e^{-2x}$ what is an equation of the normal line to the graph at $x = \ln 2$

A. $y = \frac{2}{3}(x-\ln 2)+\frac{3}{4}$ B. $y = \frac{2}{3}(x+\ln 2)-\frac{3}{4}$ C. $y = -\frac{3}{2}(x-\ln 2)+\frac{3}{4}$

D. $y = -\frac{3}{2}(x-\ln 2)-\frac{3}{4}$ E. $y = 24(x-\ln 2)+12$

128. The equation of the normal line to the graph of $y = e^{2x}$ at the point where $\frac{dy}{dx} = 2$ is

A. $y = -\frac{1}{2}x-1$ B. $y = -\frac{1}{2}x+1$ C. $y = 2x+1$

D. $y = -\frac{1}{2}\left(x-\frac{\ln 2}{2}\right)+2$ E. $y = 2\left(x-\frac{\ln 2}{2}\right)+2$

129. Find $\frac{dy}{dx}$ for $y = \ln(5-x)^6$

A. $\frac{1}{(5-x)^6}$ B. $\frac{6}{x-5}$ C. $-6(5-x)^5$ D. $6(5-x)^5$ E. *none of these*

130. The slope of the line tangent to the graph of $y = \ln(x^2)$ at $x = e^2$ is

A. $\frac{1}{e^2}$ B. $\frac{2}{e^2}$ C. $\frac{4}{e^2}$ D. $\frac{1}{e^4}$ E. $\frac{4}{e^4}$

131. If $\log_a 2^a = \frac{a}{4}$ then $a =$

A. 2 B. 4 C. 8 D. 16 E. 32

132. The slope of the line tangent to the graph of $y = \ln\left(\frac{x}{2}\right)$ at $x = 4$ is

A. $\frac{1}{8}$ B. $\frac{1}{4}$ C. $\frac{1}{2}$ D. 1 E. 4

133. If $f(x) = \log_b x$ and $g(x) = b^x$, then $f(g(x)) =$

A. 1 B. x C. x^b D. b^x E. $\log_x b$

134. Simplify: $\ln e^4 =$

A. 4 B. 10^4 C. $4(\ln 10)$ D. e^4 E. $4e$

135. If $y = \ln(x^x)$ then $y' =$

A. $1 + \ln x$ B. $y(1 + \ln x)$ C. $x + \ln x$ D. $y(x + \ln x)$ E. $x \ln x$

136. If $f(x) = x^2 \ln x$ then $f'(x) =$

A. 2 B. $x + 2\ln x$ C. $2x \ln x$ D. $1 + 2x\ln x$ E. $x + 2x\ln x$

137. Simplify: $2\ln e^{5x} =$

A. $10x$ B. $5x^2$ C. $25x^2$ D. e^{10x} E. e^{5x^2}

138. If $f(x) = e^{2x}$ and $g(x) = \ln x$ then the derivative of $y = f(g(x))$ at $x = e$ is

A. e^2 B. $2e^2$ C. $2e$ D. 2 E. *undefined*

139. If $f(x) = e^{2\ln x}$ then $f'(3) =$

A. 6 B. 9 C. e^6 D. e^9 E. $\frac{e^9}{9}$

140. If $y = e^{8x^2+1}$ then $\frac{dy}{dx} =$

A. e^{8x^2} B. e^{8x^2+1} C. $16xe^{8x^2}$ D. $16xe^{8x^2+1}$ E. $(8x^2+1)e^{8x^2}$

141. $\dfrac{d}{dx}\ln\left(\dfrac{1}{1-x}\right)=$

A. $\dfrac{1}{1-x}$ B. $\dfrac{1}{x-1}$ C. $1-x$ D. $x-1$ E. $(1-x)^2$

142. If $f(x)=x\ln(x^2)$ then $f'(x)=$

A. $\ln(x^2)+1$ B. $\ln(x^2)+2$ C. $\ln(x^2)+\dfrac{1}{x}$ D. $\dfrac{1}{x^2}$ E. $\dfrac{1}{x}$

143. $\dfrac{d}{dx}(\ln e^{3x})=$

A. 1 B. 3 C. $3x$ D. $\dfrac{1}{e^{3x}}$ E. $\dfrac{3}{e^{3x}}$

144. The slope of the line tangent to the graph of $y=\ln\sqrt{x}$ at $(e^2, 1)$ is

A. $\dfrac{e^2}{2}$ B. $\dfrac{2}{e^2}$ C. $\dfrac{1}{2e^2}$ D. $\dfrac{1}{2e}$ E. $\dfrac{1}{e}$

145. If $f(x)=e^{3\ln x^2}$ then $f'(x)=$

A. $e^{3\ln x^2}$ B. $\dfrac{3}{x^2}e^{3\ln x^2}$ C. $6(\ln x)e^{3\ln x^2}$ D. $5x^4$ E. $6x^5$

146. If $f(x)=\ln(x^x)$ then $f'(e^2)=$

A. 2 B. 3 C. $2e$ D. $3e^2$ E. *none of these*

147. If $y=e^{nx}$ then $\dfrac{d^n y}{dx^n}$ (the n^{th} derivative of y with respect to x) is

A. $n!e^x$ B. $n!e^{nx}$ C. ne^{nx} D. $n^n e^x$ E. $n^n e^{nx}$

148. The equation of the tangent to the curve $\ln y=3x^2+6x$ at the point where $x=0$ is

A. $y=6x+1$ B. $y=6x+6$ C. $y=6xy+6$ D. $y=\dfrac{6x}{y}+6$ E. *none of these*

149. If $y=x(\ln x)^2$ then $\dfrac{dy}{dx}=$

A. $3(\ln x)^2$ B. $(\ln x)(2x+\ln x)$ C. $(\ln x)(2+\ln x)$
D. $(\ln x)(2+x\ln x)$ E. $(\ln x)(1+\ln x)$

150. If $f(x)=3x\ln x$ then $f'(x)=$

A. $3+\ln(x^3)$ B. $1+\ln(x^3)$ C. $\dfrac{3}{x}+3\ln x$ D. $\dfrac{3}{x^2}$ E. $\dfrac{1}{x}$

151. $\frac{d}{dx}\ln\left(\frac{1}{x^2-1}\right)=$

A. $\frac{1}{x^2-1}$ B. $-\frac{2x}{x^2-1}$ C. $\frac{2x}{x^2-1}$ D. $2x^3-2x$ E. $2x-2x^3$

152. If $f(x)=\sqrt{e^{2x}+1}$ then $f'(0)=$

A. $-\frac{\sqrt{2}}{2}$ B. $\frac{\sqrt{2}}{4}$ C. $\frac{\sqrt{2}}{2}$ D. 1 E. $\sqrt{2}$

153. If $f(x)=e^x\ln x$ then $f'(e)=$

A. $e^{e-1}+e^e$ B. $e^{e+1}+e^e$ C. e^e+e D. $e^e+\frac{1}{e}$ E. e^{e-1}

154. If $y=\ln(3x+5)$ then $\frac{d^2y}{dx^2}=$

A. $\frac{3}{3x+5}$ B. $\frac{3}{(3x+5)^2}$ C. $\frac{9}{(3x+5)^2}$ D. $\frac{-9}{(3x+5)^2}$ E. $\frac{-3}{(3x+5)^2}$

155. The slope of the line normal to the curve $y=xe^x$ at $x=-1$ is

A. 0 B. $\frac{2}{e}$ C. $-\frac{e}{2}$ D. e E. *undefined*

156. If $x=\frac{1}{2}$ when $x=\log_y x$ then $y=$

A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. 1 D. 2 E. 4

157. If $f(x)=e^x$ and $g(x)=\frac{1}{x}$ then the derivative of $f(g(x))$, evaluated at $x=2$ is

A. $-\frac{\sqrt{e}}{2}$ B. $-\frac{\sqrt{e}}{4}$ C. $-\frac{e}{4}$ D. $\frac{e}{2}$ E. $\sqrt{e}$

158. If the function $f(x)=\ln(x^2-1)$ then $\frac{f(7)-f(5)}{f'(7)-f'(5)}=$

A. $-8\ln 2$ B. $-8\ln 24$ C. $-12\ln 2$ D. $-12\ln 24$ E. $-6\ln 24$

159. If $f(x)=x^e e^x$ then $f'(x)=$

A. $x^e e^x$ B. $x^{e-1}e^{x-1}$ C. $e^x(ex^e+x^e)$ D. $\frac{x^e(e+x)}{xe^x}$ E. $\frac{x^e e^x(x+e)}{x}$

160. If $y=x-1$ and $x>1$ then $\frac{d^2(\ln y)}{dx^2}=$

A. 0 B. $\frac{1}{x-1}$ C. $-\frac{1}{x-1}$ D. $\frac{1}{(x-1)^2}$ E. $-\frac{1}{(x-1)^2}$

161. The slope of the line ***normal*** to the curve $y = xe^{x^3}$ at $x = 1$ is

A. $-\frac{4}{e}$ B. $-\frac{e}{4}$ C. $-\frac{1}{4e}$ D. $\frac{4}{e}$ E. $4e$

162. If $f(x) = 1 + \ln(x+2)$ then $f^{-1}(x) =$

A. $e^{x-1} - 2$ B. $e^{x+1} - 2$ C. $e^{x-1} + 2$ D. $e^{x+1} + 2$ E. ***none of these***

163. If $f(x) = x \ln\sqrt{x}$ what is $f'(x) =$

A. $\ln\sqrt{x} + \frac{1}{2}$ B. $\ln\sqrt{x} + \frac{x}{2}$ C. $\ln\sqrt{x} + 1$ D. $\ln\sqrt{x} + \frac{\sqrt{x}}{2}$ E. $\ln\frac{1}{2\sqrt{x}}$

164. If $y = e^{4x^2}$ then $\frac{d(\ln y)}{dx} =$

A. $8x$ B. $4x^2$ C. $8xe^{4x^2}$ D. $8xe^{8x}$ E. $\frac{8x}{e^{4x^2}}$

165. If $f(x) = \ln(x^2 - e^{2x})$ then $f'(1) =$

A. 0 B. 1 C. 2 D. e E. ***undefined***

166. Write the equation of the line perpendicular to the tangent of the curve represented by the equation $y = e^{x+1}$ at $x = 0$

A. $y = -\frac{1}{e}x$ B. $y = -\frac{1}{e}x + e$ C. $y = ex + e$ D. $y = \frac{1}{e}x + e$ E. $y = ex$

167. The second derivative of $f(x) = \ln x$ at $x = 3$ is

A. $-\frac{1}{3}$ B. $-\frac{1}{9}$ C. $\frac{1}{9}$ D. $\frac{1}{3}$ E. $\frac{2}{3}$

168. Find the equation of the line tangent to $f(x) = 2x + 2e^x$ at $x = 0$

A. $y = 4x + 2$ B. $y = 2x + 2$ C. $y = 4x$ D. $y = 4x - 2$ E. $y = -\frac{1}{4}x + 2$

169. Find y'' for $y = x\ln x - 3x$

A. $\frac{1}{x} - 3$ B. $1 + \ln x$ C. $\ln x - 2$ D. $\frac{1}{x}$ E. $\frac{1}{x} - 2$

170. If $f(x) = e^{\frac{1}{x}}$ then $f'(x) =$

A. $-\frac{e^{\frac{1}{x}}}{x^2}$ B. $-e^{\frac{1}{x}}$ C. $\frac{e^{\frac{1}{x}}}{x}$ D. $\frac{e^{\frac{1}{x}}}{x^2}$ E. $\frac{1}{x}e^{\frac{1}{x}-1}$

171. If $f(x) = \ln\sqrt{x}$ then $f''(x) =$

A. $-\frac{2}{x^2}$ B. $-\frac{1}{2x^2}$ C. $-\frac{1}{2x}$ D. $-\frac{1}{2x^{\frac{3}{2}}}$ E. $\frac{2}{x^2}$

172. If $f(x)=(x-1)^{\frac{3}{2}}+\dfrac{e^{x-2}}{2}$ then $f'(2)=$

A. 1 B. $\dfrac{3}{2}$ C. 2 D. $\dfrac{7}{2}$ E. $\dfrac{3+e}{2}$

173. If $y=\ln(e^{-t^2}+10)$ then $\dfrac{dy}{dx}=$

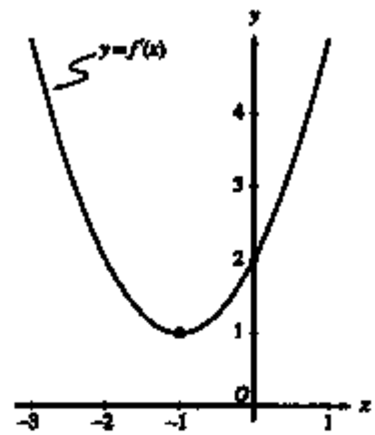
A. $-2t$ B. $\dfrac{1}{e^{-t^2}+10}$ C. $\dfrac{-2te^{-t^2}}{e^{-t^2}+10}$ D. $\dfrac{-2t}{e^{-t^2}+10}$ E. $-2t+\dfrac{1}{10}$

174. The function f defined by $f(x)=e^{3x}+6x^2+1$ has a horizontal tangent at $x=$

A. -0.144 B. -0.150 C. -0.156 D. -0.162 E. -0.168

175.

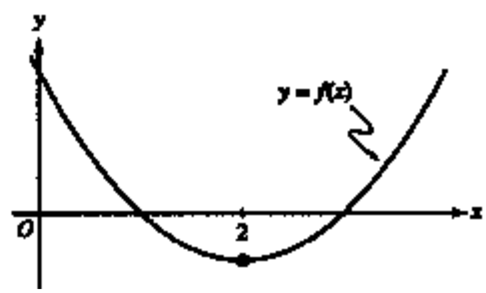
The graph of the derivative of the function f is shown in the diagram. If $f(0)=0$ then which of the following is true ?



A. $f(-1)<f'(-1)<f''(-1)$ B. $f(-1)<f''(-1)<f'(-1)$ C. $f'(-1)<f''(-1)<f(-1)$
D. $f''(-1)<f(-1)<f'(-1)$ E. $f''(-1)<f'(-1)<f(-1)$

176.

The graph of the twice differentiable function $f(x)$ is shown in the graph. Which of the following statements is true ?



A. $f(2)<f'(2)<f''(2)$ B. $f(2)<f''(2)<f'(2)$ C. $f'(2)<f(2)<f''(2)$
D. $f'(2)<f''(2)<f(2)$ E. $f''(2)<f(2)<f'(2)$

177. Simplify: $\dfrac{\ln 16}{3\ln 4-3\ln 2}=$

A. $\ln 2$ B. 2 C. $\dfrac{\ln 2}{\ln 8}$ D. $\dfrac{4}{3}$ E. *none of these*

178. Find the equation of the line perpendicular to the line tangent to $f(x)=\ln(3-2x)$ at $x=1$

A. $y=-2x+1$ B. $y=\frac{1}{2}x+1$ C. $y=\frac{1}{2}(x-1)$ D. $y=\frac{1}{2}(x+1)$ E. $y=-2x+2$

179. **Implicit Differentiation** $\rightarrow$ used when it is very difficult or impossible to isolate the variable y in terms of x Involves lots of chain rule/product rule operations.

180. If $xy + y = 3$ then $\frac{dy}{dx} =$

A. $\frac{-y}{1+x}$ B. 0 C. $\frac{3}{y}$ D. $\frac{3}{1+x}$ E. $-y$

181. If $x + y = xy$ then $\frac{dy}{dx} =$

A. $\frac{1}{x-1}$ B. $\frac{y-1}{x-1}$ C. $\frac{1-y}{x-1}$ D. $x+y-1$ E. $\frac{2-xy}{y}$

182. If $y^2 - 2xy = 16$ then $\frac{dy}{dx} =$

A. $\frac{x}{y-x}$ B. $\frac{y}{x-y}$ C. $\frac{y}{y-x}$ D. $\frac{y}{2y-x}$ E. $\frac{2y}{x-y}$

183. If $x^2 + xy + y^3 = 0$ then in terms of x and y, $\frac{dy}{dx} =$

A. $-\frac{2x+y}{x+3y^2}$ B. $-\frac{x+3y^2}{2x+y}$ C. $\frac{-2x}{1+3y^2}$ D. $\frac{-2x}{x+3y^2}$ E. $-\frac{2x+y}{x+3y^2-1}$

184. If $x^2 - 2xy + 3y^2 = 8$ then $\frac{dy}{dx} =$

A. $\frac{8+2y-2x}{6y-2x}$ B. $\frac{3y-x}{y-x}$ C. $\frac{2x-2y}{6y-2x}$ D. $\frac{1}{3}$ E. $\frac{y-x}{3y-x}$

185. Find $\frac{dy}{dx}$ if $x^2 + y^2 = -2xy$

A. 1 B. -1 C. $\frac{x-y}{x+y}$ D. $\frac{x+y}{x-y}$ E. $-\frac{x+2y}{x}$

186. Find y' if $y^2 - 3xy + x^2 = 7$

A. $\frac{2x+y}{3x-2y}$ B. $\frac{3y-2x}{2y-3x}$ C. $\frac{2x}{3-2y}$ D. $\frac{2x}{y}$ E. *none of these*

187. Given y is a differentiable function of x, find $\frac{dy}{dx}$ for $x^3 - xy + y^3 = 1$

A. $\frac{3x^2}{x-3y^2}$ B. $\frac{3x^2-1}{1-3y^2}$ C. $\frac{y-3x^2}{3y^2-x}$ D. $\frac{3x^2+3y^2-y}{x}$ E. $\frac{3x^2+3y^2}{x}$

188. If $y^2 = x + y^3$ then $y' =$

A. $1+3y^2$ B. $\dfrac{1}{2y-3y^2}$ C. $\dfrac{2x}{3-y^2}$ D. $\dfrac{1}{2y(1+y^2)}$ E. $\dfrac{1}{2y(1+3y^2)}$

189. Find $\dfrac{dy}{dx}$ for $2x^2 + xy + 3y^2 = 0$

A. $-\dfrac{4x+y}{x+6y}$ B. $-\dfrac{4x+y}{6y}$ C. $4x+y+6y$ D. $\dfrac{4x+6y}{-x}$ E. *none of these*

190. Given y is a differentiable function of x, find $\dfrac{dy}{dx}$ for $3x^2 - 2xy + 5y^2 = 1$

A. $\dfrac{3x+y}{x-5y}$ B. $\dfrac{y-3x}{5y-x}$ C. $3x+5y$ D. $\dfrac{3x+4y}{x}$ E. *none of these*

191. If $x^2 + y^3 = x^3y^2$ then $\dfrac{dy}{dx} =$

A. $\dfrac{2x+3y^2-3x^2y^2}{2x^3y}$ B. $\dfrac{2x^3y+3x^2y^2-2x}{3y^2}$ C. $\dfrac{3x^2y^2-2x}{3y^2-2x^3y}$

D. $\dfrac{3y^2-2x^3y}{3x^2y^2-2x}$ E. $\dfrac{6x^2y-2x}{3y^2}$

192. If $xy^2 - y^3 = x^2 - 5$ then $\dfrac{dy}{dx} =$

A. $\dfrac{y^2-2x}{3y^2-2xy}$ B. $\dfrac{y^2-2x+5}{3y^2-2xy}$ C. $\dfrac{2x-5}{2y-3y^2}$ D. $\dfrac{2x}{2y-3y^2}$ E. $\dfrac{x+y^2}{xy}$

193. If $x^3 + 3xy + 2y^3 = 17$ then in terms of x and y $\dfrac{dy}{dx} =$

A. $-\dfrac{x^2+y}{x+2y^2}$ B. $-\dfrac{x^2+y}{x+y^2}$ C. $-\dfrac{x^2+y}{x+2y}$ D. $-\dfrac{x^2+y}{2y^2}$ E. $-\dfrac{x^2}{1+2y^2}$

194. Find $\dfrac{dy}{dx}$ for $e^y = xy$

A. $\ln x + \ln y$ B. $\dfrac{x+y}{xy}$ C. $\dfrac{xy}{x+y}$ D. $\dfrac{xy-x}{y}$ E. $\dfrac{y}{xy-x}$

195. Find y' if $\ln xy = x + y$

A. $-\dfrac{y}{x}$ B. e^{x+y} C. $\dfrac{xy}{1-xy}$ D. $\dfrac{xy-y}{x-xy}$ E. *none of these*

196. Find y' if $xe^y + 1 = xy$

A. 0 B. $\dfrac{y-e^y}{xe^y-x}$ C. $\dfrac{y}{e^y-x}$ D. $\dfrac{e^y}{xe^y-1}$ E. *none of these*

197. Consider the curve $x + xy + 2y^2 = 6$ The slope of the line tangent to the curve at the point $(2, 1)$ is

A. $\frac{2}{3}$ B. $\frac{1}{3}$ C. $-\frac{1}{3}$ D. $-\frac{1}{5}$ E. $-\frac{3}{4}$

198. The equation of the tangent to the curve $2x^2 - y^4 = 1$ at the point $(-1, 1)$ is

A. $y = -x$ B. $y = 2 - x$ C. $4y + 5x + 1 = 0$
D. $x - 2y + 3 = 0$ E. $x - 4y + 5 = 0$

199. If $y^2 - 2xy = 21$ then $\frac{dy}{dx}$ at the point $(2, -3)$ is

A. $-\frac{6}{5}$ B. $-\frac{3}{5}$ C. $-\frac{2}{5}$ D. $\frac{3}{8}$ E. $\frac{3}{5}$

200. The slope of the curve $y^2 - xy - 3x = 1$ at the point $(0, -1)$ is

A. -1 B. -2 C. 1 D. 2 E. -3

201. What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point $(3, 2)$

A. 0 B. $\frac{4}{9}$ C. $\frac{7}{9}$ D. $\frac{6}{7}$ E. $\frac{5}{3}$

202. The slope of the line tangent to the graph of $3x^2 + 5\ln y = 12$ at $(2, 1)$ is

A. $-\frac{12}{5}$ B. $\frac{12}{5}$ C. $\frac{5}{12}$ D. 12 E. -7

203. If $y = \ln(x^2 + y^2)$ then the value of $\frac{dy}{dx}$ at the point $(1, 0)$ is

A. 0 B. $\frac{1}{2}$ C. 1 D. 2 E. *undefined*

204. Consider the curve $5x - xy + y^2 = 7$ The slope of the line tangent to the curve at the point $(1, 2)$ is

A. -2 B. -1 C. 0 D. 1 E. 2

205. If $y^2 + xy = 6$ what is $\frac{dy}{dx}$ at the point $(-1, 3)$

A. $-\frac{3}{5}$ B. $-\frac{3}{7}$ C. $\frac{3}{7}$ D. $\frac{3}{5}$ E. $\frac{6}{5}$

206. The equation of the line tangent to the curve $y^2 - 2x - 4y = 1$ at $(-2, 1)$ is

A. $y = -x - 1$ B. $-y = -x - 3$ C. $3y = -x + 1$ D. $5y = -x + 3$ E. *none of these*

207. If $xy^2 + 2xy = 8$ then at the point $(1, 2)$ $y' =$

A. $-\frac{5}{2}$ B. $-\frac{4}{3}$ C. -1 D. $-\frac{1}{2}$ E. 0

208. If $7 = xy - e^{xy}$ then $\frac{dy}{dx} =$

A. $x - e^y$ B. $y - e^x$ C. $\frac{ye^{xy} + y}{x - xe^{xy}}$ D. $\frac{-y}{x}$ E. $\frac{ye^{xy} + y}{x + xe^{xy}}$

209. Which is the slope of the line tangent to $y^2 + xy - x^2 = 11$ at $(2, 3)$

A. $-\frac{5}{2}$ B. 0 C. $\frac{1}{8}$ D. $\frac{4}{7}$ E. $\frac{9}{7}$

210. The slope of the line tangent to the curve $3x^2 - 2xy + y^2 = 11$ at the point $(1, -2)$ is

A. $-\frac{1}{6}$ B. 0 C. 1 D. $\frac{5}{3}$ E. 10

211. Find an equation of the tangent line to the graph of $x^2 + 2y^2 = 3$ at the point $(1, 1)$

A. $y - 1 = -\frac{x}{2y}(x - 1)$ B. $y + 1 = -\frac{1}{2}(x + 1)$ C. $y - 1 = \frac{1}{2}(x - 1)$

D. $x + 2y = 3$ E. *none of these*

212. Suppose $x^2 - xy + y^2 = 3$ Find $\frac{dy}{dx}$ at the point (a, b)

A. $\frac{a - 2b}{2a - b}$ B. $\frac{b - 2a}{2b - a}$ C. $\frac{a - 2b}{2a + b}$ D. $\frac{b - 2a}{2b + a}$ E. $\frac{b + 2a}{2b + a}$

213. If $(x - y)^2 = y^2 - xy$ then $\frac{dy}{dx} =$

A. $\frac{2x - y}{2y - x}$ B. $\frac{2x - y}{2x}$ C. $\frac{2x - y}{x}$ D. $\frac{2x + 3y}{x}$ E. *undefined*

214. The slope of the line tangent to the graph of $\ln(x + y) = x^2$ at the point where $x = 1$ is

A. 0 B. 1 C. $e - 1$ D. $2e - 1$ E. $e - 2$

215. The slope of the line tangent to the graph of $\ln(xy) = x$ at the point where $x = 1$ is

A. 0 B. 1 C. e D. e^2 E. $1 - e$

216. If $e^{xy} = \ln x$ then $\frac{dy}{dx} =$

A. $\frac{1}{e^{xy}}$ B. $\frac{1 - xye^{xy}}{x^2 e^{xy}}$ C. $\frac{1 - xy}{xe^{xy}}$ D. $\frac{xy - 1}{x^2 e^{xy}}$ E. $\frac{1 - xy}{x^2 e^{xy}}$

217. The curve defined by $x^3 + xy - y^2 = 10$ has a vertical tangent line when $x =$

A. $0 \text{ or } -\frac{1}{3}$ B. 1.037 C. 2.074 D. 2.096 E. 2.154

218. The slope of the line tangent to the curve $y^2 + (xy+1)^3 = 0$ at $(2, -1)$ is

A. $-\frac{3}{2}$ B. $-\frac{3}{4}$ C. 0 D. $\frac{3}{4}$ E. $\frac{3}{2}$

219. The curve $3y^2 - 3xy + 2x^3 = 7$ has vertical tangents when

A. $x = y$ B. $2x = y$ C. $x = 2y$ D. $3x = y$ E. $x = 3y$

220. If $e^{xy} = 2$ then at the point $(1, \ln 2)$ $\frac{dy}{dx} =$

A. $-\ln 2$ B. $2\ln 2$ C. $\ln 2$ D. $-2e$ E. $-4\ln 2$

221. The slope of $9x - 4x\ln y = 3$ at $\left(\frac{1}{3}, 1\right)$ is

A. $9 - 4\ln 3$ B. 5 C. 6 D. $\frac{27}{4}$ E. $9 + 4\ln 3$

222. If $2x^3 + 3xy + e^y = 6$ what is y' when $x = 0$

A. -0.896 B. 0.896 C. 1.792 D. -1.792 E. 0

223. If $y^2 - 3x = 7$ then $\frac{d^2y}{dx^2} =$

A. $\frac{-9}{4y^3}$ B. $\frac{3}{2y}$ C. 3 D. $\frac{-3}{y^3}$ E. $\frac{-6}{7y^3}$

224. If a point moves on the curve $x^2 + y^2 = 25$, then, at $(0, 5)$, $\frac{d^2y}{dx^2}$ is

A. 0 B. $\frac{1}{5}$ C. -5 D. $-\frac{1}{5}$ E. *nonexistent*

225. If $y^2 - 3x = 7$ then $\frac{d^2y}{dx^2} =$

A. $\frac{-6}{7y^3}$ B. $\frac{-3}{y^3}$ C. 3 D. $\frac{3}{2y}$ E. $\frac{-9}{4y^3}$

226. If $\frac{dy}{dx} = \sqrt{1-y^2}$, then $\frac{d^2y}{dx^2} =$

A. $-2y$ B. $-y$ C. $-\frac{y}{\sqrt{1-y^2}}$ D. y E. $\frac{1}{2}$

227.

The table gives values of f, f', g and g' at selected values of x If $h(x) = f(g(x))$ then $h'(1) =$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	6	5	3	-2
1	3	-3	-1	2
3	1	-2	2	3

A. 5 B. 6 C. 9 D. 10 E. 12

228. If $f(x) = \dfrac{4}{x-1}$ and $g(x) = 2x$ then the solution set of $f(g(x)) = g(f(x))$ is

A. $\dfrac{1}{3}$ B. 2 C. 3 D. $-1, 2$ E. $\dfrac{1}{3}, 2$

229. Let f and g be differentiable functions such that

$f(1) = 2$ $f'(1) = 3$ $f'(2) = -4$

$g(1) = 2$ $g'(1) = -3$ $g'(2) = 5$

If $h(x) = f(g(x))$ then $h'(1) =$

A. -9 B. -4 C. 0 D. 12 E. 15

230. If f and g are twice differentiable and if $h(x) = f(g(x))$, then $h''(x) =$

A. $f''(g(x))[g'(x)]^2 + f'(g(x))g''(x)$

B. $f''(g(x))g'(x) + f'(g(x))g''(x)$

C. $f''(g(x))[g'(x)]^2$

D. $f''(g(x))g''(x)$

E. $f''(g(x))$

231. Let f and g be differentiable functions such that

$f(1) = 4,\quad g(1) = 3,\quad f'(3) = -5$

$f'(1) = -4,\quad g'(1) = -3,\quad g'(3) = 2$

If $h(x) = f(g(x))$ then $h'(1) =$

A. -9 B. 15 C. 0 D. -5 E. -12

232.

The function $\mathbf{F}$ is defined by

$$\mathbf{F}(x) = G[x + G(x)]$$

where the graph of the function G is shown on the right.

The approximate value of $\mathbf{F}'(1) =$

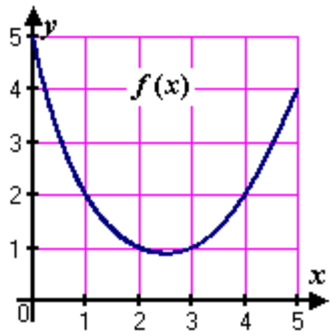
A. $\dfrac{7}{3}$ B. $\dfrac{2}{3}$ C. -2 D. -1 E. $-\dfrac{2}{3}$

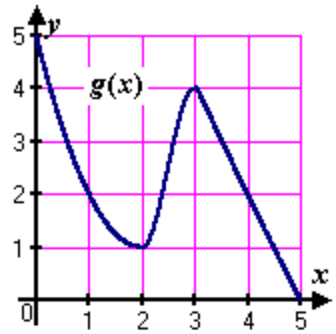
233. The graphs of functions f and g are shown on the right. If $h(x) = g[f(x)]$ which of the following statements are true about the function h

I. $h(0) = 4$

II. h is increasing at $x = 2$

III. The graph of h has a horizontal tangent at $x = 4$

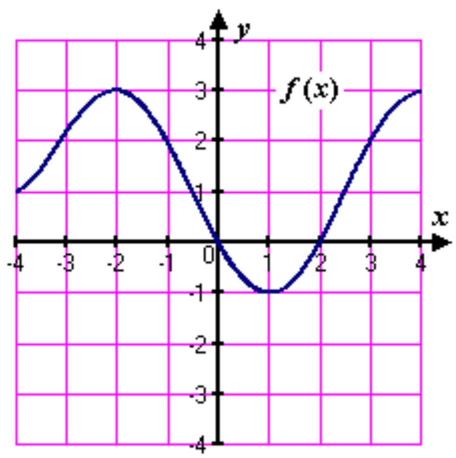


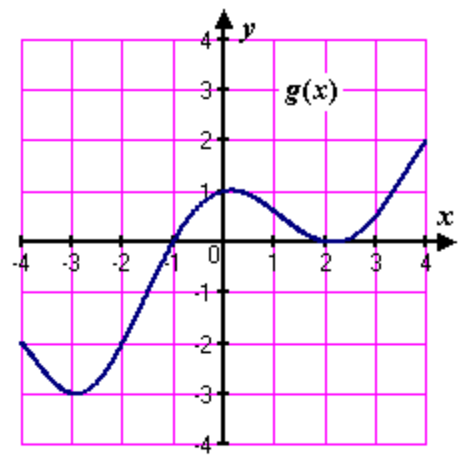


A. **I** *only* B. **II** *only* C. **I** *and* **II** *only*

D. **II** *and* **III** *only* E. **I, II** *and* **III**

234. The composite function h is defined by $h(x) = f[g(x)]$ where f and g are functions whose graphs are shown below.





The number of horizontal tangent lines to the graph of h is

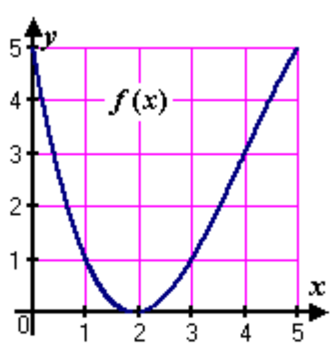
A. **3** B. **4** C. **5** D. **6** E. **7**

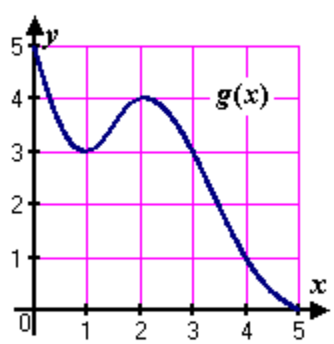
235. The graphs of functions f and g are shown at the right. If $h(x) = f[g(x)]$, which of the following statements are true about the function h

I. $h(2) = 5$

II. h is increasing at $x = 4$

III. The graph of h has a horizontal tangent at $x = 1$



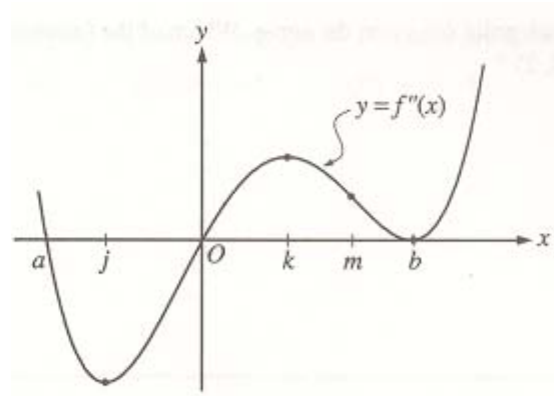


A. **I** *only* B. **II** *only* C. **III** *only*

D. **II** *and* **III** *only* E. **I, II** *and* **III**

236.

The second derivative of the function f is given by $f''(x) = x(x-a)(x-b)^2$ The graph of f'' is shown in the diagram. For what values of x does the graph of f have a point of inflection ?



A. 0 and a only B. 0 and m only C. b and j only D. 0, a and b E. b, j and k

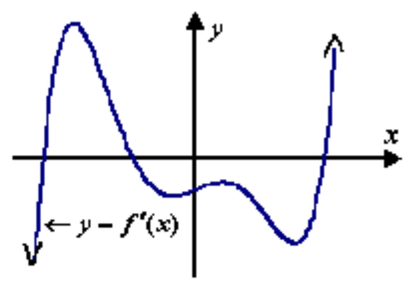
237.

The graph of $y = f(x)$ on the closed interval $[\,2,\ 7\,]$ is shown. How many points of inflection does this graph have on this interval ?

A. One B. Two C. Three D. Four E. Five

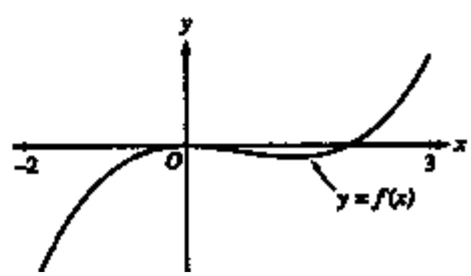
238.

The diagram shows the graph of the derivative of a function f How many points of inflection does f have in the interval shown ?



A. none B. one C. two D. three E. four

239. The function f is defined on the closed interval $[-2,\ 3]$ The graph of $y = f'(x)$ is shown in the diagram. Which of the following describes the relative extrema of f and the points of inflection of the graph of f



A. 1 relative minimum, 1 relative maximum and 1 point of inflection
B. 1 relative minimum and 2 points of inflection
C. 2 relative minima and 1 point of inflection
D. 1 relative minimum and 1 point of inflection
E. 1 relative maximum and 1 point of inflection

240. The function f is defined by $f'(x) = (x-2)^2(x-7)^3$ The graph of f has an inflection point where $x =$

A. 4 only B. 7 only C. 2 and 4 only D. 2 and 7 only E. $2,\ 4$ and 7

241. The function defined by $f(x)=(x-1)(x+2)^2$ has inflection points at $x=$

A. -2 only B. -1 only C. 0 only D. -2 and 0 only E. -2 and 1 only

242. For some key values of x, the values of $f(x)$, $f'(x)$, and $f''(x)$ are given in the table. The equation of the tangent to the curve $y=f(x)$ at the point of inflection shown in the table is:

x	-8	-6	-4	-2	0	2	4
$f(x)$	0	5	0	-2	-4	-6	-4
$f'(x)$	4	0	-4	-2	-1	0	1
$f''(x)$	-2	-6	-2	0	1	4	3

A. $y=4x$ B. $y=4x+8$ C. $y=-6x+24$ D. $y=-2x-6$ E. $y=-x+3$

243. Which of the following statements are true about the function f if its derivative f' is defined by $f'(x)=x(x-a)^3$ where $a>0$

I. The graph of f is increasing at $x=2a$

II. The function f has a local maximum at $x=0$

III. The graph of f has an inflection point at $x=a$

A. **I *only*** B. **I *and* II *only*** C. **I *and* III *only***
D. **II *and* III *only*** E. **I, II *and* III**

244. If $f'(x)=x^3(x+2)^2$ then the graph of f has inflection points when $x=$

A. -2 only B. 0 only C. -2 and 0 only
D. -2 and $-\frac{6}{5}$ only E. -2, $-\frac{6}{5}$ and 0

245. If $f'(x)=-5(x-3)^2(x-2)$ which of the following features does the graph of $f(x)$ have ?

A. a local minimum at $x=2$ and a local maximum at $x=3$
B. a local maximum at $x=2$ and a local minimum at $x=3$
C. a point of inflection at $x=2$ and a local minimum at $x=3$
D. a local minimum at $x=2$ and a point of inflection at $x=3$
E. a local maximum at $x=2$ and a point of inflection at $x=3$

246. A function $f(x)$ exists such that $f''(x)=(x-2)^2(x+1)$. How many points of inflection does $f(x)$ have ?

A. none B. one C. two
D. three E. cannot be determined

247. $f(x)=x^2-3x^3$ has a point of inflection at

A. $x=0$ B. $x=\frac{1}{9}$ C. $x=\frac{2}{9}$ D. $x=\frac{1}{3}$
E. There is no inflection point

248. The graph of $y=2x^3+5x^2-6x+7$ has a point of inflection at $x=$

A. $-\frac{5}{3}$ B. 0 C. $-\frac{5}{6}$ D. $\frac{5}{2}$ E. -2

249. The number of inflection points in the curve $f(x)=x^4-4x^2$ is

A. 0 B. 1 C. 2 D. 3 E. 4

250. An equation of the line tangent to $y=x^3+3x^2+2$ at it's point of inflection is

A. $y=-6x-6$ B. $y=-3x+1$ C. $y=2x+10$ D. $y=3x-1$ E. $y=4x+1$

251. If the graph of $y=x^3+ax^2+bx-4$ has a point of inflection at $(1,-6)$, what is the value of b

A. -3 B. 0 C. 1 D. 3

E. It cannot be determined from the information given

252. At what value of x does the graph of $y=\frac{1}{x^2}-\frac{1}{x^3}$ have a point of inflection ?

A. 0 B. 1 C. 2 D. 3 E. *at no value of* x

253. What is the value of k such that the curve $y=x^3-\frac{k}{x}$ has a point of inflection at $x=1$

A. $k=2$ B. $k=-2$ C. $k=3$ D. $k=-3$ E. *none of these*

254. The curve $y=x^5+10x^4-5$ has points of inflection at $x=$

A. 0 and -8 B. 0 and -6 C. -8 only D. -6 only E. 0 only

255. The curve $y=1-6x^2-x^4$ has inflection points at $x=$

A. $\pm\sqrt{3}$ B. 1 C. -1 D. ± 1 E. *none*

256. The slope of the line tangent to the curve $f(x)=x^3+3x^2-24x+4$ at the point of inflection is

A. -27 B. -15 C. 30 D. 32 E. *none of these*

257. The curve $y=3x^4-8x^3+6x^2-1$ has points of inflection at $x=$

A. 1 only B. -1 only C. -1 and $-\frac{1}{3}$ D. -1 and 1 E. 1 and $\frac{1}{3}$

258. The equation of the line tangent to the curve $f(x)=2x^3-3x^2$ at the point of inflection is

A. $y=0$ B. $y=x$ C. $y=-x$ D. $3x-2y=1$ E. $6x+4y=1$

259. An equation for the line tangent to the curve $f(x)=-x^3+12x+5$ at the point of inflection is

A. $12x-y=3$ B. $y-12x=3$ C. $12x-y=5$ D. $y-12x=5$ E. $12x-y=35$

260. The curve $y=3x^5-5x^4+3x-2$ has a point of inflection at

A. $(-1,-13)$ only B. $(0,-2)$ only C. $(1,-1)$ only

D. $(1,-1)$ and $(0,-2)$ E. *none of these*

261. If the graph of $f(x)=2x^2+\frac{k}{x}$ has a point of inflection at $x=-1$ then the value of k is

A. -2 B. -1 C. 0 D. 1 E. 2

262. The function $y=x^4+bx^2+8x+1$ has a horizontal tangent and a point of inflection for the same value of x What must be the value of b

A. -6 B. -1 C. 1 D. 4 E. 6

263. How many points of inflection does the graph of $y=2x^6+9x^5+10x^4-x+2$ have ?

A. none B. one C. two D. three E. four

264. If the graph of $y=x^3+ax^2+bx-8$ has a point of inflection at $(2, 0)$, what is the value of b

A. 0 B. 4 C. 8 D. 12

E. the value of b cannot be determined from the given information

265. What is the x-coordinate of the point of inflection on the graph of $y=xe^x$

A. -2 B. -1 C. 0 D. 1 E. 2

266. What is the x-coordinate of the point of inflection of the graph of $y=x^3+3x^2-45x+81$

A. -9 B. -5 C. -1 D. 1 E. 3

267. What are the x-coordinates of the points of inflection on the graph of the function $f(x)=3x^4-4x^3+6$

A. 0 only B. $\frac{2}{3}$ only C. 1 only D. 0 and $\frac{2}{3}$ E. 0 and 1

268. Given the function $h(x)=6x^3-8x^2+2$, at what x value(s) is/are the inflection point(s) ?

A. $x=\frac{4}{9}$ B. $x=0$ *and* $x=\frac{8}{9}$ C. $x=0$

D. $x=0$, $x=\frac{4}{9}$ *and* $x=\frac{8}{9}$ E. $x=0$ *and* $x=\frac{4}{9}$

269. How many inflection points does $3x^4-5x^3-9x+2$ have ?

A. 0 B. 1 C. 2 D. 3 E. 4

270. What is the x-coordinate of the point of inflection on the graph of $y=\frac{2}{3}x^3-2x^2+7$

A. -1 B. 1 C. 2 D. $\frac{13}{3}$ E. $\frac{17}{3}$

271. What is the x-coordinate of the point of inflection for the graph of $y=x^3+3x^2-1$

A. -2 B. -1 C. 0 D. 1 E. 2

272. A particle moves along the x-axis so that its position at time t is $x(t) = 2t^2 - 7t + 3$ (x in cm and t in seconds). What is the velocity (in cm/sec) at time $t = 2$ seconds ?

A. -6 B. -3 C. 1 D. 4 E. *none of these*

273. A particle moves along the x-axis according to the function $x(t) = t^2 - 4t + 3$, where x (metres) is the position of the particle at time t (seconds). At what time t does the particle have a velocity of $6\ m/s$

A. 1 B. 2 C. 5 D. 8 E. *none of these*

274. A particle moves along the x-axis so that its position at time t is $x(t) = 3t^3 + 2t^2 + 7$ where x is in meters and t is in seconds. Find the velocity at $t = 2$ seconds.

A. $26\ m/s$ B. $39\ m/s$ C. $42\ m/s$ D. $44\ m/s$ E. *none of these*

275. A particle moves along the x-axis according to the position function $x(t) = 2t^3 - 6t^2 + 9$ where x is in meters and t is in seconds. Find the value(s) of t when the particle is stationary.

A. $t = 0$ B. $t = 2$ C. $t = 0,\ t = -2$ D. $t = 0,\ t = 2$ E. *none of these*

276. A particle moves along the x-axis so that its position at time t is $x(t) = t^2 - 2t + 5$ where x is in centimeters and t is in seconds. At what time is the particle's velocity $4\ cm/s$

A. $t = 1$ B. $t = 3$ C. $t = 6$ D. $t = 13$ E. *none of these*

277. An object moves along the x-axis so that its position at time t is $x = t^2 - 3t + 5$ where x is in meters and t is in seconds. At what time(s) is its velocity $5\ m/s$

A. $t = 1$ B. $t = 4$ C. $t = 7$ D. $t = 0$ *or* 3 E. *none of these*

278. A particle moves along the x-axis according to the position function $x(t) = 2t^3 - 6t + 1$ where x is in meters and t is in seconds. For what values of t is the particle moving to the right ?

A. $-1 < t < 1$ B. $t < -1$ *or* $t > 1$ C. *all values of t* D. *no values of t*

279. The position of an object moving in a straight path is given by $x(t) = kt^2 + 12t$, where x is in meters and t is in seconds. Find the value of k if the velocity of the object is $4\ m/s$ when $t = 2$ seconds.

A. -12 B. -6 C. -3 D. -2 E. *none of these*

280. A particle moves along the x-axis according to the position function $x(t) = t^3 - 4t^2 + 3$ (x in meters, t in seconds). Determine the velocity in m/s at $t = -2$

A. -21 B. 4 C. 28 D. 31 E. *none of these*

281. A particle moves along the x-axis according to the position function $x(t) = t^2 - t$ (x in cm, t in sec). Determine the time t (in sec) when the velocity is $12\ cm/s$

A. 0.5 B. 4 C. 6.5 D. 23 E. *none of these*

282. As a particle moves along the x-axis, its distance from the origin is given by $x(t) = 3t^2 - 4t + 10$ where x is in meters and t is in seconds. At what time is the velocity $14\ m/s$

A. $\frac{2}{3}\ s$ B. $2\ s$ C. $3\ s$ D. $5\ s$ E. *none of these*

283. An object moves so that its distance in metres, at time t seconds, is given by $f(t)$. What does $f'(2)$ represent ?

A. the velocity at time $2\ s$
B. the time when the velocity is $2\ m/s$
C. the time when the distance is $2\ m$
D. the distance at time $2\ s$

284. As a particle moves along the x-axis, its distance from the origin is given by $x = t^2 - 6t + 5$ At what time t (in seconds) is the velocity of the particle zero ?

A. 2 *seconds* B. 3 *seconds* C. 5 *seconds* D. 6 *seconds* E. *none of these*

285. A particle moves along a line according to the distance function $s(t) = 2t^3 - 21t^2 + 60t + 13$ During the time interval from $t = 1$ to $t = 12$, how many times does the paticle reverse its direction of movement ?

A. 0 B. 1 C. 2 D. 3 E. 4

286. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 5$ At what time t is the particle at rest ?

A. $t = 1$ only
B. $t = 3$ only
C. $t = \frac{7}{2}$ only
D. $t = 3$ and $t = \frac{7}{2}$
E. $t = 3$ and $t = 4$

287. The position of a particle moving along a straight line at any time t is given by $s(t) = t^2 + 4t + 4$ What is the acceleration of the particle when $t = 4$

A. 0 B. 2 C. 4 D. 8 E. 12

288. A particle moves along the x-axis so that at any time t its position is given by $x(t) = te^{-2t}$ For what values of t is the particle at rest ?

A. no values B. 0 only C. $\frac{1}{2}$ only D. 1 only E. 0 and $\frac{1}{2}$

289. A particle moves along the x-axis so that at any time $t \geq 0$ its position is given by $x(t) = t^3 - 3t^2 - 9t + 1$ For what values of t is the particle at rest

A. *no values* B. 1 *only* C. 3 *only* D. 5 *only* E. 1 *and* 3

290. A particle starts at time $t=0$ and moves along a number line so that its position, at time $t \geq 0$ is given by $x(t)=(t-2)^3(t-6)$ The particle is moving to the right for

A. $0<t<5$ B. $2<t<6$ C. $t>5$ D. $t \geq 0$ E. *never*

291. The formula $x(t)=\ln t+\frac{t^2}{18}+1$ gives the position of an object moving along the x-axis during the time interval $1 \leq t \leq 5$ At the instant when the acceleration of the object is zero, the velocity is

A. 0 B. $\frac{1}{3}$ C. $\frac{2}{3}$ D. 1 E. *undefined*

292. Which of the following must be true about a particle that starts at $t=0$ and moves along a number line if its position at time t is given by $s(t)=(t-2)^3(t-6)$

I. The particle is moving to the right for $t>5$

II. The particle is at rest at $t=2$ and $t=6$

III. The particle changes direction at $t=2$

A. **I** *only* B. **II** *only* C. **III** *only*
D. **I** *and* **III** *only* E. *none*

293. A particle starts at time $t=0$ and moves along a number line so that its position, at time $t \geq 0$, is given by $x(t)=(t-2)(t-6)^3$ The particle is moving to the left for

A. $t>3$ B. $2<t<6$ C. $3<t<6$ D. $0 \leq t<3$ E. $t>6$

294. The position function of a moving particle on the x-axis is given as $s(t)=t^3+t^2-8t$ for $0 \leq t \leq 10$ For what values of t is the particle moving to the right ?

A. $t<-2$ B. $t>0$ C. $t<\frac{4}{3}$ D. $0<t<\frac{4}{3}$ E. $t>\frac{4}{3}$

295. A particle is moving along the x-axis. Its position at time $t>0$ is e^{2-t} What is its acceleration when $t=2$

A. e B. 1 C. 0 D. -1 E. $-e$

296. A particle is moving along the x-axis. Its position at time $t>0$ is $\ln(2t^{\frac{3}{2}}+1)$ What is its speed when $t=4$

A. **2.01** B. **3.06** C. **0.353** D. **4.63** E. **7.81**

297. A particle moves along the x-axis so that its position at time t is given by $x(t)=4t^3-33t^2+30t+12$, where t is measured in seconds and x is measured in meters.

a) Determine the velocity, in m/s, of the particle at time $t=2$ seconds

b) Determine the time(s), in seconds, when the particle is stationary

298. A particle moves along the x-axis such that its distance from the origin is given by $x(t)=2t^2+60t$ where x is in centimeters and t is in seconds. When the particle's velocity is $72\ cm/\sec$, determine its distance $x(t)$ from the origin.

299. A particle moves along the x-axis so that its position at time t is $x(t)=4t^3-21t^2+30t$ where t is measured in seconds, and x is measured in meters.

a) Determine the time(s) when the particle is stopped.

b) Determine when the particle is moving to the left

300. A particle moves along the x-axis so that its position at time t is $x(t)=2t^3-5t^2-4t+3$ (x in cm and t in seconds.)

a) At what time(s) is the particle stationary ?

b) At what time(s) is the particle moving to the left ?

301. Given the function $f(x)=x^3-3x+5$ determine

a) the equation of the tangent line at $x=2$

b) the x-values where the slope of the tangent line is equal to 0

302. A particle moves along the x-axis so that its position at time t is $x(t)=2t^3-9t^2+12t$ (x in cm and t in seconds)

a) Determine the time(s) when the particle is stopped

b) Determine the velocity of the particle at time $t=3$ seconds

303. A particle moves along the x-axis so that its position at time t is $x(t)=4t^3-21t^2+18t+3$ where t is measured in seconds and x is measured in meters.

a) Determine an equation for the velocity function

b) Determine the velocity at time $t=2$

c) Determine the time(s) when the particle is stationary

304. A particle moves along the x-axis in such a way that its position at time t is given by $x(t)=3t^4-16t^3+24t^2$ for $-5\le t\le 5$

a) Determine the velocity and acceleration of the particle at time t

b) At what values of t is the particle at rest ?

c) At what values of t does the particle change direction ?

d) What is the velocity when the acceleration is first zero ?

305. A particle moves along the x-axis in such a way that its position at time t for $t\ge 0$ is given by $x(t)=\frac{1}{3}t^3-3t^2+8t$

a) Show that at time $t=0$, the particle is moving to the right.

b) Find all values of t for which the particle is moving to the left.

c) What is the position of the particle at time $t=3$

d) When $t=3$, what is the total distance the particle has traveled ?

AB Calculus Jan 6/2009   Name: ______________________________

1. *Answer is C.*

Difficulty = **0.96 K**

Find the derivative of $f(x) = 4x^2 + 7x - 5$

$$f'(x) = 8x + 7$$

2. *Answer is D.*

Difficulty = **0.95 U**

Given $f(x) = 3x^2 - 4x + 5$ find $f'(x)$

$$f'(x) = \boxed{6x - 4}$$

3. *Answer is C.*

Difficulty = **0.93 U**

If $y = 3x^3 - 4x^2 + 5$ find $\frac{dy}{dx}$

$$\frac{dy}{dx} = 3(3x^2) - 4(2x) + 0 = \boxed{9x^2 - 8x}$$

4. *Answer is A.*

Difficulty = **0.89 U**

Find $\frac{dy}{dx}$ if $y = -3x^2 + 6x$

$$\frac{dy}{dx} = -3(2x^1) + 6(1x^0) = \boxed{-6x^2 + 6}$$

5. *Answer is A.*

Difficulty = **0.84 K**

Find $f'(x)$ if $f(x) = 3$

$$f'(x) = \boxed{0}$$

6. *Answer is A.*

Difficulty = **0.79 K**

Given that r is any real number, determine $\frac{d}{dx}(x^r)$

$$\frac{dy}{dx} = \boxed{rx^{r-1}}$$

7. *Answer is A.*

Difficulty = **0.69 U**

If $f(x)=\dfrac{3}{x}$ then $f'(x)=$

$$f(x)=\frac{3}{x}=3x^{-1}$$

$$f(x)=3(-1x^{-2})=\boxed{-\frac{3}{x^2}}$$

8. *Answer is A.*

Difficulty = **0.64 U**

Find $\dfrac{dy}{dx}$ if $y=2\sqrt{x}$

$$y=2\sqrt{x}=2x^{\frac{1}{2}}$$

$$y=2(\tfrac{1}{2}x^{-\frac{1}{2}})=\boxed{\frac{1}{\sqrt{x}}}$$

9. *Answer is C.*

Difficulty = **0.63 U**

If $f(x)=\sqrt{x}$ determine the value of $f'(x)$ at $(16,\ 4)$

$$f(x)=\sqrt{x}=x^{\frac{1}{2}}$$

$$f'(x)=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}}$$

$$f'(16)=\frac{1}{2\sqrt{16}}=\boxed{\frac{1}{8}}$$

10. *Answer is D.*

Difficulty = **0.46 H**

If $f(x)=k\sqrt{x}$ determine the value of the constant k so that $f'(4)=6$

$$f(x)=k\sqrt{x}=kx^{\frac{1}{2}}$$

$$f'(x)=k(\tfrac{1}{2}x^{-\frac{1}{2}})=\frac{k}{2\sqrt{x}}$$

$$f'(4)=\frac{k}{2\sqrt{4}}=6 \qquad \leftarrow f'(4)=6$$

$$\frac{k}{4}=6$$

$$\boxed{k=24}$$

11. *Answer is C.*

For the curve $y = x^k$ $(k \neq 0)$, the slope of the tangent is equal to $16k$ when $x = 2$
Determine the value of k

$$y' = kx^{k-1}$$
$$y'(2) = k(2)^{k-1} = 16k$$
$$(2)^{k-1} = 16$$
$$(2)^{k-1} = 2^4$$
$$k - 1 = 4$$
$$\boxed{k = 5}$$

12. *Answer is B.*

Given $f(x) = \dfrac{5}{x^2}$ determine $f'(x)$

$$f(x) = \frac{5}{x^2} = 5x^{-2}$$
$$f'(x) = 5(-2x^{-3}) = \boxed{-\frac{10}{x^3}}$$

13. *Answer is B.*

Given $y = \dfrac{1}{x^3}$ determine $\dfrac{dy}{dx}$

$$y = \frac{1}{x^3} = x^{-3}$$
$$\frac{dy}{dx} = -3x^{-4} = \boxed{-\frac{3}{x^4}}$$

14. *Answer is B.*

Find y' if $y = x^{\frac{3}{2}}$

$$y' = \frac{3}{2}x^{\frac{3}{2}-\frac{2}{2}} = \boxed{\frac{3}{2}x^{\frac{1}{2}}}$$

15. *Answer is A.*

Which of the following represents the ***slope*** of the tangent to $f(x)$ at $x = 2$

slope of the tangent at ***any*** point x is $f'(x)$
slope of the tangent at $x = 2$ is $f'(2)$

16. *Answer is A.*

Given $f(x)=\frac{1}{x}$ determine $f'(x)$

$$f(x)=\frac{1}{x}=x^{-1}$$

$$f'(x)=-1x^{-2}=\boxed{-\frac{1}{x^2}}$$

17. *Answer is A.*

If $y=7$ determine $\frac{dy}{dx}$

$$\frac{dy}{dx}=\boxed{0}$$

18. *Answer is A.*

Evaluate the derivative of the function $f(x)=3x^2-2x-1$ at the point where $x=0$

$$f'(x)=3(2x)-2-0=6x-2$$

$$f'(0)=6(0)-2=\boxed{-2}$$

19. *Answer is D.*

Evaluate the derivative of $f(x)=2x^2-3x+2$ at the point where $x=2$

$$f'(x)=2(2x)-3=4x-3$$

$$f'(2)=4(2)-3=\boxed{5}$$

20. *Answer is D.*

Given $f(x)=(2x-3)^2$ determine $f'(x)$

$f(x)=4x^2-12x+9$ ← or use ***chain*** rule if you know it

$$f'(x)=\boxed{8x-12}$$

21. *Answer is A.*

Given the function $f(x)=\sqrt{2}$ determine $f'(x)$

$$f'(x)=\boxed{0}$$

22. *Answer is A.*

If $f(x)=6g(x)$ then $f'(x)$ equals

$$f'(x)=6(g'(x))=\boxed{6g'(x)}$$

23. *Answer is C.*

For what condition is $f(x)$ increasing ?

$y = f(x)$ is ***increasing*** $\Rightarrow$ $f'(x)$ is ***positive*** $\leftarrow$ **MUST** know !!!

24. *Answer is B.*

Difficulty = **0.72 U**

Find k such that the function $f(x) = kx^2 + 12x - 4$ has a $\underbrace{\textit{critical point}}_{f'(x)=0}$ at $x = 4$

$$f'(x) = 2kx + 12$$
$$f'(4) = 2k(4) + 12 = 0$$
$$8k = -12$$
$$k = \frac{-12}{8} = \boxed{-\frac{3}{2}}$$

25. *Answer is C.*

Difficulty = **0.58 U**

Determine all values of x such that the function $f(x) = x^3 - 3x^2 + 5$ is $\underbrace{\text{decreasing}}_{f'(x)=negative}$.

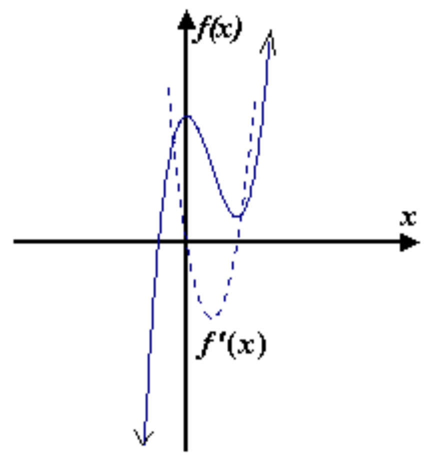
$f'(x) = 3x^2 - 6x = 0$ (parabola opening up)

$3x(x-2) = 0$

$x = 0 \mid x = 2$ $\leftarrow$ zeros of $f'(x)$

$f'(x)$ is ***negative*** on interval $0 < x < 2$ so

$f(x)$ is ***decreasing*** on the interval $0 < x < 2$



26. *Answer is B.*

Difficulty = **0.56 U**

Find the x-value of the point on the graph of $y = x^2 - x$ where the $\underbrace{\textit{slope of the tangent}}_{y'=2}$ is **2**

$y' = 2x - 1 = 2$

$2x = 3$

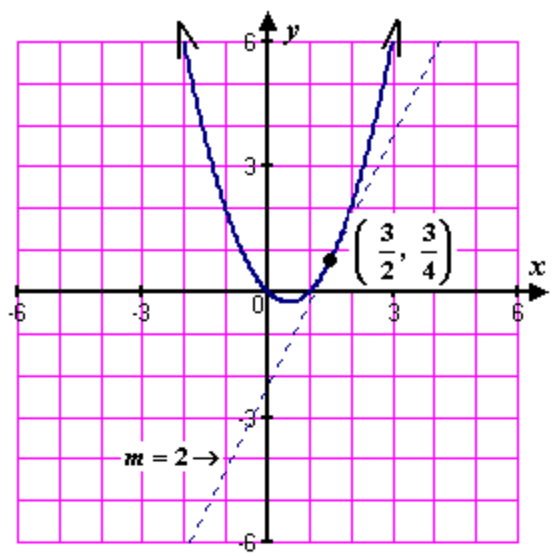
$\boxed{x = \frac{3}{2}}$

$y = x^2 - x$

$y(\frac{3}{2}) = (\frac{3}{2})^2 - (\frac{3}{2})$

$y(\frac{3}{2}) = \frac{9}{4} - \frac{6}{4} = \frac{3}{4}$

At the ***point*** $(\underbrace{\tfrac{3}{2}}_{x}, \tfrac{3}{4})$ the slope of the tangent $m = 2$



27. *Answer is D.*

Difficulty = **0.54 U**

> Find all values of x such that the function $f(x) = 2x^3 - 3x^2$ is $\underbrace{\textit{increasing}}_{f'(x)>0}$

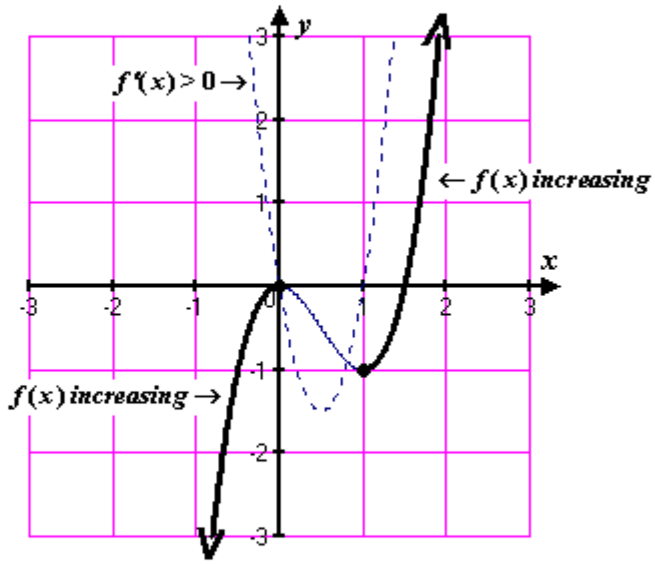
$$f'(x) = 6x^2 - 6x = 0 \quad \leftarrow \text{opening } \textit{up}$$

$$6x(x-1) = 0$$

$$x = 0 \quad | \quad x = 1 \quad \leftarrow \text{zeros of } f'(x)$$

Function is increasing if $f'(x) > 0$

$$\overbrace{\underbrace{x < 0}_{f(x)\ \textit{increasing}}}^{f'(x)>0} \quad \textit{or} \quad \overbrace{\underbrace{x > 1}_{f(x)\ \textit{increasing}}}^{f'(x)>0}$$



28. *Answer is D.*

Difficulty = **0.53 U**

> Give all values of x where the function $f(x) = x^3 - 3x + 4$ is increasing

$$f'(x) = 3x^2 - 3 = 0 \quad \leftarrow \text{opening } \textit{up}$$

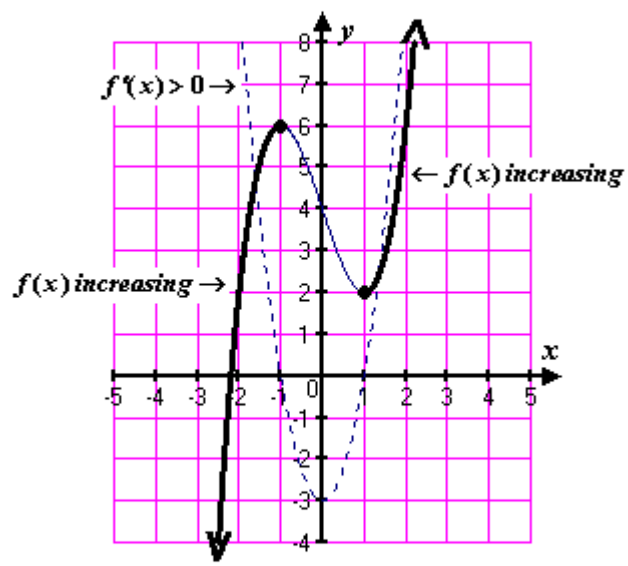
$$3(x^2 - 1) = 0$$

$$3(x-1)(x+1) = 0$$

$$x = 1 \quad | \quad x = -1 \quad \leftarrow \text{zeros of } f'(x)$$

Function is increasing if $f'(x) > 0$

$$\overbrace{\underbrace{x < -1}_{f(x)\ \textit{increasing}}}^{f'(x)>0} \quad \textit{or} \quad \overbrace{\underbrace{x > 1}_{f(x)\ \textit{increasing}}}^{f'(x)>0}$$



29. *Answer is C.*

Difficulty = **0.50 U**

At which of the following values of x is the function $g(x)=x^3-4x^2$ ***decreasing*** ?

$g'(x)=3x^2-8x=0$ ← opening ***up***

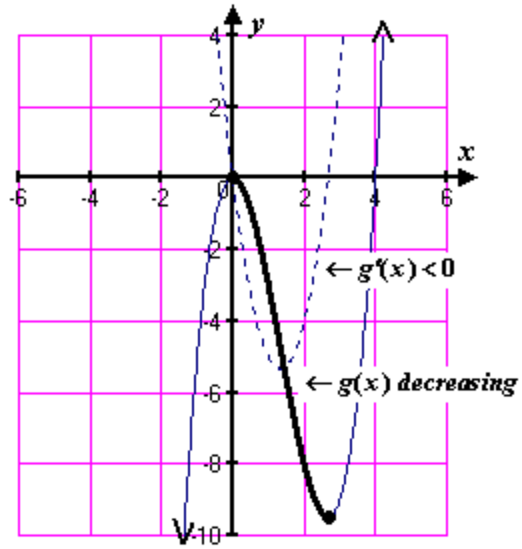
$x(3x-8)=0$

$x=0 \quad | \quad x=\frac{8}{3}$ ← zeros of $g'(x)$

Function is ***decreasing*** if $g'(x)<0$

$$\overbrace{\underbrace{0<x<\tfrac{8}{3}}_{g(x)\ decreasing}}^{g'(x)<0}$$

$\boxed{x=2}$ is only point where $g(x)$ ***decreasing***



30. *Answer is A.*

Difficulty = **0.45 H**

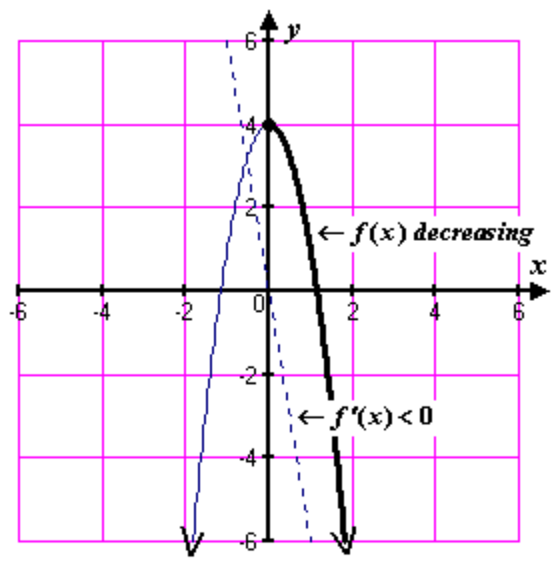
If $f'(x)=-6x$ determine all values of x such that $f(x)$ is $\underbrace{\textit{decreasing}}_{f'(x)<0}$

Function is ***decreasing*** if $f'(x)<0$

Example $f(x)=-3x^2+4$

$f'(x)=-6x<0$ ← divide by ***negative*** **6** and change direction of inequality

$$\overbrace{\underbrace{\boxed{x>0}}_{f(x)\ decreasing}}^{f'(x)<0}$$



31. *Answer is A.*

Determine the x-values of the critical points for the function $f(x)=x^3+3x^2-24x$

$f(x)=x^3+3x^2-24x$

$f'(x)=3x^2+6x-24=0$

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x=-4 \quad | \quad x=2$ ← x-values of the critical points

32. **Determine all values of x such that the function $f(x) = x^4 - 18x^2 + 8$ is *decreasing*.**

$$f(x) = x^4 - 18x^2 + 8$$

$$f'(x) = 4x^3 - 36x = 0$$

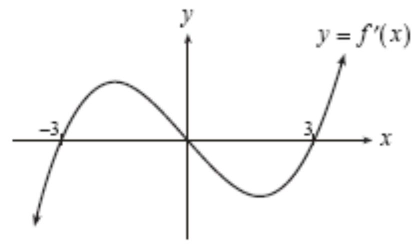
$$4x(x^2 - 9) = 0$$

$$4x(x-3)(x+3) = 0$$

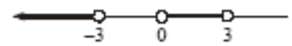
$x = -3$ | $x = 0$ | $x = 3$ ← ***critical numbers***

$$f'(x) = 4x(x-3)(x+3) < 0$$

Graph of $f'(x)$



$x < -3$ **or** $0 < x < 3$



33. **Determine all values of x such that the function $f(x) = x^4 - 8x^2 - 9$ is increasing**

$$f'(x) = 4x^3 - 16x = 0$$

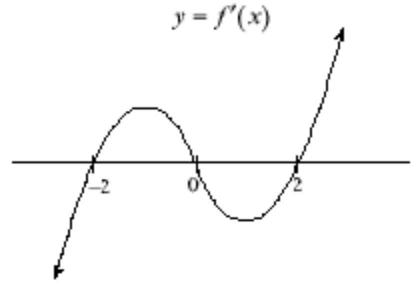
$$4x(x^2 - 4) = 0$$

$$4x(x-2)(x+2) = 0$$

$\boxed{x = 0}$ | $\boxed{x = 2}$ | $\boxed{x = -2}$ ← ***critical*** numbers

$f'(x) = 4x(x-2)(x+2) > 0$ ← $f(x)$ increasing

$y = f'(x)$



∴ $f(x)$ is increasing on

or

$-2 < x < 0$ or $x > 2$

34.

a) Determine the x values of the critical points of $f(x) = x^4 - 8x^2$

$$f'(x) = 4x^3 - 16x = 0$$

$$4x(x^2 - 4) = 0$$

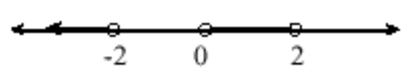
$$4x(x-2)(x+2) = 0$$

$\boxed{x = 0}$ | $\boxed{x = 2}$ | $\boxed{x = -2}$

b) For what values of x is $f(x) = x^4 - 8x^2$ **decreasing**?

$$f'(x) = 4x(x-2)(x+2) < 0$$

$f(x)$ is decreasing for:



$x < -2$ or $0 < x < 2$

or

$-2 < x < 0$ or $x > 2$

35. Given the function $f(x)=2x^3-3x^2-12x+4$, *a*) determine the coordinates of the critical points

$$f'(x)=6x^2-6x-12=0$$
$$x^2-x-2=0$$
$$(x-2)(x+1)=0$$
$\boxed{x=2}$ | $\boxed{x=-1}$ ← critical ***numbers***

$$f(2)=2(2)^3-3(2)^2-12(2)+4=-16$$
$$f(-1)=2(-1)^3-3(-1)^2-12(-1)+4=11$$

Critical ***points*** are $(2,-16)$ and $(-1,\ 11)$

b) determine where $f(x)$ is increasing.

Sketch graph $f'(x)=x^2-x-2$

$f(x)$ is increasing whenever

$$f'(x)=x^2-x-2>0$$
$x<-1$ or $x>2$

36. *Answer is C.*

Difficulty = **0.49 U**

For the function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-6x$, find the x-coordinate of the critical point where the local ***minimum*** point occurs.

$$f'(x)=\frac{1}{3}(3x^2)+\frac{1}{2}(2x)-6(1)$$
$$f'(x)=x^2+x-6=0$$
$$(x+3)(x-2)=0$$
$x=-3$ | $x=2$ ← critical numbers

$f''(x)=2x+1$ (at ***critical*** numbers)

$f''(-3)=2(-3)+1=-5$

At $x=-3$ $f(x)$ is ***concave*** down → Max

$f''(2)=2(2)+1=+5$

At $\boxed{x=2}$ $f(x)$ is ***concave*** up → **Min**

37. *Answer is B.*

Difficulty = **0.46 U**

Find the minimum value of the function $f(x)=2x^2-12x+6$

$$f(x)=2x^2-12x+6$$
$$f'(x)=4x-12=0$$
$$4x=12$$
Critical number → $x=3$

$f''(x)=4$ (positive)

∴ at critical number $x=3$ there is a ***minimum*** (concave up)

$$f(x)=2x^2-12x+6$$
$$f(3)=2(3)^2-12(3)+6$$
$f(3)=\boxed{-12}$ ← minimum value

38. *Answer is B.*

Difficulty = **0.45 U**

Determine the ***minimum*** value of the function $f(x)=3x^2-12x+13$

$$f'(x)=6x-12=0$$
$$6x=12$$
Critical number → $x=2$

$f''(x)=6$ (positive)

∴ concave up

$$f(x)=3x^2-12x+13$$
$f(2)=3(2)^2-12(2)+13=\boxed{1}$ ← minimum value

Vertex of parabola $(2,\ 1)$

Parabola opens ***up*** so minimum value is **1**

39. *Answer is C.*

Difficulty = **0.43 U**

Determine the ***minimum*** value of the function $g(x) = 2x^2 - 12x + 25$

$f'(x) = 4x - 12 = 0$

$4x = 12$

Critical number $\rightarrow x = 3$

$f''(x) = 4$ (positive)

$\therefore$ concave up

$f(x) = 2x^2 - 12x + 25$

$f(3) = 2(3)^2 - 12(3) + 25 = \boxed{7} \leftarrow$ minimum value

Vertex of parabola $(3, 7)$

Parabola opens ***up*** so minimum value is **7**

40. *Answer is B.*

Difficulty = **0.43 U**

Determine the ***minimum*** value of the function $y = 3x^2 - 24x - 7$

$f'(x) = 6x - 24 = 0$

$6x = 24$

Critical number $\rightarrow x = 4$

$f''(x) = 6$ (positive)

$\therefore$ concave up

$f(x) = 3x^2 - 24x - 7$

$f(4) = 3(4)^2 - 24(4) - 7 = \boxed{-55} \leftarrow$ minimum value

Vertex of parabola $(4, -55)$

Parabola opens ***up*** so minimum value is -55

41. *Answer is C.*

Difficulty = **0.41 U**

Find the ***maximum*** value of the function $y = -13 - 6x - x^2$

$y' = -2x - 6 = 0$

$-6 = 2x$

Critical number $\rightarrow -3 = x$

$y'' = -2$ (negative)

$\therefore$ concave down

$y = -x^2 - 6x - 13$

$y(-3) = -(-3)^2 - 6(-3) - 13 = \boxed{-4}$

Vertex of parabola $(-3, -4)$

Parabola opens ***down*** so maximum value is -4

42. *Answer is C.*

Difficulty = **0.38 H**

If $y = 2ax + bx^2$ and a and b are positive constants, determine the minimum value of y

Parabola opening ***up***

$y' = 2a + 2bx = 0$

$2bx = -2a$

$x = -\frac{a}{b}$

Find coordinates of vertex and y-value is minimum

$y(x) = 2ax + bx^2$

$y(-\frac{a}{b}) = 2a\left(-\frac{a}{b}\right) + b\left(-\frac{a}{b}\right)^2$

$y(-\frac{a}{b}) = \frac{-2a^2}{b} + \frac{a^2}{b} = \boxed{-\frac{a^2}{b}}$

Vertex $\left(-\frac{a}{b}, \underbrace{-\frac{a^2}{b}}_{minimum}\right)$

43. *Answer is D.*

Difficulty = **0.31 U**

Determine the ***maximum*** value of the function $f(x) = -2x^2 - x + 6$

$f'(x) = -4x - 1 = 0$

$-1 = 4x$

Critical number $\rightarrow -\frac{1}{4} = x$

$f''(x) = -4$ (negative)

$\therefore$ concave down

$f(x) = -2x^2 - x + 6$

$f(-\frac{1}{4}) = -2(-\frac{1}{4})^2 - (-\frac{1}{4}) + 6 = \boxed{6.125}$ ← maximum value

Vertex of parabola $(-\frac{1}{4},\ 6.125)$

Parabola opens ***down*** so maximum value is **6.125**

44. *Answer is C.*

Find the ***minimum*** value of the function $f(x) = 2x^2 - 12x + 25$

$f'(x) = 4x - 12 = 0$

$4x = 12$

Critical number $\rightarrow x = 3$

$f''(x) = 4$ (positive)

$\therefore$ concave up

$f(x) = 2x^2 - 12x + 25$

$f(3) = 2(3)^2 - 12(3) + 25 = \boxed{7}$ ← minimum value

Vertex of parabola $(\ 3,\ 7)$

Parabola opens ***up*** so minimum value is **7**

45. *Answer is A.*

If $f(x) = x^4 + kx^2$ has a minimum at $x = 1$, then determine the value of the constant k

$f'(x) = 4x^3 + 2kx = 0$

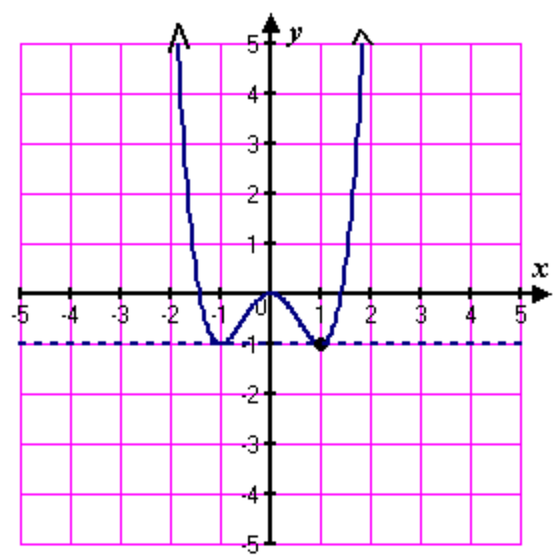
$4(1)^3 + 2k(1) = 0$

$4 + 2k = 0$

$2k = -4$

$\boxed{k = -2}$

$f(x) = x^4 - 2x^2$ has a local minimum at $(\ 1, -1)$



46. *Answer is D.*

Determine the ***maximum*** value of the function $f(x) = 2 - 18x - 3x^2$

$f'(x) = -6x - 18 = 0$

$-18 = 6x$

Critical number $\rightarrow -3 = x$

$f''(x) = -6$ (negative)

$\therefore$ concave down

$f(x) = 2 - 18x - 3x^2$

$f(-3) = 2 - 18(-3) - 3(-3)^2 = \boxed{29}$ ← maximum value

Vertex of parabola $(-3,\ 29)$

Parabola opens ***down*** so maximum value is **29**

47. *Answer is D.*

What is the maximum value of the function $f(x) = 4 + 8x - x^2$

$$f'(x) = -2x + 8 = 0$$

$$8 = 2x$$

Critical number $\rightarrow 4 = x$

$f''(x) = -2$ (negative)

$\therefore$ concave down

$$f(x) = 4 + 8x - x^2$$

$f(4) = 4 + 8(4) - (4)^2 = \boxed{20}$ $\leftarrow$ maximum value

Vertex of parabola $(4,\ 20)$

Parabola opens ***down*** so maximum value is **20**

48. *Answer is A.*

Difficulty = **0.74 U**

Find the slope of the line tangent to the graph of $f(x) = x^2 + 3$ at the point where $x = -1$

$$f(x) = x^2 + 3$$
$$f'(x) = 2x$$
$$f'(-1) = 2(-1) = \boxed{-2}$$

49. *Answer is A.*

Difficulty = **0.71 U**

Find the slope of the tangent to $y = x^3 - 2x^2 + 6$ at $(2,\ 6)$

$$y'(x) = 3x^2 - 4x$$
$$y'(2) = 3(2)^2 - 4(2) = 12 - 8 = \boxed{4}$$

50. *Answer is C.*

Difficulty = **0.71 U**

Find the slope of the line tangent to the graph of $y = x^3 - 4x^2 + 2$ at the point where $x = 2$

$$y'(x) = 3x^2 - 8x$$
$$y'(2) = 3(2)^2 - 8(2) = 12 - 16 = \boxed{-4}$$

51. *Answer is C.*

Difficulty = **0.70 H**

If $y = -3x + 1$ is tangent to the curve $f(x)$ at $x = a$ which must be true ?

$$\underbrace{y = -3x + 1}_{slope=-3} \text{ is } \underbrace{\textit{tangent} \text{ to the curve } f(x) \text{ at } x = a}_{\textit{then derivative of } f(x) \text{ at } x=a \text{ must be } f'(a)=-3}$$

52. *Answer is B.*

Difficulty = **0.69 U**

Given the function $f(x) = 3x^2 - 4x + 3$ for what value(s) of x is the slope of the tangent line equal to **2**

$$f'(x) = 6x - 4 = 2 \quad \leftarrow \text{slope}$$
$$6x = 6$$
$$\boxed{x = 1}$$

53. *Answer is C.*

Difficulty = **0.68 U**

Determine the slope of the line tangent to $y = \frac{6}{x}$ at $(2, 3)$

$$y(x) = \frac{6}{x} = 6x^{-1}$$

$$y'(x) = 6(-1x^{-2}) = -\frac{6}{x^2}$$

$$y'(2) = -\frac{6}{(2)^2} = -\frac{6}{4} = \boxed{-\frac{3}{2}}$$

54. *Answer is A.*

Difficulty = **0.62 U**

Find the ***point*** on $y = 2x^2 + 6x - 1$ where the slope of the tangent line is **2**

$$y'(x) = 4x + 6 = 2 \leftarrow \textit{slope}$$

$$4x = 2 - 6$$

$$\boxed{x = -1}$$

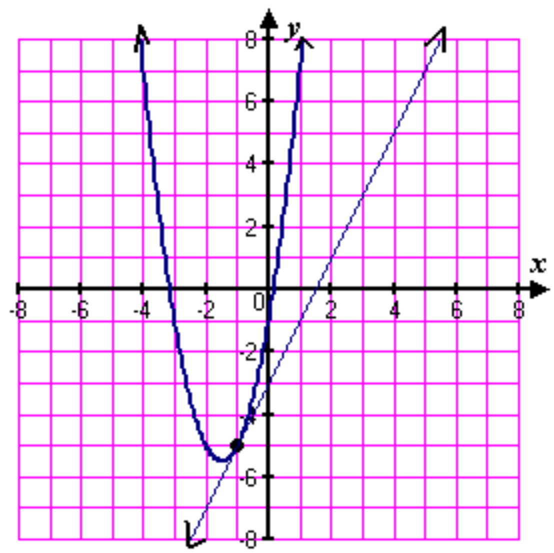
When $x = -1$

$$y(x) = 2x^2 + 6x - 1$$

$$= 2(-1)^2 + 6(-1) - 1 = -5$$

$$\therefore \textit{ at point } \boxed{(-1, -5)}$$

slope of the tangent line = **2**



55. *Answer is B.*

Difficulty = **0.61 U**

Determine the slope of the line tangent to the graph of $y = \frac{1}{x}$ at $x = 4$

$$y(x) = \frac{1}{x} = x^{-1}$$

$$y'(x) = -1x^{-2} = -\frac{1}{x^2}$$

$$y'(4) = -\frac{1}{(4)^2} = \boxed{-\frac{1}{16}}$$

56. *Answer is D.*

Difficulty = **0.59 U**

Find an ***equation*** of the line tangent to the graph of $y = x^3 - 3x^2 + 3x + 2$ at $(0,\ 2)$

$y(x) = x^3 - 3x^2 + 3x + 2$
$y'(x) = 3x^2 - 6x + 3$
$y'(0) = 3(0)^2 - 6(0) + 3 = 3$

At point $(0,\ 2)$ slope $m = 3$

so tangent line is $\boxed{y = 3x + 2}$

57. *Answer is B.*

Difficulty = **0.54 H**

At what ***point*** on the curve $y = x^2 - 4$ is the tangent ***parallel*** to the line $6x + y = 4$

Line $6x + y = 4$
$y = -6x + 4$
slope of line $m = -6 \rightarrow$

$y(x) = x^2 - 4$
$y'(x) = 2x = -6 \leftarrow m$
$2x = -6$
$x = -3 \quad \rightarrow$

When $x = -3$
$y(x) = x^2 - 4$
$y(-3) = (-3)^2 - 4 = 5$

At point $\boxed{(-3,\ 5)}$ tangent line is ***parallel*** to the line $6x + y = 4$

58. *Answer is A.*

Difficulty = **0.53 U**

Determine the slope of the line tangent to the graph of $f(x) = \sqrt{x}$ at $x = 9$

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}$$
$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$
$$f'(9) = \frac{1}{2\sqrt{9}} = \boxed{\frac{1}{6}}$$

59. *Answer is B.*

Difficulty = **0.41 H**

The line $\underbrace{y = -4x + 18}_{slope=-4}$ is tangent to the parabola $y = ax^2 + bx$ at the point where $\underbrace{x = 3}_{y'(3)=-4}$

If the parabola has a $\underbrace{\textit{critical} \text{ point}}_{y'(x)=0}$ at $\underbrace{x = 2}_{y'(2)=0}$ determine the value of a

Point of tangency $(3,\ 6)$
$y(x) = ax^2 + bx$
$y'(x) = 2ax + b$
$y'(3) = 2a(3) + b = -4$
$6a + b = -4$

At critical point $(2,\ ?)$
$y(x) = ax^2 + bx$
$y'(x) = 2ax + b$
$y'(2) = 2a(2) + b = 0$
$4a + b = 0$

Solve system

$$\boxed{\begin{aligned} 6a + b &= -4 \\ 4a + b &= 0 \end{aligned}} \Rightarrow \boxed{\begin{aligned} 6a + \cancel{b} &= -4 \\ -4a - \cancel{b} &= 0 \end{aligned}}$$

$\therefore\ 2a = -4$

$\boxed{a = -2}$

60. *Answer is D.*

Difficulty = **0.46 H**

What are the coordinates of the point on the graph of $y=\sqrt{x}$ where the slope of the tangent is $\frac{1}{8}$

$$y(x)=\sqrt{x}=x^{\frac{1}{2}}$$
$$y'(x)=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}}=\frac{1}{8}$$
$$2\sqrt{x}=8$$
$$\sqrt{x}=4$$
$$x=16$$

When $x=16$

$$y(x)=\sqrt{x}$$
$$y(16)=\sqrt{16}=4\Rightarrow(16,\ 4)$$

At point $\boxed{(16,\ 4)}$ on the graph of $y=\sqrt{x}$ the slope of the tangent is $\frac{1}{8}$

61. *Answer is C.*

Determine the slope of the line tangent to the graph of $y=x^3-x^2$ at the point where $x=2$

$$y'(x)=3x^2-2x$$
$$y'(2)=3(2)^2-2(2)=\boxed{8}$$

62. *Answer is C.*

Determine the slope of the tangent line to $f(x)=-\frac{2}{x}$ at the point where $x=2$

$$f(x)=-\frac{2}{x}=-2x^{-1}$$
$$f'(x)=-2(-1x^{-2})=\frac{2}{x^2}$$
$$f'(2)=\frac{2}{(2)^2}=\frac{2}{4}=\boxed{\frac{1}{2}}$$

63. *Answer is C.*

What is the slope of the tangent line to the graph of $y=-x^2+2x-3$ at the point $(2,-3)$

$$y'(x)=-2x+2$$
$$y'(2)=-2(2)+2=\boxed{-2}$$

64. *Answer is A.*

What is the slope of the tangent line to the function $y=3-x$

$$y(x)=3-x$$
$$y'(x)=\boxed{-1}$$

The original function is a ***straight line*** with slope -1

65. *Answer is E.*

The equation of the normal line to the curve $y = x^4 + 3x^3 + 2$ at the point where $x = 0$ is

Point of tangency/normal

$y = x^4 + 3x^3 + 2$

$y(0) = 2$

$(0,\ 2)$

Slope of tangent at $x = 0$

$y = x^4 + 3x^3 + 2$

$y' = 4x^3 + 9x^2$

$y'(0) = 4x^3 + 9x^2 = 0$ ← ***horizontal*** tangent at $x = 0$

Slope of normal is ***undefined*** at $x = 0$ ← Vertical ***line*** $\boxed{x = 0}$

66. *Answer is C.*

The line **L** is perpendicular to the parabola $y = kx^2$ at the point **(1, 5)** What is the equation of **L**

$y = kx^2$

$5 = k(1)^2$

$5 = k$

$y = 5x^2$

$y' = 10x$

$y'(1) = 10(1) = 10$

Slope of ***tangent*** at the point **(1, 5)** is **10**

Slope of ***normal*** is $-\frac{1}{10}$

Equation of normal

$$\text{Slope} = \frac{rise}{run} = \frac{-1}{10} = \frac{y-5}{x-1}$$

$10y - 50 = -x + 1$

$\boxed{x + 10y = 51}$

67. *Answer is A.*

If $x + 7y = 29$ is an equation of the line ***normal*** to the graph of f at the point **(1, 4)**, then $f'(1) =$

$y = -\frac{1}{7}x + \frac{29}{7}$

Slope of normal $= -\frac{1}{7}$ at the point **(1, 4)**

Slope of tangent $= 7$ at the point **(1, 4)**

$\therefore f'(1) = 7$ ← slope of tangent when $x = 1$

68. *Answer is B.*

The line perpendicular to the tangent of the curve represented by the equation $y = x^2 + 6x + 4$ at the point **(−2,−4)** also intersects the curve at $x =$

$y = x^2 + 6x + 4$

$y' = 2x + 6$

$y'(-2) = 2(-2) + 6 = 2$

→ tangent $m = 2$ at point **(−2,−4)**

→ normal $m = -\frac{1}{2}$ at point **(−2,−4)**

Equation of normal

$$m = \frac{-1}{2} = \frac{y+4}{x+2}$$

$2y + 8 = -x - 2$

$2y = -x - 10$

$y = -\frac{1}{2}x - 5$

Normal curve intersection

$-\frac{1}{2}x - 5 = x^2 + 6x + 4$

$0 = x^2 + 6.5x + 9$

$0 = 2x^2 + 13x + 18$

$0 = (2x + 9)(x + 2)$

$\boxed{x = -\frac{9}{2}}$ | $x = -2$

69. *Answer is D.*

An equation of the line ***normal*** to the graph of $y = x^4 - 3x^2 + 1$ at the point where $x = 1$ is

$y = x^4 - 3x^2 + 1$

$y' = 4x^3 - 6x$

$y'(1) = 4(1)^3 - 6(1) = -2$

Slope of ***tangent*** to → graph at $x = 1$

$y = x^4 - 3x^2 + 1$

$y(1) = (1)^4 - 3(1)^2 + 1 = -1$

Point of tangency $(1, -1)$

Slope of ***normal*** is $m = \frac{1}{2}$

Equation of ***normal*** at $(1, -1)$

$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{1}{2} = \frac{y+1}{x-1}$

$2y + 2 = x - 1$

$-x + 2y + 3 = 0$

$\boxed{x - 2y - 3 = 0}$

70. *Answer is A.*

An equation of the line normal to the graph of $y = 7x^4 + 2x^3 + x^2 + 2x + 5$ at the point where $x = 0$ is

$y = 7x^4 + 2x^3 + x^2 + 2x + 5$

$y' = 28x^3 + 6x^2 + 2x + 2$

$y'(0) = 2$

Slope of tangent $m = \frac{2}{1}$

Slope of normal $m = -\frac{1}{2}$

Point/slope $(0, 5)$ $m = -\frac{1}{2}$

Equation of normal

$\text{Slope} = \frac{rise}{run} = \frac{-1}{2} = \frac{y-5}{x-0}$

$2y - 10 = -x$

$\boxed{x + 2y = 10}$

71. *Answer is B.*

Find the equation of the line ***normal*** to $y = 4x^2 + 2x + 9$ at the point where $x = 1$

$y = 4x^2 + 2x + 9$

$y' = 8x + 2$

$y'(1) = 8(1) + 2 = 10$

at $x = 1$: tangent $m = 10$; normal $m = -\frac{1}{10}$

$y = 4x^2 + 2x + 9$

$y(1) = 4(1)^2 + 2(1) + 9 = 15$

Normal at point $(1, 15)$ $m = -\frac{1}{10}$

$\text{Slope} = \frac{-1}{10} = \frac{y-15}{x-1}$

$10y - 150 = -x + 1$

$\boxed{x + 10y = 151}$

72. *Answer is C.*

The coordinates of the point where the normal to the curve $y = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x$ at $x = 1$ intersects the y-axis are

$y = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x$

$y' = x^2 + x + 1$

$y'(1) = (1)^2 + (1) + 1 = 3$

Slope of tangent $m = \frac{3}{1}$

Slope of normal $m = -\frac{1}{3}$

Point/slope $(1, \frac{11}{6})$ $m = -\frac{1}{3}$

Equation of normal

$\text{Slope} = \frac{rise}{run} = \frac{-1}{3} = \frac{y - \frac{11}{6}}{x-1}$

$3y - \frac{33}{6} = -x + 1$

$18y - 33 = -6x + 6$

$6x + 18y - 39 = 0$

Intersects the y-axis $(x = 0)$

$6x + 18y - 39 = 0$

$18y = 39$

$6y = 13$

$y = \frac{13}{6}$

Point $\boxed{(0, \frac{13}{6})}$

73. *Answer is C.*

The line normal to the curve $y = x^2$ at $(2,\ 4)$ intersects the curve at $x =$

$y = x^2$
$y' = 2x$
$y'(2) = 2(2) = 4$
Slope of tangent $m = \frac{4}{1}$
Slope of normal $m = -\frac{1}{4}$

Point/slope $(2,\ 4)$ $m = -\frac{1}{4}$
Equation of normal
$$\text{Slope} = \frac{rise}{run} = \frac{-1}{4} = \frac{y-4}{x-2}$$
$$4y - 16 = -x + 2$$
$$y = -\tfrac{1}{4}x + \tfrac{9}{2}$$

Normal intersects the curve
$x^2 = y \qquad y = -\frac{1}{4}x + \frac{9}{2}$
$$x^2 = -\tfrac{1}{4}x + \tfrac{9}{2}$$
$$4x^2 = -x + 18$$
$$4x^2 + x - 18 = 0$$
$$(4x+9)(x-2) = 0$$
$\boxed{x = -\frac{9}{4}}$ | $x = 2$

74. Find the value of x at which the normal to the curve $y = x^2 + 1$ at $x = 3$ intersects the curve again.

$y = x^2 + 1$
$y' = 2x$
$y'(3) = 2(3) = 6$
Slope of tangent $m = \frac{6}{1}$
Slope of normal $m = -\frac{1}{6}$

Point/slope $(3,\ 10)$ $m = -\frac{1}{6}$
Equation of normal
$$\text{Slope} = \frac{rise}{run} = \frac{-1}{6} = \frac{y-10}{x-3}$$
$$6y - 60 = -x + 3$$
$$y = -\tfrac{1}{6}x + \tfrac{63}{6}$$

Intersects the curve again
$1 + x^2 = y \qquad y = -\frac{1}{6}x + \frac{63}{6}$
$$1 + x^2 = -\tfrac{1}{6}x + \tfrac{63}{6}$$
$$6 + 6x^2 = -x + 63$$
$$6x^2 + x - 57 = 0$$
$$(6x+19)(x-3) = 0$$
$\boxed{x = -\frac{19}{6}}$ | $x = 3$

75. The line normal to the function $f(x) = 4 - x^2$ at $x = -1$ intersects the curve again. Find the value of the function at that point.

$f(x) = 4 - x^2$
$f'(x) = -2x$
$f'(-1) = -2(-1) = 2$
Slope of tangent $m = \frac{2}{1}$
Slope of normal $m = -\frac{1}{2}$

Point/slope $(-1,\ 3)$ $m = -\frac{1}{2}$
Equation of normal
$$\text{Slope} = \frac{rise}{run} = \frac{-1}{2} = \frac{y-3}{x+1}$$
$$2y - 6 = -x - 1$$
$$y = -\tfrac{1}{2}x + \tfrac{5}{2}$$

$$f(\tfrac{3}{2}) = 4 - (\tfrac{3}{2})^2 = \tfrac{16}{4} - \tfrac{9}{4} = \boxed{\tfrac{7}{4}}$$

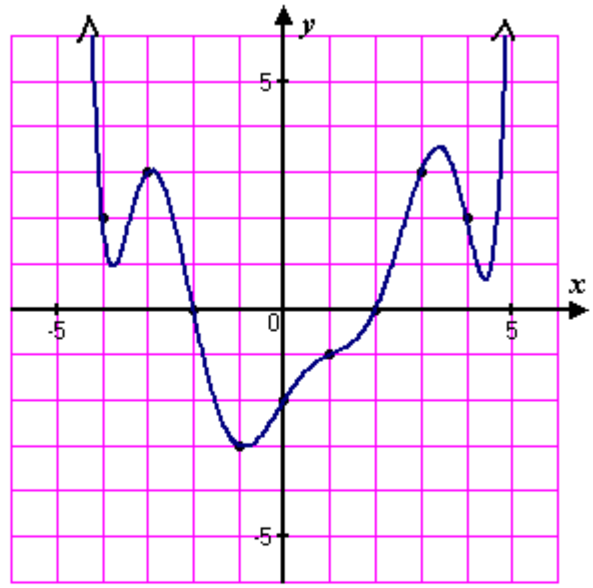
Intersects the curve again
$4 - x^2 = y \qquad y = -\frac{1}{2}x + \frac{5}{2}$
$$4 - x^2 = -\tfrac{1}{2}x + \tfrac{5}{2}$$
$$8 - 2x^2 = -x + 5$$
$$0 = 2x^2 - x - 3$$
$$0 = (2x-3)(x+1)$$
$\leftarrow$ $\boxed{x = \frac{3}{2}}$ | $x = -1$

76. *Answer is A.*

Difficulty = **0.52**

x	-4	-3	-2	-1	0	1	2	3	4
$g'(x)$	2	3	0	-3	-2	-1	0	3	2

The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals ?



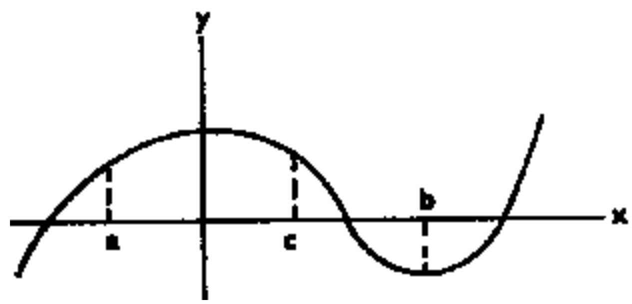
$g(x)$ is ***decreasing*** whenever

$g'(x) < 0 \quad \rightarrow \quad \boxed{-2 < g < 2}$

77. *Answer is E.*

Given the function shown above, ***how many*** of the following statements are true ?

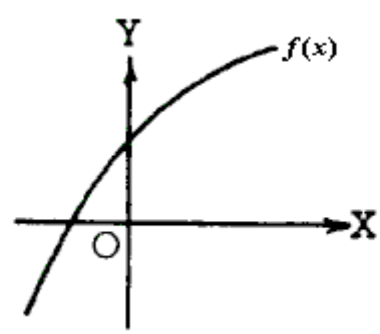
I. $f'(b) = 0$
II. $f''(a) < 0$
III. $f''(c) < 0$
IV. $f''(b) > 0$



I. $f'(b) = 0$ ☑ True, minimum point, horizontal tangent
II. $f''(a) < 0$ ☑ True, concave downwards
III. $f''(c) < 0$ ☑ True, concave downwards
IV. $f''(b) > 0$ ☑ True, concave upwards

78. *Answer is B.*

If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $f(x)$



$y' > 0 \rightarrow f(x)$ ***increasing***
$y'' < 0 \rightarrow f(x)$ concave ***downwards***
$\therefore f(x) \rightarrow$ ***increasing*** and concave ***downwards***

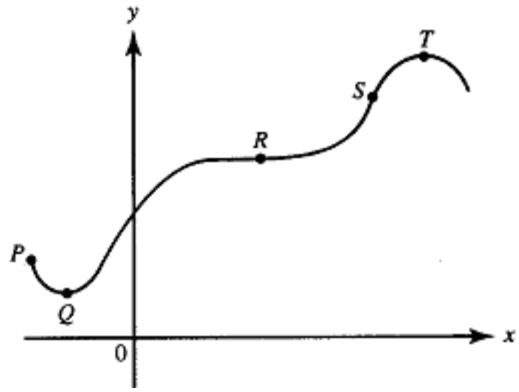
79. *Answer is C.*

> Use the graph on the right for this and the <u>next two</u> questions.
>
> At which labelled point do both $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ equal zero ?

$\frac{dy}{dx} = 0$ → horizontal tangent

$\frac{d^2y}{dx^2} = 0$ → ***possible*** change concavity

Point **R** → inflection point fits



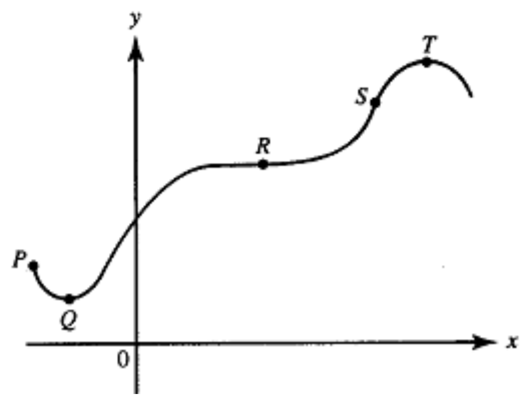
80. *Answer is D.*

> At which labelled point is $\frac{dy}{dx}$ positive and $\frac{d^2y}{dx^2}$ equal to zero ?

$\frac{dy}{dx} = \textit{positive}$ → function ***increasing***

$\frac{d^2y}{dx^2} = 0$ → ***possible*** change of concavity

Point **S** → function increasing ***and*** inflection point



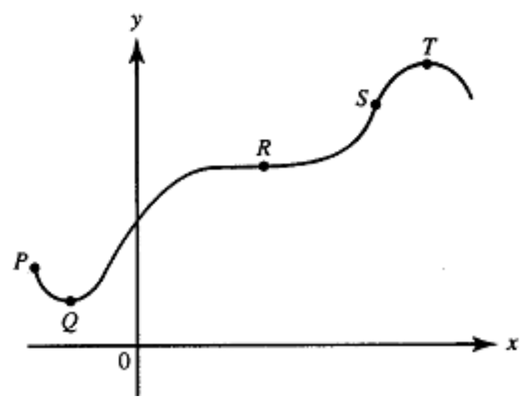
81. *Answer is E.*

> At which labelled point is $\frac{dy}{dx}$ equal to zero and $\frac{d^2y}{dx^2}$ negative ?

$\frac{dy}{dx} = 0$ → horizontal tangent

$\frac{d^2y}{dx^2} = \textit{negative}$ → concave ***down***

Point **T** → ***maximum*** point



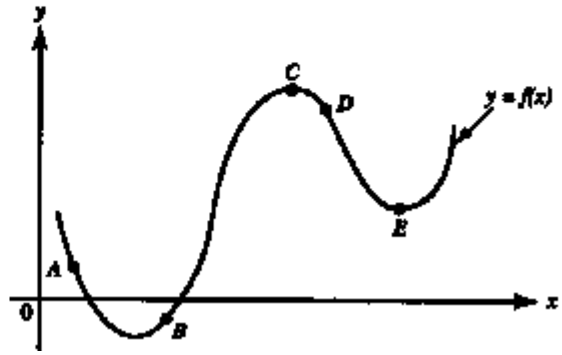
82. *Answer is A.*

At which point on the graph of $y = f(x)$ is $f'(x) < 0$ and $f''(x) > 0$

$f'(x) < 0 \rightarrow$ function decreasing

$f''(x) > 0 \rightarrow$ concave upwards

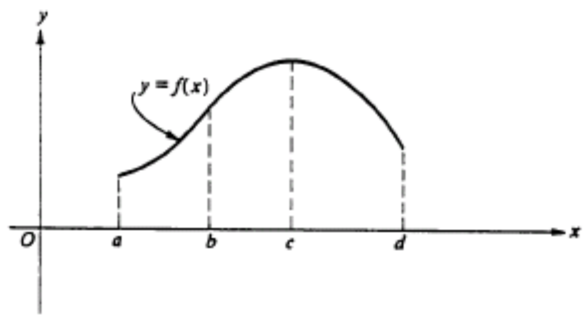
Point **A** $\rightarrow$ function decreasing/concave up



83. *Answer is B.*

Difficulty = **0.59**

The graph of $y = f(x)$ is shown in the diagram. On which of the following intervals are $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$ **I.** $a < x < b$ **II.** $b < x < c$ **III.** $c < x < d$



$\frac{dy}{dx} > 0 \rightarrow$ function increasing

$\frac{d^2y}{dx^2} < 0 \rightarrow$ concave downwards

$\Rightarrow$ Open interval $\Rightarrow$ **II.** $b < x < c$

84. *Answer is D.*

Difficulty = **0.33**

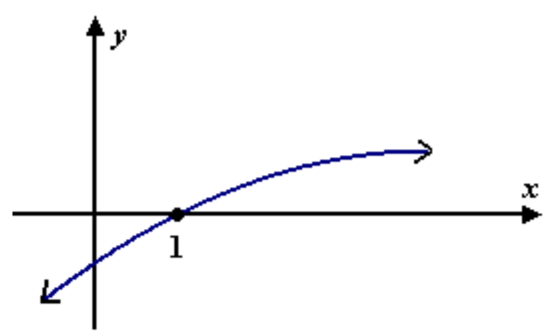
The graph of a twice-differentiable function f is shown in the figure on the right. Which of the following is true ?

$f(1) = 0 \rightarrow x$-intercept

$f'(1) = \textbf{positive} \rightarrow$ function is ***increasing***

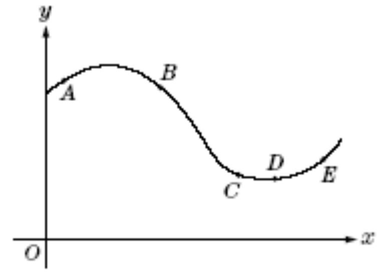
$f''(1) = \textbf{negative} \rightarrow$ function is concave ***down***

$\therefore \underbrace{f''(1)}_{negative} < \underbrace{f(1)}_{zero} < \underbrace{f'(1)}_{positive}$



85. *Answer is B*

At which of the five points on the graph in the figure at the right are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both negative ?



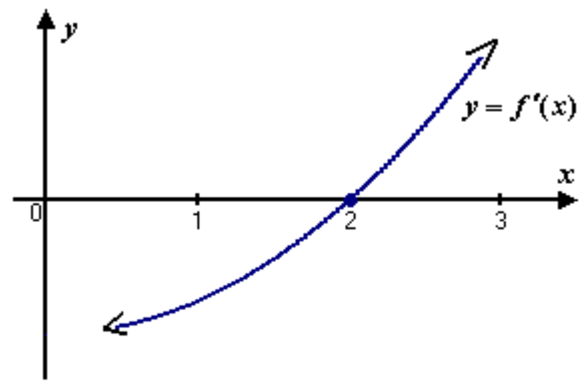
$\frac{dy}{dx}$ negative $\rightarrow$ function is ***decreasing***

$\frac{d^2y}{dx^2}$ negative $\rightarrow$ function is ***concave down***

Point $\boldsymbol{B}$ $\rightarrow$ function is ***decreasing/concave down***

86. *Answer is A.*

The graph of the derivative of a twice differentiable function f is shown in the graph. If $f(1) = -2$ which of the following is true ?



New twist $\rightarrow$ be careful !!!

$f''(2) > 0$ (positive) $\leftarrow$ concave up

$f'(2) = 0$ (zero) $\leftarrow$ ***minimum*** point

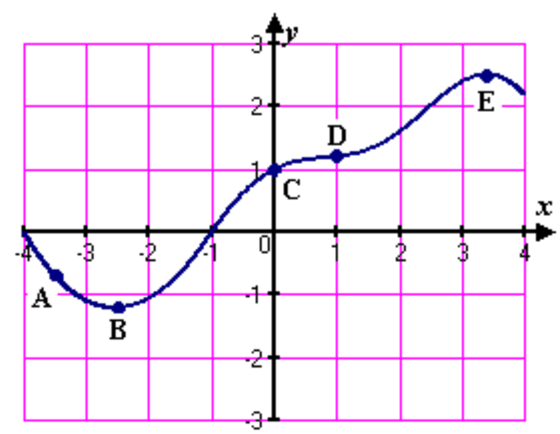
$f(1) = -2$ and f value is decreasing between $1 < x < 2$ until minimum point

$f(2) < -2$ (negative)

$\boxed{f(2) < f'(2) < f''(2)}$

87. *Answer is E.*

At which point on the graph of $y = g(x)$ on the right is $g'(x) = 0$ and $g''(x) < 0$



$g'(x) = 0$ $\leftarrow$ horizontal tangent

$g''(x) < 0$ $\leftarrow$ concave down

Point $\boxed{\textbf{E}}$ is the only point

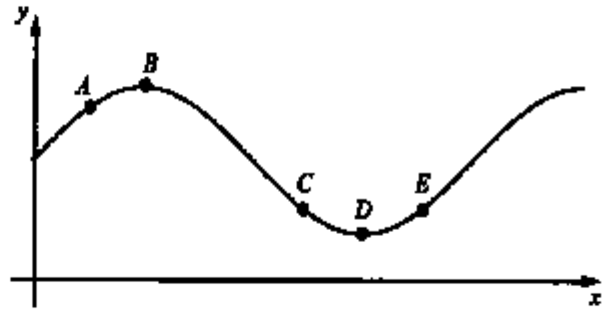
88. *Answer is E.*

> The graph of a function f is shown. At which of the marked points are both f' and f'' positive ?

f' positive $\leftarrow$ function increasing

f'' positive $\leftarrow$ concave up

Point $\boxed{\mathbf{E}}$ is the only point



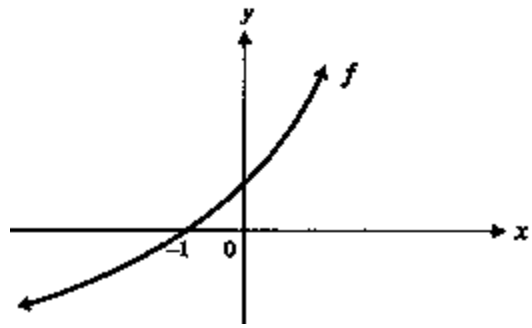
89. *Answer is A.*

> The graph of f is shown in the diagram and f is twice differentiable. Which of the following has the smallest value ?
>
> **I.** $f(-1)$
>
> **II.** $f'(-1)$
>
> **III.** $f''(-1)$

I. $\boxed{f(-1)=0}$ $\rightarrow$ x-intercept *** ***smallest*** value

II. $f'(-1) = \textit{positive} \rightarrow$ function increasing

III. $f''(-1) = \textit{positive} \rightarrow$ concave upwards



90. *Answer is A.*

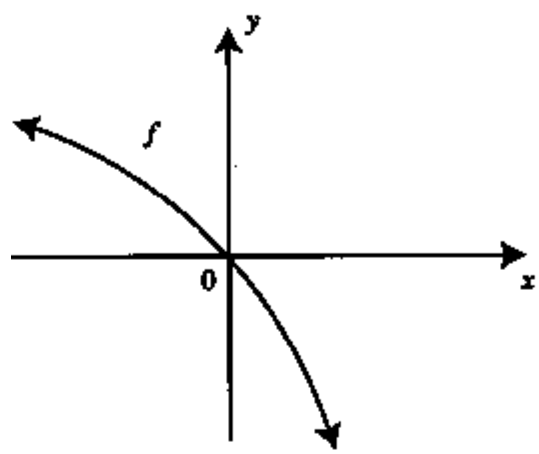
> The graph of f is shown on the right and f is twice differentiable. Which of the following has the largest value $f(0)$, $f'(0)$ or $f''(0)$

$f(0) = 0$ $\rightarrow$ x-intercept

$f'(0) = \textit{negative}$ $\rightarrow$ function ***decreasing***

$f''(0) = \textit{negative}$ $\rightarrow$ function concave ***down***

$\therefore$ $f(0)$ has the ***largest*** value



91. *Answer is D.*

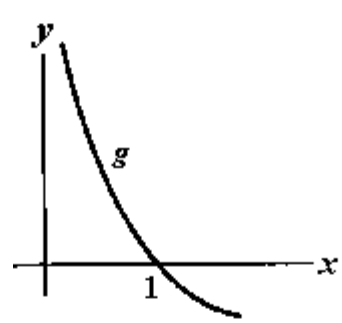
> The graph of g, a twice-differentiable function is shown in the diagram. Choose the correct order for the values of $g(1)$, $g'(1)$ and $g''(1)$

$g(1) = 0$ $\rightarrow$ x-intercept

$g''(1) = \textit{negative}$ $\rightarrow$ function ***decreasing***

$g''(1) = \textit{positive}$ $\rightarrow$ function concave ***up***

$$\therefore \quad \underbrace{g'(1)}_{\textit{negative}} < \underbrace{g(1)}_{\textit{zero}} < \underbrace{g''(1)}_{\textit{positive}}$$



92.

Derivatives of	$y = e^u$	and	$y = \ln u$
	$y' = e^u \dfrac{du}{dx}$		$y' = \dfrac{1}{u}\dfrac{du}{dx} = \dfrac{u'}{u}$

93. *Answer is A.*

If $f(x) = \ln x^3$ then $f''(3) =$

$$f(x) = 3\ln x$$
$$f'(x) = \frac{3}{x} = 3x^{-1}$$
$$f''(x) = 3(-1x^{-2)}) = \frac{-3}{x^2}$$
$$f''(3) = \frac{-3}{3^2} = \boxed{\frac{-1}{3}}$$

94. *Answer is D.*

If $y = e^x(x-1)$ then $y''(0) =$

$$y' = e^x(1) + (x-1)e^x \quad \leftarrow \text{product rule}$$
$$y' = e^x(1 + x - 1)e^x = xe^x$$
$$y'' = xe^x + e^x(1) = e^x(x+1) \quad \leftarrow \text{product rule second time}$$
$$y''(0) = e^0(0+1) = \boxed{1}$$

95. *Answer is E.*

The domain of the function defined by $f(x) = \ln(x^2 - x - 6)$ is the set of all real numbers x such that

$$f(x) = \ln(x^2 - x - 6)$$
$$(x+2)(x-3) > 0$$

$x = -2$	$x = 3$

$\leftarrow$ endpoints

$$\boxed{-2 < x \ \ or \ \ x > 3}$$

Sketch parabola, to get intervals

96. *Answer is D.*

Find y' given $y = \ln\left(x\sqrt{x^2+1}\right)$

$$y = \ln x + \ln(x^2+1)^{\frac{1}{2}} \quad \leftarrow \text{log rules}$$
$$y = \ln x + \tfrac{1}{2}\ln(x^2+1)$$
$$y' = \frac{1}{x} + \frac{\cancel{2}x}{\cancel{2}(x^2+1)} = \frac{1}{x}\left(\frac{x^2+1}{x^2+1}\right) + \frac{x}{(x^2+1)}\left(\frac{x}{x}\right) = \boxed{\frac{2x^2+1}{x(x^2+1)}}$$

97. *Answer is A.*

$\log_{\frac{1}{b}} x =$

$$\log_{\frac{1}{b}} \frac{x}{1} = \log_{\frac{b}{1}} \frac{1}{x} = \log_b x^{-1} = \boxed{-\log_b x} \quad \leftarrow \text{log shortcuts}$$

98. *Answer is A.*

If $f(x) = 2e^x + e^{2x}$ then $f'''(0) =$

$$f'(x) = 2e^x + e^{2x}(2)$$
$$f''(x) = 2e^x + 2e^{2x}(2)$$
$$f'''(x) = 2e^x + 4e^{2x}(2) = 2e^x + 8e^{2x}$$
$$f'''(0) = 2e^0 + 8e^{2(0)} = \boxed{10}$$

99. *Answer is B.*

If $e^{g(x)} = 2x+1$ then $g'(x) =$

$$g(x) = \ln(2x+1) \quad \leftarrow \text{ln both sides}$$
$$g'(x) = \boxed{\frac{2}{2x+1}}$$

100. *Answer is B.*

If $f(x) = (x+1)^{\frac{3}{2}} - e^{x^2-9}$ then $f'(3) =$

$$f'(x) = \tfrac{3}{2}(x+1)^{\frac{1}{2}} - e^{x^2-9}(2x) = \tfrac{3}{2}\sqrt{x+1} - 2xe^{x^2-9}$$
$$f'(3) = \tfrac{3}{2}\sqrt{3+1} - 2(3)e^{3^2-9} = 3 - 6 = \boxed{-3}$$

101. *Answer is A.*

Simplify: $\ln 2 + \ln 5 - \ln 8 - \ln 15 =$

$$\ln(2)(5) - \ln 8 - \ln 15 = \ln\frac{10}{8} - \ln 15 = \ln\frac{10}{8(15)} = \ln\frac{1}{12} = \ln 12^{-1} = \boxed{-\ln 12}$$

102.

Let $f(x)=\ln(x^2-x-6)$
$a)$ the domain of $f(x)$ is
$b)$ find $f(5)$
$c)$ find $f'(-3)$

$$f(x)=\ln(x^2-x-6)$$

$$(x+2)(x-3)>0$$

$$x=-2 \quad | \quad x=3$$

$a)$ the domain of $f(x)$ is $\boxed{x<-2 \text{ or } x>3}$

$b)$ find $f(5)=\ln(5^2-5-6)=\boxed{\ln 14}$

$c)$ find $f'(x)=\dfrac{2x-1}{x^2-x-6}$

$$f'(-3)=\frac{2(-3)-1}{(-3)^2-(-3)-6}=\boxed{\frac{-7}{6}}$$

103. *Answer is C.*

If $y=f(x)=x^3+\ln x$ then $y'=$

$$f'(x)=\boxed{3x^2+\frac{1}{x}}$$

104. *Answer is D.*

Solve: $\log_9 x^2=9$

$$x^2=9^9 \quad \leftarrow \text{exponentiate both sides base } 9$$

$$(x^2)^{\frac{1}{2}}=(9^9)^{\frac{1}{2}} \quad \leftarrow \text{exchange exponent order to square root first}$$

$$x=(9^{\frac{1}{2}})^9=(\sqrt{9})^9=(\pm 3)^9=\boxed{\pm 3^9}$$

105. *Answer is C.*

If $f(x)=x\ln x$ then $f'''(e)=$

$$f'(x)=x\left(\frac{1}{x}\right)+\ln x(1)=1+\ln x \quad \leftarrow \text{product rule}$$

$$f''(x)=0+\frac{1}{x}=x^{-1}$$

$$f'''(x)=-1x^{-2}=\frac{-1}{x^2}$$

$$f'''(e)=\boxed{-\frac{1}{e^2}}$$

106. *Answer is E.*

If $e^{g(x)} = \frac{x^x}{x^2-1}$ then $g(x) =$

$$g(x) = \ln \frac{x^x}{x^2-1} \quad \leftarrow \text{ln both sides}$$

$$g(x) = \ln x^x - \ln(x^2-1) = \boxed{x \ln x - \ln(x^2-1)}$$

107. *Answer is C.*

Difficulty = **0.64**

If $\ln x - \ln\left(\frac{1}{x}\right) = 2$, then $x =$

$$\ln \frac{x}{\frac{1}{x}} = 2$$

$$\ln x^2 = 2$$

$$x^2 = e^2$$

$$\boxed{x = e}$$

108. *Answer is C.*

Difficulty = **0.88**

If $y = x^2 e^x$ then $\frac{dy}{dx} =$

$$y' = x^2 e^x + e^x(2x) = \boxed{xe^x(x+2)} \quad \leftarrow \text{product rule}$$

109. *Answer is E.*

If $y = \ln[(x+1)(x+2)]$, then $\frac{dy}{dx} =$

$$y = \ln(x+1) + \ln(x+2) \quad \leftarrow \text{log rules}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{x+1} + \frac{1}{x+2}}$$

110. *Answer is E.*

Solve: $2x = 7^{1+\log_7 4}$

$$2x = (7^1)(\cancel{7}^{\cancel{\log_7} 4})$$

$$2x = (7)(4)$$

$$2x = 28$$

$$\boxed{x = 14}$$

111. *Answer is D.*

What is x when $6 = e^{5x}$

$$\ln 6 = \ln e^{5x} \quad \leftarrow \text{ln both sides}$$

$$\frac{\ln 6}{5} = \frac{5x}{5}$$

$$\boxed{\frac{\ln 6}{5} = x}$$

112. *Answer is B.*

$\ln_e 10 =$

$$\ln_e 10 = \boxed{\frac{1}{\ln_{10} e}} \quad \leftarrow \text{log shortcut}$$

113. *Answer is B.*

The tangent to the curve of $y = xe^{-x}$ is horizontal when x is equal to

$$y' = x(-e^{-x}) + e^{-x}(1) \quad \leftarrow \text{product rule}$$

$$y' = -xe^{-x} + e^{-x} = e^{-x}(1-x) = 0 \quad \leftarrow \text{looking for critical numbers}$$

$$e^{-x} \neq 0 \quad \Big| \quad \boxed{x = 1}$$

Critical number $x = 1$ with horizontal tangent

114. *Answer is C.*

Find $\frac{dy}{dx}$ for $y = \ln\sqrt{x^2+4}$

$$y = \ln(x^2+4)^{\frac{1}{2}} = \tfrac{1}{2}\ln(x^2+4)$$

$$y' = \frac{1}{2}\left(\frac{2x}{x^2+4}\right) = \boxed{\frac{x}{x^2+4}}$$

115. *Answer is D.*

Find an equation for the tangent line to the graph of $f(x) = \ln(x^2-1)$ at the point where $x = 2$

Slope of tangent

$$f(x) = \ln(x^2-1)$$

$$f'(x) = \frac{2x}{x^2-1}$$

$$f'(2) = \frac{2(2)}{2^2-1} = \frac{4}{3}$$

Point of tangency

$$f(x) = \ln(x^2-1)$$

$$f(2) = \ln(2^2-1) = \ln 3$$

$$point \rightarrow (\ 2,\ \ln 3)$$

Equation of tangent at $(\ 2,\ \ln 3)$

$$\frac{rise}{run} = \frac{4}{3} = \frac{y - \ln 3}{x-2}$$

$$4x - 8 = 3y - 3\ln 3$$

$$4x - 3y = 8 - \ln 3^3$$

$$\boxed{4x - 3y = 8 - \ln 27}$$

116. *Answer is B.*

If $f(x)=e^{-2x}$, then $f^{(4)}(x)=$

$$f'(x)=-2e^{-2x}$$
$$f''(x)=4e^{-2x}$$
$$f'''(x)=-8e^{-2x}$$
$$f^{(4)}(x)=\boxed{16e^{-2x}}$$

117. *Answer is E.*

If $\log_b(3^b)=\frac{b}{2}$, then $b=$

$$\log_b 3^b=\frac{b}{2}$$
$$b\log_b 3=\frac{b}{2}$$
$$\frac{\cancel{b}\log_b 3}{\cancel{b}}=\frac{b}{2b}$$
$$\log_b 3=\frac{1}{2}$$

$$\log_b 3=\frac{1}{2}$$
$$3=b^{\frac{1}{2}}$$
$$\boxed{9=b}$$

118. *Answer is E.*

Find $\frac{dy}{dx}$ if $y=x\ln^3 x$

$y=x\ln^3 x=x(\ln x)^3$ ← rearrange exponent, means the same

$y'=\cancel{x}(3)(\ln x)^2\frac{1}{\cancel{x}}+(\ln x)^3(1)$ ← product rule

$$y'=3(\ln x)^2+(\ln x)^3$$
$$y'=\boxed{\ln^2 x(3+\ln x)}$$

119. *Answer is E.*

If $y=\frac{e^{\ln u}}{u}$, then $\frac{dy}{du}=$

$y=\frac{\cancel{e^{\ln u}}}{u}=\frac{u}{u}=1$ ← ln shortcut

$$y'=\boxed{0}$$

120. *Answer is D.*

What is the slope of the tangent line to the curve $y = \ln \frac{x^2}{\sqrt{x^2+1}}$ at the point where $x = 2$

$$y = \ln \frac{x^2}{\sqrt{x^2+1}} = \ln x^2 - \ln \sqrt{x^2+1} = 2\ln x - \frac{1}{2}\ln(x^2+1)$$

$$y' = \frac{2}{x} - \frac{1}{\cancel{2}}\left(\frac{\cancel{2}x}{x^2+1}\right) = \frac{2}{x} - \left(\frac{x}{x^2+1}\right)$$

$$y'(2) = \frac{2}{2} - \left(\frac{2}{2^2+1}\right) = 1 - \frac{2}{5} = \boxed{\frac{3}{5}}$$

121. *Answer is E.*

What is the slope of the tangent line to the curve $y = \ln(x^2+1)$ when $x = 3$

$$y' = \frac{2x}{x^2+1}$$

$$y'(3) = \frac{2(3)}{3^2+1} = \frac{6}{10} = \boxed{\frac{3}{5}}$$

122. *Answer is B.*

The derivative of $f(x) = \ln(x^2+2x+1)$ is

$$f'(x) = \frac{2x+2}{x^2+2x+1} = \frac{2\cancel{(x+1)}}{\cancel{(x+1)}(x+1)} = \boxed{\frac{2}{x+1}}$$

123. *Answer is A.*

Difficulty = **0.44**

If $f(x) = \ln(x+4+e^{-3x})$ then $f'(0) =$

$$f'(x) = \frac{1-3e^{-3x}}{x+4+e^{-3x}}$$

$$f'(0) = \frac{1-3e^{0}}{0+4+e^{0}} = \frac{1-3}{0+4+1} = \boxed{-\frac{2}{5}}$$

124. *Answer is E.*

If $6y = 3e^{2x}$ then $y' =$

$$y = \tfrac{1}{2}e^{2x}$$

$$y' = \tfrac{1}{2}e^{2x}(2) = \boxed{e^{2x}}$$

125. *Answer is B.*

If $f(x) = x^2 \ln x^3$ then $f'(x) =$

$$f'(x) = x^2\left(\frac{3x^2}{x^3}\right) + \ln x^3(2x) \quad \leftarrow \text{product rule}$$

$$f'(x) = 3x + 6x\ln x = 3x(1 + 2\ln x) = \boxed{3x(1 + \ln x^2)}$$

126. *Answer is D.*

If $y = e^{\frac{1}{2}\ln(x^2-4x+7)}$ then $\dfrac{dy}{dx} =$

$$y = \cancel{e}^{\cancel{\ln}(x^2-4x+7)^{\frac{1}{2}}} \quad \leftarrow \text{simplify logs}$$

$$y' = \tfrac{1}{\cancel{2}}(x^2 - 4x + 7)^{-\frac{1}{2}}\cancel{2}(x-2) = \boxed{\frac{x-2}{\sqrt{x^2-4x+7}}}$$

127. *Answer is A.*

Given the equation $y = 3e^{-2x}$ what is an equation of the normal line to the graph at $x = \ln 2$

$$y = 3e^{-2x}$$

$$y' = 3(e^{-2x})(-2) = -6e^{-2x}$$

$$y'(\ln 2) = -6e^{-2\ln 2} = -6\cancel{e}^{\cancel{\ln}2^{-2}} = -6(\tfrac{1}{4}) = -\frac{3}{2}$$

Slope of tangent $= -\frac{3}{2}$

Slope of normal $= \frac{2}{3}$

$$y(\ln 2) = 3e^{-2\ln 2} = 3\cancel{e}^{\cancel{\ln}2^{-2}} = 3(\tfrac{1}{4}) = \tfrac{3}{4}$$

Point on curve $(\ln 2,\ \frac{3}{4})$

$$\text{Slope} = \frac{rise}{run} = \frac{2}{3} = \frac{y - \frac{3}{4}}{x - \ln 2}$$

$$3y - \tfrac{9}{4} = 2(x - \ln 2)$$

$$3y = 2(x - \ln 2) + \tfrac{9}{4}$$

$$y = \frac{2}{3}(x - \ln 2) + \frac{9}{3(4)}$$

$$\boxed{y = \frac{2}{3}(x - \ln 2) + \frac{3}{4}}$$

128. *Answer is B.*

The equation of the normal line to the graph of $y = e^{2x}$ at the point where $\dfrac{dy}{dx} = 2$ is

$$y = e^{2x}$$

$$y' = 2e^{2x} = 2 \quad \leftarrow \text{given}$$

$$e^{2x} = 1$$

$$2x = \ln 1 = 0$$

$$x = 0$$

$$y(0) = e^{2(0)} = 1 \quad \leftarrow \text{point } (0,\ 1)$$

Slope of tangent $= 2$ $\leftarrow$ given

Slope of normal $= -\frac{1}{2}$

Point on curve $(0,\ 1)$

$$\text{Slope} = \frac{rise}{run} = \frac{-1}{2} = \frac{y-1}{x-0}$$

$$2y - 2 = -x$$

$$2y = -x + 2$$

$$\boxed{y = -\frac{1}{2}x + 1}$$

129. *Answer is B.*

Find $\frac{dy}{dx}$ for $y = \ln(5-x)^6$

$$y = 6\ln(5-x) \quad \leftarrow \text{log shortcut}$$

$$y' = \frac{6(-1)}{5-x} = \boxed{\frac{6}{x-5}}$$

130. *Answer is B.*

The slope of the line tangent to the graph of $y = \ln(x^2)$ at $x = e^2$ is

$$y = 2\ln x$$

$$y' = \frac{2}{x}$$

$$y'(e^2) = \boxed{\frac{2}{e^2}}$$

131. *Answer is D.*

If $\log_a 2^a = \frac{a}{4}$ then $a =$

$$a\log_a 2 = \frac{a}{4} \quad \leftarrow \text{log rule}$$

$$\log_a 2 = \frac{1}{4} \quad \leftarrow \text{divide both sides by } a$$

$$2 = a^{\frac{1}{4}} \quad \leftarrow \text{exponentiate both sides base } a \text{ then } 4^{th} \text{ power both sides}$$

$$\boxed{16 = a}$$

132. *Answer is B.*

The slope of the line tangent to the graph of $y = \ln\left(\frac{x}{2}\right)$ at $x = 4$ is

$$y = \ln x - \ln 2$$

$$y' = \frac{1}{x} - 0$$

$$y'(4) = \boxed{\frac{1}{4}}$$

133. *Answer is B.*

If $f(x) = \log_b x$ and $g(x) = b^x$ then $f(g(x)) =$

$$f(g(x)) = \log_b(b^x) = \cancel{\log_b b}^{\,x} = \boxed{x} \quad \leftarrow \text{log rule}$$

134. *Answer is A.*

Simplify: $\ln e^4 =$

$\ln e^4 = \cancel{\ln e}^4 = \boxed{4}$ ← log rule

135. *Answer is A.*

If $y = \ln(x^x)$ then $y' =$

$y = x \ln x$ ← rearrange by log rule

$y' = x(\frac{1}{x}) + (\ln x)(1)$ ← product rule

$y' = \boxed{1 + \ln x}$

136. *Answer is E.*

If $f(x) = x^2 \ln x$ then $f'(x) =$

$f'(x) = x^2\left(\frac{1}{x}\right) + \ln x(2x)$ ← product rule

$f'(x) = \boxed{x + 2x \ln x}$

137. *Answer is A.*

Simplify: $2 \ln e^{5x} =$

$2 \ln e^{5x} = 2\cancel{\ln e}^{5x} = 2(5x) = \boxed{10x}$

138. *Answer is C.*

If $f(x) = e^{2x}$ and $g(x) = \ln x$ then the derivative of $y = f(g(x))$ at $x = e$ is

$y = f(g(x)) = e^{2g(x)} = e^{2\ln x} = \cancel{e}^{\cancel{\ln} x^2} = x^2$ ← composite function and log rules

$y = x^2$

$y'(x) = 2x$

$y'(e) = \boxed{2e}$

139. *Answer is A.*

If $f(x) = e^{2\ln x}$ then $f'(3) =$

$f(x) = e^{2\ln x} = \cancel{e}^{\cancel{\ln} x^2} = x^2$ ← and log rules

$f(x) = x^2$

$f'(x) = 2x$

$f'(3) = 2(3) = \boxed{6}$

140. *Answer is D.*

If $y = e^{8x^2+1}$ then $\frac{dy}{dx} =$

$y' = e^{8x^2+1}(16x) = \boxed{16xe^{8x^2+1}}$

141. *Answer is A.*

$$\frac{d}{dx}\ln\left(\frac{1}{1-x}\right)=$$

$$\frac{d}{dx}\ln\left(\frac{1}{1-x}\right)=\frac{d}{dx}\left[\ln 1-\ln(1-x)\right]=\left[0-\frac{-1}{(1-x)}\right]=\boxed{\frac{1}{1-x}}$$

142. *Answer is B.*

If $f(x)=x\ln(x^2)$ then $f'(x)=$

$f(x)=2x\ln x$ ← log rules

$f'(x)=2x\left(\frac{1}{x}\right)+\ln x(2)=2+2\ln x=\boxed{2+\ln(x^2)}$ ← product rule

143. *Answer is B.*

$$\frac{d}{dx}(\ln e^{3x})=$$

$$\frac{d}{dx}(\cancel{\ln e}^{3x})=\frac{d}{dx}(3x)=\boxed{3}$$

144. *Answer is C.*

The slope of the line tangent to the graph of $y=\ln\sqrt{x}$ at $(e^2,\ 1)$ is

$$y=\ln\sqrt{x}=\ln x^{\frac{1}{2}}=\tfrac{1}{2}\ln x$$

$$y'=\frac{1}{2x}$$

$$y'(e^2,\ 1)=\boxed{\frac{1}{2e^2}}$$

145. *Answer is E.*

Difficulty = **0.30**

If $f(x)=e^{3\ln x^2}$ then $f'(x)=$

$f(x)=e^{3\ln x^2}=\cancel{e}^{\cancel{\ln}(x^2)^3}=x^6$

$f'(x)=\boxed{6x^5}$

146. *Answer is B.*

If $f(x)=\ln(x^x)$ then $f'(e^2)=$

$f(x)=x\ln x$ ← log rules

$f'(x)=x\left(\frac{1}{x}\right)+\ln x(1)=1+\ln x$ ← product rule

$f'(e^2)=1+\cancel{\ln e}^2=\boxed{3}$

147. *Answer is E.*

If $y = e^{nx}$ then $\dfrac{d^n y}{dx^n}$ (the n^{th} derivative of y with respect to x) is

$$y' = e^{nx}(n) = ne^{nx}$$
$$y'' = ne^{nx}(n) = n^2 e^{nx}$$
$$y''' = n^2 e^{nx}(n) = n^3 e^{nx} \quad \leftarrow \text{observe pattern}$$
$$f^{(n)}(x) = \boxed{n^n e^{nx}}$$

148. *Answer is A.*

The equation of the tangent to the curve $\ln y = 3x^2 + 6x$ at the point where $x = 0$ is

$$\ln y = 3x^2 + 6x$$
$$y = e^{3x^2+6x} \quad \leftarrow \text{exponentiate both sides base } e$$
$$y' = e^{3x^2+6x}(6x+6) = (6x+6)e^{3x^2+6x}$$
$$y'(0) = (6(0)+6)e^{3(0)^2+6(0)} = 6$$
$$y(0) = e^{3(0)^2+6(0)} = 1 \quad \leftarrow \text{point } (0,\ 1)$$

Equation of the tangent through $(0,\ 1)$

$$\text{Slope} = \frac{rise}{run} = \frac{6}{1} = \frac{y-1}{x-0}$$
$$y - 1 = 6x$$
$$\boxed{y = 6x + 1}$$

149. *Answer is C.*

If $y = x(\ln x)^2$ then $\dfrac{dy}{dx} =$

$$y' = \frac{x}{1}\left[2(\ln x)^1\left(\frac{1}{x}\right)\right] + (\ln x)^2(1) \quad \leftarrow \text{product rule}$$
$$y' = 2\ln x + (\ln x)^2 = \boxed{(\ln x)(2 + \ln x)}$$

150. *Answer is A.*

If $f(x) = 3x\ln x$ then $f'(x) =$

$$f'(x) = 3\left[x\left(\frac{1}{x}\right) + (\ln x)(1)\right] \quad \leftarrow \text{product rule}$$
$$f'(x) = 3[1 + \ln x] = 3 + 3\ln x = \boxed{3 + \ln(x^3)}$$

151. *Answer is B.*

$\dfrac{d}{dx}\ln\left(\dfrac{1}{x^2-1}\right) =$

$$\frac{d}{dx}\ln\left(\frac{1}{x^2-1}\right) = \frac{d}{dx}\left[\ln 1 - \ln(x^2-1)\right] = 0 - \frac{2x}{(x^2-1)} = \boxed{\frac{-2x}{x^2-1}}$$

152. *Answer is C.*

If $f(x)=\sqrt{e^{2x}+1}$ then $f'(0)=$

$$f(x)=\sqrt{e^{2x}+1}=(e^{2x}+1)^{\frac{1}{2}} \leftarrow \text{rearrange}$$

$$f'(x)=\tfrac{1}{\cancel{2}}(e^{2x}+1)^{-\frac{1}{2}}(e^{2x})(\cancel{2}) \leftarrow \text{power rule and chain rule twice}$$

$$f'(x)=\frac{e^{2x}}{\sqrt{e^{2x}+1}}$$

$$f'(0)=\frac{e^{2(0)}}{\sqrt{e^{2(0)}+1}}=\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)=\boxed{\frac{\sqrt{2}}{2}}$$

153. *Answer is A.*

If $f(x)=e^x\ln x$ then $f'(e)=$

$$f'(x)=e^x\frac{1}{x}+(\ln x)(e^x)=\frac{e^x}{x}+e^x\ln x \qquad \leftarrow \text{product rule}$$

$$f'(e)=\frac{e^e}{e^1}+e^e\cancel{\ln e}^{\,1}=\boxed{e^{e-1}+e^e}$$

154. *Answer is D.*

If $y=\ln(3x+5)$ then $\frac{d^2y}{dx^2}=$

$$y'=\frac{3}{3x+5}=3(3x+5)^{-1}$$

$$y''=3(-1)(3x+5)^{-2}(3)=\boxed{\frac{-9}{(3x+5)^2}}$$

155. *Answer is E.*

The ***slope*** of the line ***normal*** to the curve $y=xe^x$ at $x=-1$ is

$$y(x)=xe^x$$

$$y'(x)=xe^x+e^x(1) \quad \leftarrow \text{product rule}$$

$$y'(-1)=(-1)e^{(-1)}+e^{(-1)}=-\frac{1}{e}+\frac{1}{e}=\boxed{\frac{0}{1}} \leftarrow \text{slope of tangent}$$

Slope of normal (negative reciprocal) $=\boxed{\frac{-1}{0}}$ ← undefined !!!

156. *Answer is A.*

If $x = \frac{1}{2}$ when $x = \log_y x$ then $y =$

$$\frac{1}{2} = \frac{\log \frac{1}{2}}{\log y} \quad \leftarrow \text{log rule}$$

$$\log y = 2\log \tfrac{1}{2} \quad \leftarrow \text{cross multiply}$$

$$\log y = \log(\tfrac{1}{2})^2$$

$$y = (\tfrac{1}{2})^2 = \boxed{\tfrac{1}{4}}$$

157. *Answer is B.*

If $f(x) = e^x$ and $g(x) = \frac{1}{x}$ then the ***derivative*** of $f(g(x))$, evaluated at $x = 2$ is

$$f(x) = e^x$$

$$f(g(x)) = e^{\frac{1}{x}}$$

$$\frac{d}{dx} f(g(x)) = e^{\frac{1}{x}}(-1x^{-2}) = \frac{-e^{\frac{1}{x}}}{x^2}$$

$$\frac{d}{dx} f(g(2)) = \frac{-e^{\frac{1}{2}}}{(2)^2} = \boxed{\frac{-\sqrt{e}}{4}}$$

158. *Answer is A.*

If the function $f(x) = \ln(x^2 - 1)$ then $\frac{f(7) - f(5)}{f'(7) - f'(5)} =$

$$f(x) = \ln(x^2 - 1)$$

$$f'(x) = \frac{2x}{x^2 - 1}$$

$$\frac{f(7) - f(5)}{f'(7) - f'(5)} = \frac{\ln 48 - \ln 24}{\frac{14}{48} - \frac{10}{24}} = \frac{\ln \frac{48}{24}}{\frac{-6}{48}} = \frac{\ln 2}{\frac{-1}{8}} = \boxed{-8\ln 2}$$

159. *Answer is E.*

If $f(x) = x^e e^x$ then $f'(x) =$

$$f'(x) = x^e e^x + e^x(ex^{e-1}) = x^e e^x + e^x ex^{e-1} = x^e e^x + x^{e-1}e^{x+1} \quad \leftarrow \text{product rule}$$

$$f'(x) = x^{e-1}e^x(x + e) = \boxed{\frac{x^e e^x(x+e)}{x}}$$

160. *Answer is E.*

If $y = x - 1$ and $x > 1$ then $\dfrac{d^2(\ln y)}{dx^2} =$

$$\ln y = \ln(x-1) \qquad \leftarrow \text{ln both sides}$$

$$\frac{d(\ln y)}{dx} = \frac{1}{x-1} = (x-1)^{-1}$$

$$\frac{d^2(\ln y)}{dx^2} = -1(x-1)^{-2} = \boxed{\frac{-1}{(x-1)^2}}$$

161. *Answer is C.*

The slope of the line ***normal*** to the curve $y = xe^{x^3}$ at $x = 1$ is

$$y'(x) = xe^{x^3}(3x^2) + e^{x^3}(1) \qquad \leftarrow \text{product rule}$$

slope of ***tangent*** → $y'(1) = (1)e^{(1)^3}(3(1)^2) + e^{(1)^3} = e(3) + e = 4e$

slope of ***normal*** → negative reciprocal $\boxed{\dfrac{-1}{4e}}$

162. *Answer is A.*

If $f(x) = 1 + \ln(x+2)$ then $f^{-1}(x) =$

$$y = 1 + \ln(x+2)$$

$$x = 1 + \ln(y+2)$$

$$(x-1) = \ln(y+2)$$

$$e^{(x-1)} = y + 2$$

$$e^{x-1} - 2 = y$$

$$f^{-1}(x) = e^{x-1} - 2$$

163. *Answer is A.*

If $f(x) = x \ln\sqrt{x}$ what is $f'(x) =$

$$f(x) = x\ln\sqrt{x} = x\ln x^{\frac{1}{2}} = \tfrac{1}{2}x\ln x \qquad \leftarrow \text{ln rules}$$

$$f'(x) = \tfrac{1}{2}\left[x\left(\frac{1}{x}\right) + \ln x(1)\right] = \tfrac{1}{2}\left[1 + \ln x\right] = \frac{1}{2} + \ln x^{\frac{1}{2}} = \boxed{\frac{1}{2} + \ln\sqrt{x}} \qquad \leftarrow \text{product rule}$$

164. *Answer is A.*

If $y = e^{4x^2}$ then $\frac{d(\ln y)}{dx} =$

$$\ln y = \cancel{\ln e}^{4x^2}$$

$$\ln y = 4x^2$$

$$\frac{d(\ln y)}{dx} = \boxed{8x}$$

165. *Answer is C.*

If $f(x) = \ln(x^2 - e^{2x})$ then $f'(1) =$

$$f'(x) = \frac{2x - 2e^{2x}}{x^2 - e^{2x}}$$

$$f'(1) = \frac{2(1) - 2e^{2(1)}}{(1)^2 - e^{2(1)}} = \frac{2 - 2e^2}{1 - e^2} = \frac{2\cancel{(1 - e^2)}}{\cancel{1 - e^2}} = \boxed{2}$$

166. *Answer is B.*

Write the equation of the line perpendicular to the tangent of the curve represented by the equation $y = e^{x+1}$ at $x = 0$

$$y' = e^{x+1}$$

$$y'(0) = e^{0+1} = e$$

Slope of tangent $= e$

Slope of normal $= \frac{-1}{e}$

$$y(0) = e^{0+1} = e$$

Point of tangency $(0, e)$

Equation of the normal at point $(0, e)$

Slope $= \frac{rise}{run} = \frac{-1}{e} = \frac{y - e}{x - 0}$

$$ey - e^2 = -x$$

$$ey = -x + e^2$$

$$\boxed{y = -\frac{1}{e}x + e}$$

167. *Answer is B.*

The second derivative of $f(x) = \ln x$ at $x = 3$ is

$$f'(x) = \frac{1}{x} = x^{-1}$$

$$f''(x) = -x^{-2} = \frac{-1}{x^2}$$

$$f''(3) = \frac{-1}{(3)^2} = \boxed{\frac{-1}{9}}$$

168. *Answer is A.*

Find the equation of the line tangent to $f(x)=2x+2e^x$ at $x=0$

$f(x)=2x+2e^x$

$f'(x)=2+2e^x$

$f'(0)=2+2e^0=4$

Point $(0,2)$ $m=4$

Line $\rightarrow$ $\boxed{y=4x+2}$

169. *Answer is D.*

Find y'' for $y=x\ln x-3x$

$y'=x(\tfrac{1}{x})+\ln x(1)-3$ ← product rule

$y'=\ln x-2$

$y''=\boxed{\dfrac{1}{x}}$

170. *Answer is A.*

If $f(x)=e^{\frac{1}{x}}=e^{x^{-1}}$ then $f'(x)=$

$f'(x)=e^{\frac{1}{x}}(-1x^{-2})=\boxed{\dfrac{-e^{\frac{1}{x}}}{x^2}}$

171. *Answer is B.*

If $f(x)=\ln\sqrt{x}$ then $f''(x)=$

$f(x)=\ln x^{\frac{1}{2}}=\tfrac{1}{2}\ln x$

$f'(x)=\tfrac{1}{2}\left(\dfrac{1}{x}\right)=\dfrac{1}{2}x^{-1}$

$f''(x)=\dfrac{1}{2}(-1x^{-2})=\boxed{\dfrac{-1}{2x^2}}$

172. *Answer is C.*

Difficulty = **0.88**

If $f(x)=(x-1)^{\frac{3}{2}}+\dfrac{e^{x-2}}{2}$ then $f'(2)=$

$f'(x)=\dfrac{3}{2}(x-1)^{\frac{1}{2}}+\dfrac{1}{2}(1)e^{x-2}$

$f'(2)=\dfrac{3}{2}\sqrt{(2-1)}+\dfrac{1}{2}e^{2-2}=\dfrac{3}{2}+\dfrac{1}{2}e^0=\boxed{2}$

173. *Answer is C.*

If $y = \ln(e^{-t^2} + 10)$ then $\dfrac{dy}{dx} =$

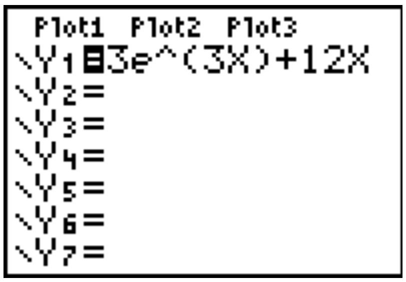
$$y' = \frac{e^{-t^2}(-2t)}{e^{-t^2}+10} = \boxed{\frac{-2te^{-t^2}}{e^{-t^2}+10}}$$

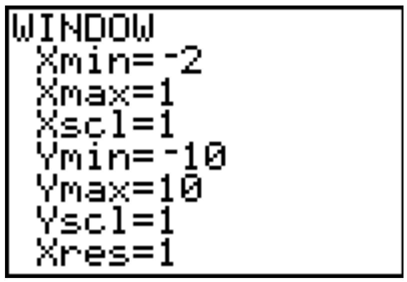
174. *Answer is B.*

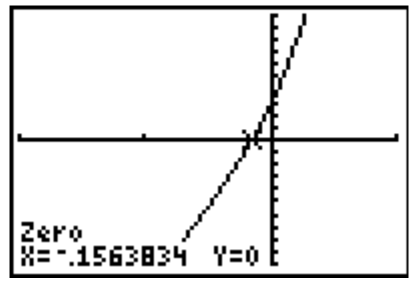
The function f defined by $f(x) = e^{3x} + 6x^2 + 1$ has a horizontal tangent at $x =$

$$f'(x) = 3e^{3x} + 12x = 0$$

$$x = \boxed{-0.1563834}$$

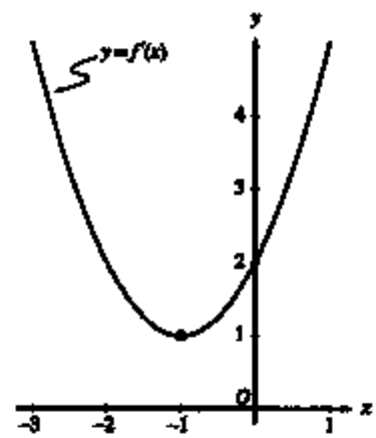






175. *Answer is B.*

The graph of the derivative of the function f is shown in the diagram. If $f(0) = 0$ then which of the following is true ?



CAREFUL !!! $\rightarrow$ graph of $f'(x)$

$f'(x)$ always ***positive*** $\leftarrow$ $f(x)$ is always ***increasing***

$f(0) = 0$ $\leftarrow$ given point $(0, 0)$ on $f(x)$

$f(-1) < 0$ $\leftarrow$ $f(x)$ always increasing and through $(0, 0)$

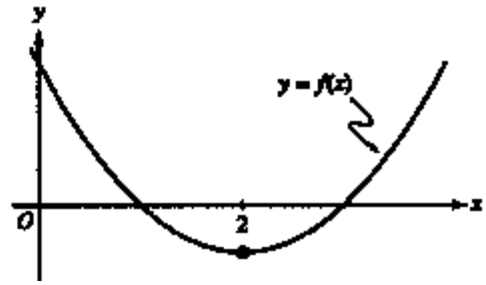
$f'(-1) = 1$ $\leftarrow$ point $(-1, 1)$ on $f'(x)$ graph

$f''(-1) = 0$ $\leftarrow$ horizontal tangent on $f'(x)$ graph

$$\underbrace{f(-1) < 0}_{negative} < \underbrace{f''(-1) = 0}_{zero} < \underbrace{f'(-1) = 1}_{positive}$$

176. *Answer is A.*

The graph of the twice differentiable function $f(x)$ is shown in the graph. Which of the following statements is true ?



$f(2) = \textbf{negative} \rightarrow$ below x-axis

$f'(2) = \textbf{zero} \rightarrow$ horizontal tangent

$f''(2) = \textbf{positive} \rightarrow$ concave upwards

$\therefore \quad \underbrace{f(2)}_{negative} < \underbrace{f'(2)}_{zero} < \underbrace{f''(2)}_{positive}$

177. *Answer is D.*

Simplify: $\dfrac{\ln 16}{3\ln 4 - 3\ln 2} =$

$$\frac{\ln 16}{3\ln 4 - 3\ln 2} = \frac{\ln 2^4}{3\ln 2^2 - 3\ln 2} = \frac{4\ln 2}{6\ln 2 - 3\ln 2} = \frac{4\cancel{\ln 2}}{3\cancel{\ln 2}} = \boxed{\frac{4}{3}}$$

178. *Answer is C.*

Find the equation of the line perpendicular to the line tangent to $f(x) = \ln(3-2x)$ at $x = 1$

$f(x) = \ln(3-2x)$

$f'(x) = \dfrac{-2}{3-2x}$

$f'(1) = \dfrac{-2}{3-2(1)} = -2$

Slope of normal $= \dfrac{1}{2}$

Point $(1,0)$ $\quad m = \dfrac{1}{2}$

Normal line $\rightarrow \textit{slope} = \dfrac{rise}{run} = \dfrac{1}{2} = \dfrac{y-0}{x-1}$

$2y = x - 1$

$\boxed{y = \tfrac{1}{2}(x-1)}$

179.

Implicit Differentiation $\rightarrow$ used when it is very difficult or impossible to isolate the variable y in terms of x Involves lots of chain rule/product rule operations.

180. *Answer is A.*

If $xy + y = 3$ then $\dfrac{dy}{dx} =$

$xy + y = 3$

$xy' + y + y' = 0$

$y'[x+1] = -y$

$$\boxed{\frac{dy}{dx} = \frac{-y}{x+1}}$$

181. *Answer is C.*

If $x + y = xy$ then $\dfrac{dy}{dx} =$

$$1 + y' = xy' + y$$
$$y' - xy' = y - 1$$
$$y'(1 - x) = y - 1$$
$$y' = \frac{y-1}{1-x} = \boxed{\frac{1-y}{x-1}}$$

182. *Answer is C.*

If $y^2 - 2xy = 16$ then $\dfrac{dy}{dx} =$

$$2yy' - 2[xy' + y] = 0$$
$$2yy' - 2xy' = 2y$$
$$2y'(y - x) = 2y$$
$$2y'(y - x) = \frac{2y}{2(y-x)} = \boxed{\frac{y}{y-x}}$$

183. *Answer is A.*

If $x^2 + xy + y^3 = 0$ then in terms of x and y, $\dfrac{dy}{dx} =$

$$x^2 + xy + y^3 = 0$$
$$2x + [xy' + y] + 3y^2 y' = 0$$
$$2x + xy' + y + 3y^2 y' = 0$$
$$y'\left[x + 3y^2\right] = -2x - y$$
$$\frac{dy}{dx} = \frac{-2x - y}{x + 3y^2} = \boxed{-\frac{2x + y}{x + 3y^2}}$$

184. *Answer is E.*

If $x^2 - 2xy + 3y^2 = 8$ then $\dfrac{dy}{dx} =$

$$2x - 2[xy' + y] + 6yy' = 0$$
$$2x - 2xy' - 2y + 6yy' = 0$$
$$y'[6y - 2x] = 2y - 2x$$
$$\frac{dy}{dx} = \frac{2y - 2x}{6y - 2x} = \frac{2(y - x)}{2(3y - x)} = \boxed{\frac{y - x}{3y - x}}$$

185. *Answer is B.*

Find $\dfrac{dy}{dx}$ if $x^2 + y^2 = -2xy$

$$2x + 2yy' = -2[xy' + y]$$
$$2x + 2yy' = -2xy' - 2y$$
$$2yy' + 2xy' = -2y - 2x$$
$$y'(2y + 2x) = -2(y + x)$$
$$y' = \frac{-\cancel{2}\,\cancel{(y+x)}}{\cancel{2}\,\cancel{(y+x)}} = \boxed{-1}$$

186. *Answer is B.*

Find y' if $y^2 - 3xy + x^2 = 7$

$$y^2 - 3xy + x^2 = 7$$
$$2yy' - 3[xy' + y] + 2x = 0$$
$$2yy' - 3xy' - 3y + 2x = 0$$
$$y'[2y - 3x] = 3y - 2x$$
$$y' = \boxed{\frac{3y - 2x}{2y - 3x}}$$

187. *Answer is C.*

Given y is a differentiable function of x, find $\dfrac{dy}{dx}$ for $x^3 - xy + y^3 = 1$

$$3x^2 - [xy' + y] + 3y^2y' = 0$$
$$3x^2 - xy' - y + 3y^2y' = 0$$
$$y'[3y^2 - x] = y - 3x^2$$
$$\boxed{\frac{dy}{dx} = \frac{y - 3x^2}{3y^2 - x}}$$

188. *Answer is B.*

If $y^2 = x + y^3$ then $y' =$

$$2yy' = 1 + 3y^2y'$$
$$2yy' - 3y^2y' = 1$$
$$y'\left[2y - 3y^2\right] = 1$$
$$\boxed{y' = \frac{1}{2y - 3y^2}}$$

189. *Answer is A.*

Find $\frac{dy}{dx}$ for $2x^2 + xy + 3y^2 = 0$

$$4x + \left[xy' + y\right] + 6yy' = 0$$
$$xy' + 6yy' = -4x - y$$
$$y'\left[x + 6y\right] = -4x - y$$
$$\boxed{y' = -\frac{4x + y}{x + 6y}}$$

190. *Answer is B.*

Given y is a differentiable function of x, find $\frac{dy}{dx}$ for $3x^2 - 2xy + 5y^2 = 1$

$$6x - 2\left[xy' + y\right] + 10yy' = 0$$
$$6x - 2xy' - 2y + 10yy' = 0$$
$$y'\left[10y - 2x\right] = 2y - 6x$$
$$y' = \frac{2(y - 3x)}{2(5y - x)}$$
$$\boxed{\frac{dy}{dx} = \frac{y - 3x}{5y - x}}$$

191. *Answer is C.*

If $x^2 + y^3 = x^3y^2$ then $\frac{dy}{dx} =$

$$2x + 3y^2y' = x^3 2yy' + y^2(3x^2)$$
$$3y^2y' - 2x^3yy' = 3x^2y^2 - 2x$$
$$y'(3y^2 - 2x^3y) = 3x^2y^2 - 2x$$
$$y' = \boxed{\frac{3x^2y^2 - 2x}{3y^2 - 2x^3y}}$$

192. *Answer is A.*

If $xy^2 - y^3 = x^2 - 5$ then $\frac{dy}{dx} =$

$$\left[x2yy' + y^2\right] - 3y^2y' = 2x - 0$$

$$2xyy' - 3y^2y' = 2x - y^2$$

$$y'(2xy - 3y^2) = 2x - y^2$$

$$y' = \frac{2x - y^2}{2xy - 3y^2} = \boxed{\frac{y^2 - 2x}{3y^2 - 2xy}}$$

193. *Answer is A.*

Difficulty = **0.66**

If $x^3 + 3xy + 2y^3 = 17$ then in terms of x and y $\frac{dy}{dx} =$

$$x^3 + 3xy + 2y^3 = 17$$

$$3x^2 + 3\left[xy' + y\right] + 6y^2y' = 0$$

$$3x^2 + 3xy' + 3y + 6y^2y' = 0$$

$$y'\left[6y^2 + 3x\right] = -3x^2 - 3y$$

$$\frac{dy}{dx} = \frac{-3x^2 - 3y}{6y^2 + 3x} = \frac{-\cancel{3}(x^2 + y)}{\cancel{3}(2y^2 + x)} = \boxed{-\frac{x^2 + y}{x + 2y^2}}$$

194. *Answer is E.*

Find $\frac{dy}{dx}$ for $e^y = xy$

$$e^y y' = xy' + y$$

$$e^y y' - xy' = y$$

$$y'(e^y - x) = y$$

$$y' = \frac{y}{\underbrace{e^y}_{xy} - x} = \boxed{\frac{y}{xy - x}}$$

195. *Answer is D.*

Find y' if $\ln xy = x + y$

$$\ln x + \ln y = x + y$$

$$\frac{1}{x} + \frac{y'}{y} = 1 + y'$$

$$\tfrac{y'}{y} - y' = 1 - \tfrac{1}{x}$$

$$y'\left(\tfrac{1}{y} - 1\right) = \frac{x-1}{x}$$

$$y' = \left[\frac{\frac{x-1}{x}}{\frac{1-y}{y}}\right] = \left(\frac{x-1}{x}\right)\left(\frac{y}{1-y}\right) = \boxed{\frac{xy-y}{x-xy}}$$

196. *Answer is B.*

Find y' if $xe^y + 1 = xy$

$$\left[xe^y y' + e^y\right] + 0 = \left[xy' + y\right]$$

$$xe^y y' - xy' = y - e^y$$

$$y'\left[xe^y - x\right] = y - e^y$$

$$\boxed{y' = \frac{y - e^y}{xe^y - x}}$$

197. *Answer is C.*

Consider the curve $x + xy + 2y^2 = 6$ The slope of the line tangent to the curve at the point $(\,2,\ 1)$ is

$$1 + \left[xy' + y\right] + 2(2)yy' = 0$$

$$1 + xy' + y + 4yy' = 0$$

$$y'(x + 4y) = -y - 1$$

$$y' = \frac{-y-1}{x+4y}$$

$$y'(\,2,\ 1) = \frac{-1-1}{2+4(1)} = \frac{-2}{6} = \boxed{\frac{-1}{3}}$$

198. *Answer is A.*

The equation of the tangent to the curve $2x^2 - y^4 = 1$ at the point $(-1,\ 1)$ is

$$2x^2 - y^4 = 1$$
$$4x - 4y^3 y' = 0$$
$$4x = 4y^3 y'$$
$$\frac{x}{y^3} = y'$$

Slope of tangent at $(-1,\ 1)$

$$y' = \frac{x}{y^3}$$
$$y'(-1,\ 1) = \frac{-1}{(1)^3} = -1$$

Equation of tangent at $(-1,\ 1)$

$$\text{Slope} = \frac{rise}{run} = \frac{-1}{1} = \frac{y-1}{x+1}$$
$$y - 1 = -1x - 1$$
$$\boxed{y = -x}$$

199. *Answer is E.*

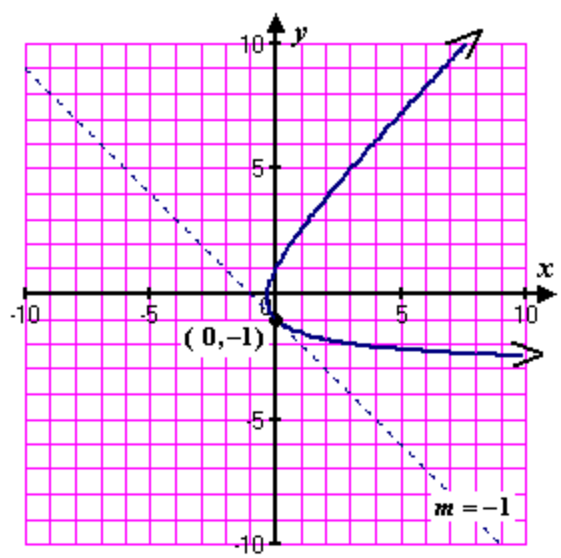
If $y^2 - 2xy = 21$ then $\frac{dy}{dx}$ at the point $(\ 2,-3)$ is

$$2yy' - 2[xy' + y] = 0$$
$$2yy' - 2xy' - 2y = 0$$
$$y'(2y - 2x) = 2y$$
$$y' = \frac{\not{2}y}{\not{2}(y-x)} = \frac{y}{y-x}$$
$$y'(2,-3) = \frac{-3}{-3-2} = \frac{-3}{-5} = \boxed{\frac{3}{5}}$$

200. *Answer is A.*   *Diagram is to help in the learning process only !*

The slope of the curve $y^2 - xy - 3x = 1$ at the point $(\ 0,-1)$ is

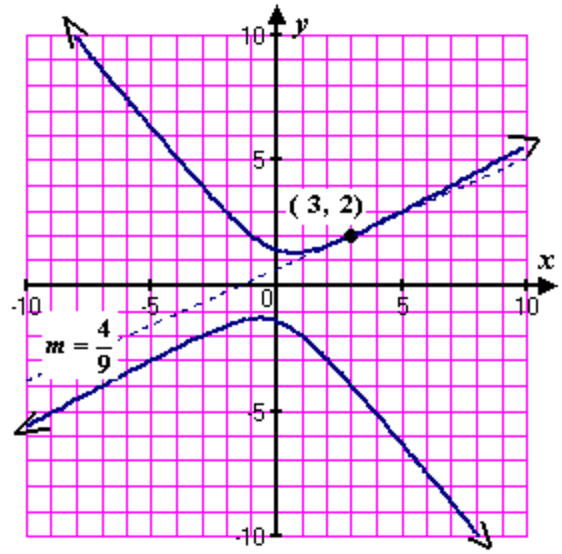
$$2yy' - [xy' + y(1)] - 3 = 0$$
$$2yy' - xy' - y - 3 = 0$$
$$y'[2y - x] = y + 3$$
$$y' = \frac{y+3}{2y-x}$$
$$\frac{dy}{dx}(\ 0,-1) = \frac{-1+3}{2(-1)-0} = \frac{2}{-2} = \boxed{-1}$$



201. *Answer is B.* *Diagram is to help in the learning process only !* Difficulty = **0.58**

What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point $(3, 2)$

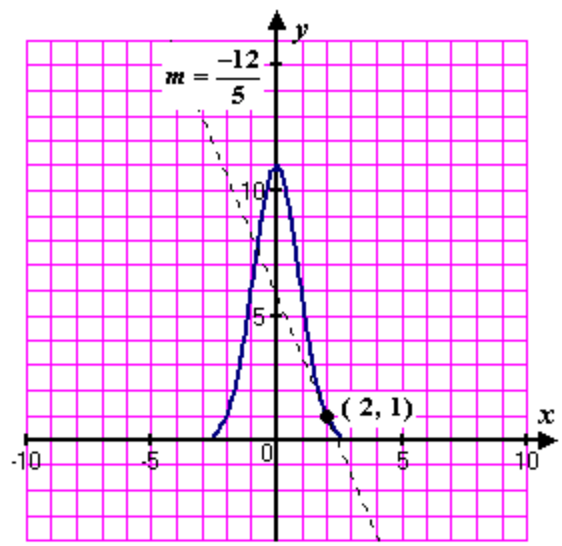
$$3y^2 - 2x^2 = 6 - 2xy$$
$$6yy' - 4x = -2[xy' + y]$$
$$6yy' - 4x = -2xy' - 2y$$
$$6yy' + 2xy' = 4x - 2y$$
$$2y'(3y + x) = 2(2x - y)$$
$$y' = \frac{2x - y}{3y + x}$$
$$y'(3, 2) = \frac{2(3) - 2}{3(2) + 3} = \boxed{\frac{4}{9}}$$



202. *Answer is A.* *Diagram is to help in the learning process only !*

The slope of the line tangent to the graph of $3x^2 + 5\ln y = 12$ at $(2, 1)$ is

$$3x^2 + 5\ln y = 12$$
$$6x + 5\frac{y'}{y} = 0$$
$$6xy + 5y' = 0$$
$$y' = \frac{-6xy}{5}$$
$$y'(2, 1) = \frac{-6(2)(1)}{5} = \boxed{-\frac{12}{5}}$$



203. *Answer is D.*

If $y = \ln(x^2 + y^2)$ then the value of $\frac{dy}{dx}$ at the point $(1,\ 0)$ is

$$y = \ln(x^2 + y^2)$$
$$y' = \frac{2x + 2yy'}{x^2 + y^2}$$
$$y'(x^2 + y^2) = 2x + 2yy'$$
$$y'(x^2 + y^2) - 2yy' = 2x$$
$$y'(x^2 + y^2 - 2y) = 2x$$
$$y' = \frac{2x}{x^2 + y^2 - 2y}$$
$$y'(1,\ 0) = \frac{2(1)}{(1)^2 + (0)^2 - 2(0)} = \boxed{2}$$

204. *Answer is B.*

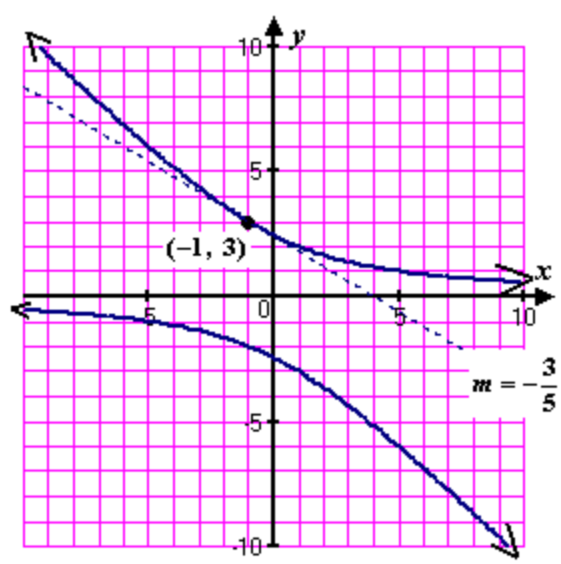
Consider the curve $5x - xy + y^2 = 7$ The slope of the line tangent to the curve at the point $(1,\ 2)$ is

$$5 - [xy' + y] + 2yy' = 0$$
$$5 - xy' - y + 2yy' = 0$$
$$2yy' - xy' = y - 5$$
$$y'(2y - x) = y - 5$$
$$y' = \frac{y - 5}{2y - x}$$
$$y'(1,2) = \frac{2 - 5}{2(2) - 1} = \frac{-3}{3} = \boxed{-1}$$

205. *Answer is A.* *Diagram is to help in the learning process only !*

If $y^2 + xy = 6$ what is $\frac{dy}{dx}$ at the point $(-1,\ 3)$

$$y^2 + xy = 6$$
$$2yy' + [xy' + y] = 0$$
$$2yy' + xy' + y = 0$$
$$y'[2y + x] = -y$$
$$y' = \frac{-y}{2y + x}$$
$$y'(-1,\ 3) = \frac{-3}{2(3) + (-1)} = \boxed{\frac{-3}{5}}$$



206. *Answer is A.*

The equation of the line tangent to the curve $y^2 - 2x - 4y = 1$ at $(-2,\ 1)$ is

$$y^2 - 2x - 4y = 1$$
$$2yy' - 2 - 4y' = 0$$
$$y'(2y-4) = 2$$
$$y' = \frac{2}{2(y-2)} = \frac{1}{y-2}$$
$$y'(-2,\ 1) = \frac{1}{1-2} = -1$$

Point $(-2,\ 1)$ and slope $m = -1$

$$\text{Slope} = \frac{rise}{run} = \frac{-1}{1} = \frac{y-1}{x+2}$$
$$y - 1 = -x - 2$$
$$\boxed{y = -x - 1}$$

207. *Answer is B.*

If $xy^2 + 2xy = 8$ then at the point $(\ 1,\ 2)$ $y' =$

$$\left[x2yy' + y^2\right] + 2\left[xy' + y\right] = 0$$
$$2xyy' + y^2 + 2xy' + 2y = 0$$
$$y'(2xy + 2x) = -2y - y^2$$
$$y' = \frac{-2y - y^2}{2xy + 2x}$$
$$y'(\ 1,\ 2) = \frac{-2(2) - (2)^2}{2(1)(2) + 2(1)} = \frac{-4-4}{4+2} = \frac{-8}{6} = \boxed{-\frac{4}{3}}$$

208. *Answer is D.*

If $7 = xy - e^{xy}$ then $\frac{dy}{dx} =$

$$0 = \left[xy' + y\right] - e^{xy}\left[xy' + y\right]$$
$$0 = xy' + y - xy'e^{xy} - ye^{xy}$$
$$ye^{xy} - y = y'(x - xe^{xy})$$
$$\boxed{\frac{-y}{x}} = \frac{-y(1-e^{xy})}{x(1-e^{xy})} = \frac{y(e^{xy}-1)}{x(1-e^{xy})} = \frac{ye^{xy} - y}{x - xe^{xy}} = y'$$

209. *Answer is C.*

Which is the slope of the line tangent to $y^2 + xy - x^2 = 11$ at $(2,\ 3)$

$$2yy' + [xy' + y] - 2x = 0$$
$$2yy' + xy' = 2x - y$$
$$y'(2y + x) = 2x - y$$
$$y' = \frac{2x - y}{2y + x}$$
$$y'(2,\ 3) = \frac{2(2) - (3)}{2(3) + (2)} = \boxed{\frac{1}{8}}$$

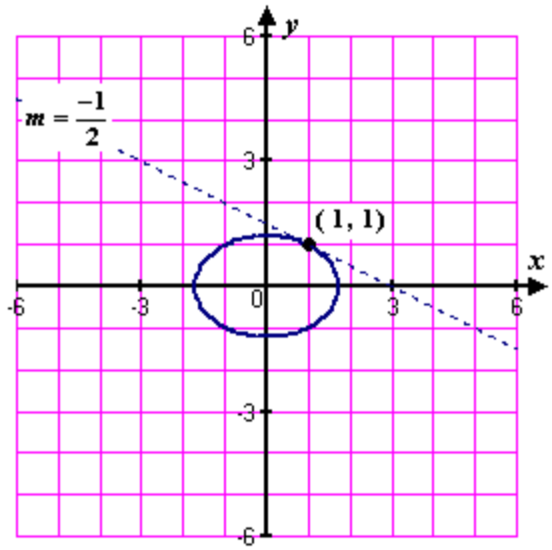
210. *Answer is D.*

The slope of the line tangent to the curve $3x^2 - 2xy + y^2 = 11$ at the point $(1, -2)$ is

$$3(2x) - 2[xy' + y] + 2yy' = 0$$
$$6x - 2xy' - 2y + 2yy' = 0$$
$$2y'(y - x) = 2y - 6x$$
$$y' = \frac{2(y - 3x)}{2(y - x)} = \frac{y - 3x}{y - x}$$
$$y'(1, -2) = \frac{(-2) - 3(1)}{(-2) - (1)} = \frac{-5}{-3} = \boxed{\frac{5}{3}}$$

211. *Answer is D.* *Diagram is to help in the learning process only !*

Find an equation of the tangent line to the graph of $x^2+2y^2=3$ at the point $(1,\ 1)$



$$x^2+2y^2=3$$
$$2x+2(2y)y'=0$$
$$2x+4yy'=0$$
$$y'=-\frac{2x}{4y}=-\frac{x}{2y}$$
$$y'(1,\ 1)=-\frac{1}{2(1)}=-\frac{1}{2}$$

Line with slope $m=-\frac{1}{2}$ through point $(1,\ 1)$

$$\text{Slope}=\frac{rise}{run}=\frac{-1}{2}=\frac{y-1}{x-1}$$
$$2y-2=-x+1$$
$$2y=-x+3$$
$$\boxed{x+2y=3}$$

212. *Answer is B.*

Suppose $x^2-xy+y^2=3$ Find $\frac{dy}{dx}$ at the point $(a,\ b)$

$$2x-[xy'+y]+2yy'=0$$
$$2x-xy'-y+2yy'=0$$
$$y'[2y-x]=y-2x$$
$$\frac{dy}{dx}=\frac{y-2x}{2y-x}$$
$$\frac{dy}{dx}(a,\ b)=\boxed{\frac{b-2a}{2b-a}}$$

213. *Answer is C.*

If $(x-y)^2 = y^2 - xy$ then $\frac{dy}{dx} =$

$$2(x-y)^1(1-y') = 2yy' - [xy' + y]$$

$$2x - 2xy' - 2y + \cancel{2yy'} = \cancel{2yy'} - xy' - y$$

$$-2xy' + xy' = -y + 2y - 2x$$

$$-xy' = y - 2x$$

$$y' = \frac{y-2x}{-x} = \boxed{\frac{2x-y}{x}}$$

214. *Answer is D.*

The slope of the line tangent to the graph of $\ln(x+y) = x^2$ at the point where $x = 1$ is

$$\ln(x+y) = x^2$$

$$\frac{1+y'}{(x+y)} = 2x$$

$$1 + y' = 2x(x+y)$$

$$y' = 2x^2 + 2xy - 1$$

$$y'(1, e-1) = 2(1)^2 + 2(1)(e-1) - 1$$

$$y'(1, e-1) = \boxed{2e-1}$$

Find point of tangency where $x = 1$

$$\ln(x+y) = x^2$$

$$\ln(1+y) = 1$$

$$1 + y = e^1$$

$$y = e - 1 \rightarrow \text{point } (1,\ e-1)$$

215. *Answer is A.*

The slope of the line tangent to the graph of $\ln(xy) = x$ at the point where $x = 1$ is

$$\ln x + \ln y = x$$

$$\frac{1}{x} + \frac{y'}{y} = 1$$

$$\frac{y'}{y} = 1 - \frac{1}{x}$$

$$y'(1, e) = e\left(\frac{1-1}{1}\right) = \boxed{0}$$

$$\ln(xy) = x$$

$$x = 1 \rightarrow \quad \ln(y) = 1$$

$$y = e$$

$$\text{Point} \rightarrow (1, e)$$

216. *Answer is B.*

If $e^{xy} = \ln x$, then $\frac{dy}{dx} =$

$$e^{xy}\left[xy' + y\right] = \frac{1}{x}$$

$$xy'e^{xy} + ye^{xy} = \frac{1}{x}$$

$$xy'e^{xy} = \frac{1}{x} - ye^{xy} = \frac{1 - xye^{xy}}{x}$$

$$y' = \boxed{\frac{1 - xye^{xy}}{x^2 e^{xy}}}$$

217. *Answer is C.*

The curve defined by $x^3 + xy - y^2 = 10$ has a vertical tangent line when $x =$

$$3x^2 + \left[xy' + y\right] - 2yy' = 0$$

$$3x^2 + xy' + y - 2yy' = 0$$

$$y'\left[x - 2y\right] = -y - 3x^2$$

$$y' = \frac{-y - 3x^2}{x - 2y}$$

Vertical tangent $y' = \frac{-y - 3x^2}{x - 2y} = \textit{undefined}$

$$x - 2y = 0$$

$$x = 2y$$

$$\tfrac{x}{2} = y$$

$$x^3 + xy - y^2 = 10$$

$$x^3 + x(\tfrac{x}{2}) - (\tfrac{x}{2})^2 = 10$$

$$x^3 + \frac{x^2}{2} - \frac{x^2}{4} = 10$$

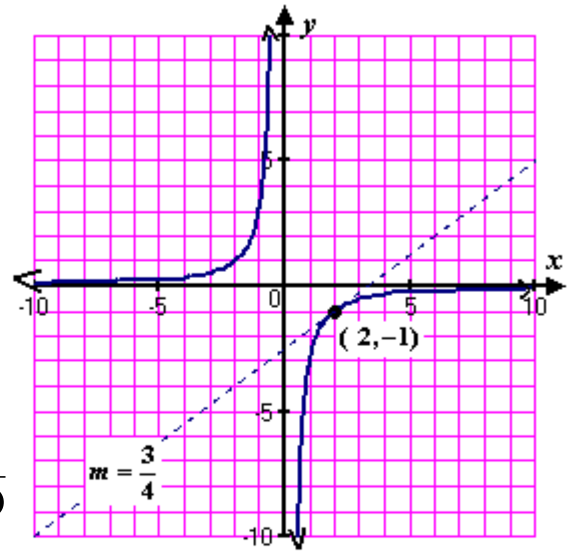
$$4x^3 + 2x^2 - x^2 = 40$$

$$4x^3 + x^2 - 40 = 0$$

$$\boxed{x = 2.0742}$$

218. *Answer is D.* *Diagram is to help in the learning process only !* Difficulty = **0.58**

The slope of the line tangent to the curve $y^2+(xy+1)^3=0$ at $(2,-1)$ is



$$y^2+(xy+1)^3=0$$
$$2yy'+3(xy+1)^2\left[xy'+y+0\right]=0$$
$$2yy'+3x(xy+1)^2y'+3y(xy+1)^2=0$$
$$y'\left[2y+3x(xy+1)^2\right]=-3y(xy+1)^2$$
$$y'=\frac{-3y(xy+1)^2}{2y+3x(xy+1)^2}$$
$$y'(2,-1)=\frac{-3(-1)(2(-1)+1)^2}{2(-1)+3(2)(2(-1)+1)}$$
$$=\frac{3(-1)^2}{-2+6(-1)^2}=\boxed{\frac{3}{4}}$$

219. *Answer is C.*

The curve $3y^2-3xy+2x^3=7$ has vertical tangents when

$$3(2yy')-3\left[xy'+y\right]+2(3x^2)=0$$
$$6yy'-3xy'-3y+6x^2=0$$
$$y'(6y-3x)=3y-6x^2$$
$$y'=\frac{\cancel{3}(y-2x^2)}{\cancel{3}(2y-x)}$$
$$y'=\frac{y-2x^2}{2y-x}$$

Vertical tangent $\rightarrow$ $y'=\textit{undefined}$

$$y'=\frac{y-2x^2}{2y-x}=\textit{undefined}$$
$$2y-x=0$$
$$\boxed{2y=x}$$

220. *Answer is A.*

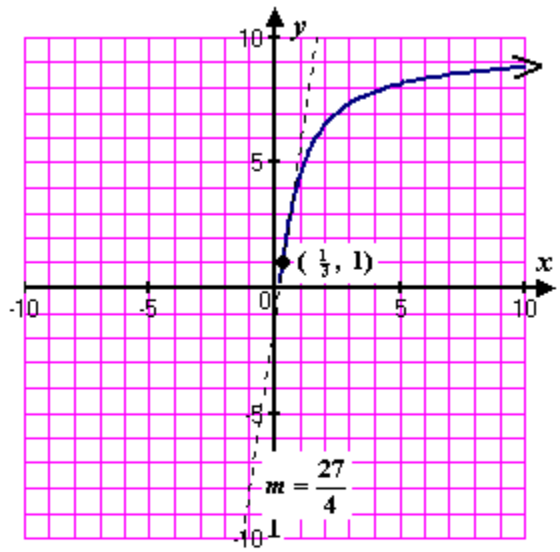
If $e^{xy}=2$ then at the point $(1,\ \ln 2)$ $\dfrac{dy}{dx}=$

$$e^{xy}\left[xy'+y(1)\right]=0 \qquad \leftarrow \text{product rule}$$
$$xy'e^{xy}+ye^{xy}=0$$
$$y'=\frac{-ye^{xy}}{xe^{xy}}=\frac{-y}{x}$$
$$y'(1,\ln 2)=\frac{-\ln 2}{1}=\boxed{-\ln 2}$$

221. *Answer is D.* *Diagram is to help in the learning process only !*

The slope of $9x-4x\ln y=3$ at $\left(\frac{1}{3},\ 1\right)$ is

$$9x-4x\ln y=3$$
$$9-4\left[x\frac{y'}{y}+\ln y\right]=0$$
$$9=4x\frac{y'}{y}+4\ln y$$
$$9y=4xy'+4y\ln y$$
$$9y-4y\ln y=4xy'$$
$$y'=\frac{9y-4y\ln y}{4x}$$
$$y'(\tfrac{1}{3},1)=\frac{9(1)-4(1)\ln(1)}{4(\frac{1}{3})}$$
$$y'(\tfrac{1}{3},1)=\frac{9-4(0)}{\frac{4}{3}}=\frac{9}{1}\times\frac{3}{4}=\boxed{\frac{27}{4}}$$



222. *Answer is A.* *Diagram is to help in the learning process only !*

If $2x^3+3xy+e^y=6$ what is y' when $x=0$

When $x=0$
$$2x^3+3xy+e^y=6$$
$$2(0)^3+3(0)y+e^y=6$$
$$e^y=6$$
$$y=\ln 6$$
point $(\ 0,\ \ln 6)$

$$2x^3+3xy+e^y=6$$
$$6x^2+3\left[xy'+y\right]+e^y y'=0$$
$$6x^2+3xy'+3y+e^y y'=0$$
$$y'(3x+e^y)=-3y-6x^2$$
$$y'=\frac{-3y-6x^2}{3x+e^y}$$
$$y'(\ 0,\ \ln 6)=\frac{-3\ln 6-6(0)^2}{3(0)+e^{\ln 6}}=\frac{-3\ln 6}{6}\approx\boxed{-0.8958}$$

223. *Answer is A.*

If $\frac{dy}{dx}=1+y^2$ then $\frac{d^2y}{dx^2}=$

$$y'=1+y^2$$
$$y''=0+2yy'=2y\underbrace{(y')}_{1+y^2}=\boxed{2y(1+y^2)}$$

224. *Answer is D.*

If a point moves on the curve $x^2 + y^2 = 25$, then, at $(0,\ 5)$, $\dfrac{d^2y}{dx^2}$ is

$$x^2 + y^2 = 25$$
$$2x + 2yy' = 0$$
$$2yy' = -2x$$
$$y' = -\frac{\not{2}x}{\not{2}y} = -\frac{x}{y} \quad \to$$
$$y'(0,\ 5) = -\frac{0}{5} = 0 \quad \to$$

$$yy' = -x$$
$$\left[yy'' + y'y'\right] = -1$$
$$yy'' + (y')^2 = -1$$
$$y'' = \frac{-1-(y')^2}{y}$$
$$y''(0,\ 5) = \frac{-1-(0)^2}{5} = \boxed{-\frac{1}{5}}$$

225. *Answer is E.*

If $y^2 - 3x = 7$ then $\dfrac{d^2y}{dx^2} =$

$$y^2 - 3x = 7$$
$$2yy' - 3 = 0 \quad \leftarrow y' = \tfrac{3}{2y}$$
$$2\left[yy'' + y'y'\right] - 0 = 0$$
$$2yy'' + 2y'y' = 0$$
$$y'' = \frac{-2y'y'}{2y} = \frac{-(\frac{3}{2y})(\frac{3}{2y})}{\frac{y}{1}} = \frac{-9}{4y^2} \times \frac{1}{y} = \boxed{\frac{-9}{4y^3}}$$

226. *Answer is B.*

Difficulty = **0.25**

If $\dfrac{dy}{dx} = \sqrt{1-y^2}$, then $\dfrac{d^2y}{dx^2} =$

$$y' = (1-y^2)^{\frac{1}{2}}$$
$$y'' = \frac{1}{2}(1-y^2)^{-\frac{1}{2}}(-2yy')$$
$$y'' = \frac{1(-\not{2}yy')}{\not{2}(1-y^2)^{\frac{1}{2}}} = \frac{-yy'}{y'} = \boxed{-y}$$

227. *Answer is D.*

Difficulty = **0.79**

The table gives values of f, f', g and g' at selected values of x If $h(x) = f(g(x))$ then $h'(1) =$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	6	5	3	-2
1	3	-3	-1	2
3	1	-2	2	3

$$h'(x) = f'(g(x))g'(x)$$
$$h'(1) = f'(g(1))g'(1)$$
$$h'(1) = f'(-1)[2]$$
$$h'(1) = 5[2] = \boxed{10}$$

228. *Answer is A.*

If $f(x) = \dfrac{4}{x-1}$ and $g(x) = 2x$ then the solution set of $f(g(x)) = g(f(x))$ is

$$\frac{4}{2x-1} = 2\left(\frac{4}{x-1}\right)$$
$$\frac{4}{2x-1} = \frac{8}{x-1}$$
$$16x - 8 = 4x - 4$$
$$12x = 4$$
$$\boxed{x = \tfrac{1}{3}}$$

229. *Answer is D.*

Let f and g be differentiable functions such that

$f(1) = 2$	$f'(1) = 3$	$f'(2) = -4$
$g(1) = 2$	$g'(1) = -3$	$g'(2) = 5$

If $h(x) = f(g(x))$ then $h'(1) =$

$$h'(x) = f'(g(x))g'(x)$$
$$h'(1) = f'(g(1))g'(1)$$
$$h'(1) = f'(2)[-3]$$
$$h'(1) = -4[-3] = \boxed{12}$$

230. *Answer is A.*

Difficulty = **0.60**

If f and g are twice differentiable and if $h(x) = f(g(x))$, then $h''(x) =$

$$h(x) = f(g(x)),$$
$$h'(x) = f'(g(x))g'(x) = [f'(g(x))][g'(x)] \quad \leftarrow \text{product rule}$$
$$h''(x) = f'(g(x))g''(x) + f''(g(x))g'(x)g'(x)$$
$$h''(x) = \boxed{f'(g(x))g''(x) + f''(g(x))[g'(x)]^2}$$

231. *Answer is B.*

Let f and g be differentiable functions such that

$$f(1)=4,\quad g(1)=3,\quad f'(3)=-5$$
$$f'(1)=-4,\quad g'(1)=-3,\quad g'(3)=2$$

If $h(x)=f(g(x))$ then $h'(1)=$

$$h'(x)=f'(g(x))g'(x)$$
$$h'(1)=f'(g(1))g'(1)=$$
$$h'(1)=f'(3)(-3)=(-5)(-3)=\boxed{15}$$

232. *Answer is E.*

The function **F** is defined by

$$\mathbf{F}(x)=G\left[x+G(x)\right]$$

where the graph of the function G is shown on the right.

The approximate value of $\mathbf{F}'(1)=$

$$\mathbf{F}(x)=G\left[x+G(x)\right]$$
$$\mathbf{F}'(x)=G'\left[x+G(x)\right](1+G'(x))$$
$$\mathbf{F}'(1)=G'\left[1+G(1)\right](1+G'(1))$$
$$\mathbf{F}'(1)=G'\left[1+3\right](1+(-2))$$
$$\mathbf{F}'(1)=G'\left[4\right](-1)$$
$$\mathbf{F}'(1)=\tfrac{2}{3}(-1)=\boxed{-\tfrac{2}{3}}$$

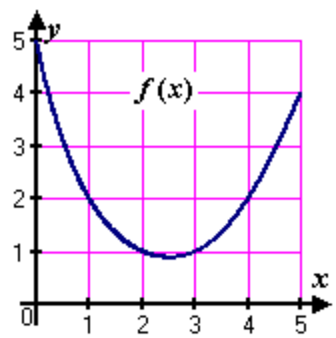
233. *Answer is D.*

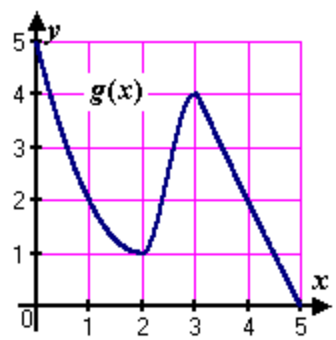
The graphs of functions f and g are shown on the right. If $h(x)=g\left[f(x)\right]$ which of the following statements are true about the function h

I. $h(0)=4$

II. h is increasing at $x=2$

III. The graph of h has a horizontal tangent at $x=4$





I. $h(0)=g\left[f(0)\right]=g\left[5\right]=0$ ☒ False $\neq 4$

$$h'(x)=g'\left[f(x)\right]f'(x)$$

II. $h'(2)=g'\left[f(2)\right]f'(2)=g'\left[1\right](-\tfrac{1}{2})=-2(-\tfrac{1}{2})=$ *positive*

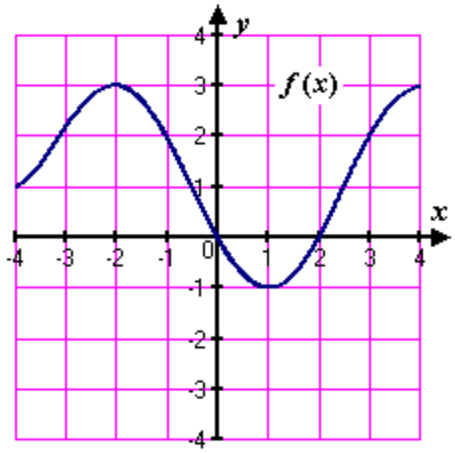
h is increasing at $x=2$ ☑ True $h'(2)>0$

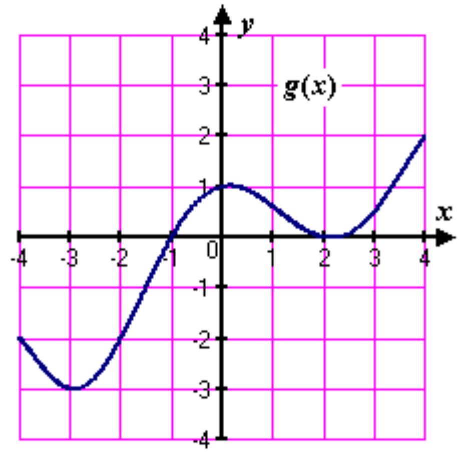
III. $h'(4)=g'\left[f(4)\right]f'(4)=g'\left[2\right](1)=0(1)=0$

The graph of h has a horizontal tangent at $x=4$ ☑ True $h'(4)=0$

234. *Answer is D.*

The composite function h is defined by $h(x) = f\left[g(x)\right]$ where f and g are functions whose graphs are shown below.





The number of horizontal tangent lines to the graph of h is

$$h(x) = f(g(x))$$
$$h'(x) = f'(g(x))g'(x) = 0 \quad \leftarrow \text{horizontal tangent}$$
$$\begin{bmatrix} f'(g(x) = -2) \rightarrow x = -2, -4 \\ f'(g(x) = 1) \rightarrow x = 0, 3.4 \end{bmatrix}\left[x = -3, 0, 2\right] = 0$$
$$x = 0 \text{ is duplicated } \begin{bmatrix} x = -2, -4 \\ x = 0, 3.4 \end{bmatrix}\left[x = -3, 0, 2\right] = 0 \leftarrow \boxed{6} \text{ horizontal tangent lines}$$

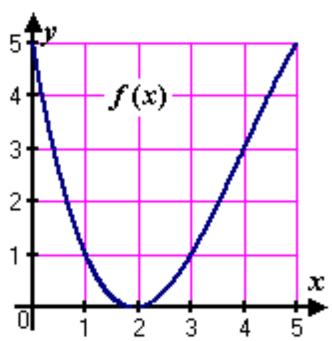
235. *Answer is D.*

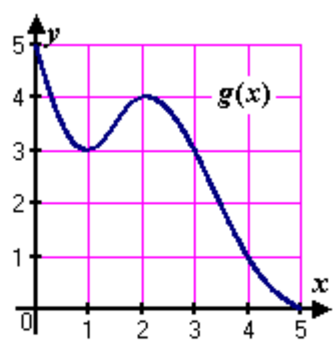
The graphs of functions f and g are shown at the right. If $h(x) = f\left[g(x)\right]$, which of the following statements are true about the function h

I. $h(2) = 5$

II. h is increasing at $x = 4$

III. The graph of h has a horizontal tangent at $x = 1$





I. $h(2) = f\left[g(2)\right] = f\left[4\right] = 3 \neq 5$ ☒ False

$h'(x) = f'\left[g(x)\right]g'(x)$

$h'(4) = f'\left[g(4)\right]g'(4) = f'\left[1\right](-1) = (-1)(-1) = \textit{positive}$

II. h is increasing at $x = 4$ ☑ True $h'(4) > 0$

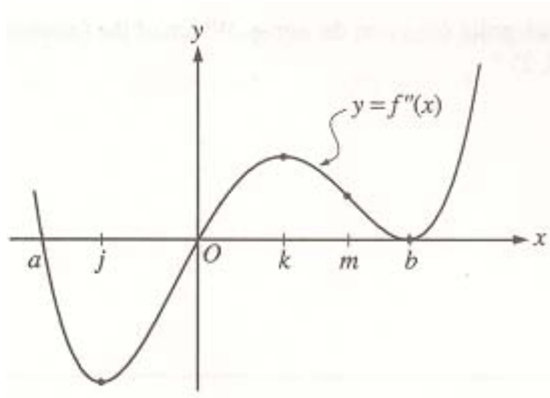
$h'(1) = f'\left[g(1)\right]g'(1) = f'\left[3\right](0) = (1)(0) = 0$

III. The graph of h has a horizontal tangent at $x = 1$ ☑ True $h'(4) = 0$

236. *Answer is A.*

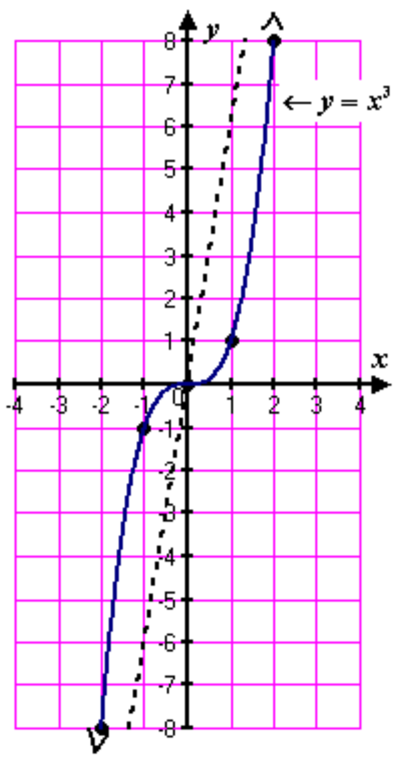
Difficulty = **0.39**

The second derivative of the function f is given by $f''(x) = x(x-a)(x-b)^2$ The graph of f'' is shown in the diagram. For what values of x does the graph of f have a point of inflection ?



$\underbrace{\text{Point of inflection } \textbf{\textit{definition}}}_{\text{when } f''(x) \text{ changes sign}}$

$\underbrace{a \text{ and } \mathbf{O} \text{ because graph of } f''(x) \text{ changes sign}}_{\text{(crosses } x\text{-axis)}}$



$y = x^3$

$y' = 3x^2$

$y'' = 6x$ cross

$y''(0) = 0$

$\leftarrow$ ***Yes*** **(0,0)**

Inflection points

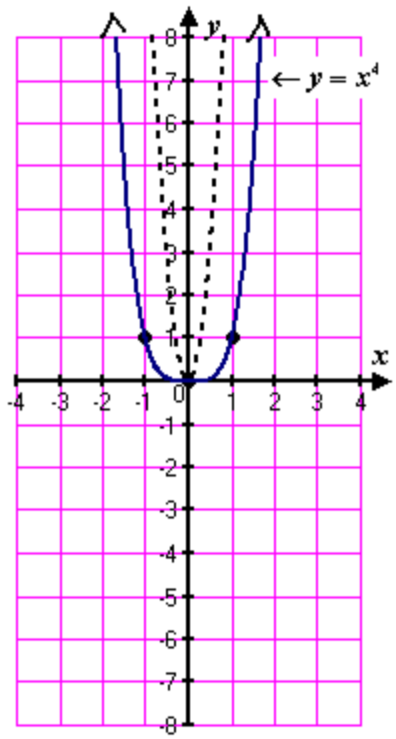
$y = x^4$

$y' = 4x^3$

$y'' = 12x^2$ bounce

$y''(0) = 0$

(0,0) ***No*** $\rightarrow$

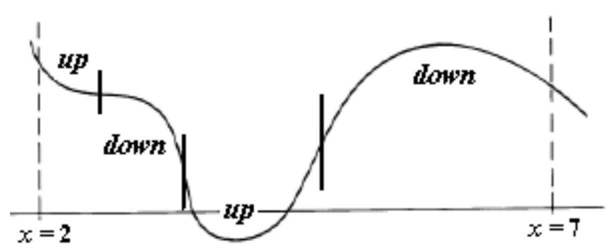


237. *Answer is C.*

The graph of $y = f(x)$ on the closed interval $[\,2,\ 7\,]$ is shown. How many points of inflection does this graph have on this interval ?

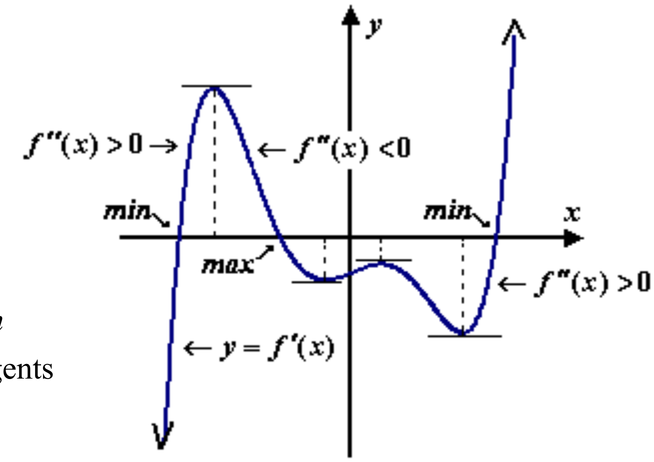
Points of inflection occur where the concavity ***changes***; marked on the graph on the right with a vertical line.

3 points of inflection



238. *Answer is E.*

> The diagram shows the graph of the ***derivative*** of a function f
> How many points of ***inflection*** does f have in the interval shown ?



Be very careful !!!

Inflection points $\rightarrow$ ***concavity*** changes ***sign***

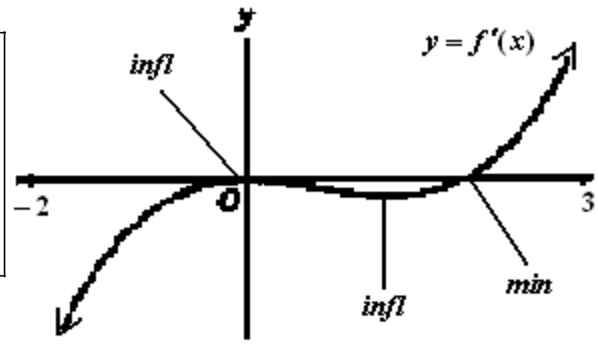
$\rightarrow$ on $f'(x)$ graph occur at horizontal tangents

$\rightarrow$ x-values of dashed lines !!!

$\rightarrow$ on this graph, **4** inflection points

239. *Answer is B.*

> The function f is defined on the closed interval $[-2, 3]$ The graph of $y = f'(x)$ is shown in the diagram. Which of the following describes the relative extrema of f and the points of inflection of the graph of f



On a ***derivative*** graph

$\rightarrow$ $f(x)$ minimum occur when $f'(x)$ changes from negative to positive

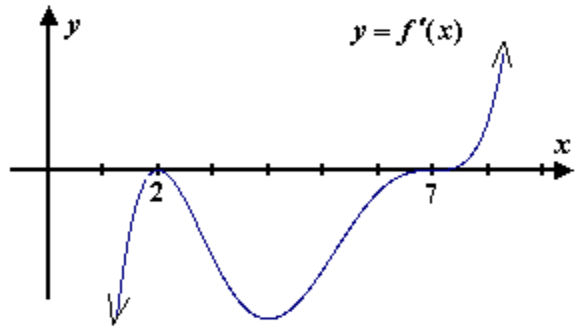
$\rightarrow$ $f(x)$ maximum occur when $f'(x)$ changes from positive to negative

$\rightarrow$ $f(x)$ inflection points occur when slope $f'(x) = 0$

So the $f(x)$ graph would have **1** ***minimum*** and **2** ***inflection*** points

240. *Answer is C.*

The function f is defined by $f'(x)=(x-2)^2(x-7)^3$ The graph of f has an inflection point where $x=$



$$f'(x)=(x-2)^2(x-7)^3$$
$$f''(x)=(x-2)^2 3(x-7)^2+(x-7)^3 2(x-2)$$
$$f''(x)=(x-2)(x-7)^2\left[3(x-2)+2(x-7)\right]$$
$$f''(x)=(x-2)(x-7)^2\left[3x-6+2x-14\right]$$
$$f''(x)=(x-2)(x-7)^2\left[5x-20\right]$$
$$f''(x)=5(x-2)^1(x-4)^1(x-7)^2$$

$f''(x)$ degree $\mathbf{4}$ with a bounce at $x=7$ and cross at $\underbrace{\boxed{x=2,4}}_{\textit{inflection points}}$

Answer could be estimated from the $y=f'(x)$ graph

241. *Answer is B.*

The function defined by $f(x)=(x-1)(x+2)^2$ has inflection points at $x=$

$f(x)=(x-1)(x+2)^2$ $\leftarrow y=f(x)$ degree $\mathbf{3}$ (maximum of one inflection point)

$$f(x)=(x-1)(x^2+4x+4)$$
$$f(x)=x^3+3x^2-4$$
$$f'(x)=3x^2+6x$$
$$f''(x)=6x+6=0$$
$$6x=-6$$
$$\boxed{x=-1}$$

Inflection number

242. *Answer is D.*

For some key values of x, the values of $f(x)$, $f'(x)$, and $f''(x)$ are given in the table. The equation of the tangent to the curve $y=f(x)$ at the point of inflection shown in the table is:

x	-8	-6	-4	$\underbrace{-2}_{x\text{-value}}$	0	2	4
$f(x)$	0	5	0	$\underbrace{-2}_{y\text{-value}}$	-4	-6	-4
$f'(x)$	4	0	-4	$\underbrace{-2}_{\text{slope}}$	-1	0	1
$f''(x)$	-2	-6	-2	$\underbrace{0}_{\text{change concavity}}$	1	4	3

Inflection number $\rightarrow x=-2$

($f''(x)$ changes sign)

Inflection point $\rightarrow f(-2)=-2\rightarrow(-2,-2)$

Slope at $(-2,-2)\rightarrow f'(-2)=-2$

Slope at $(-2,-2)$ and $m=-2$ $\rightarrow$ Slope $=\dfrac{rise}{run}=\dfrac{-2}{1}=\dfrac{y+2}{x+2}$

$$y+2=-x-4$$
$$\boxed{y=-x-6}$$

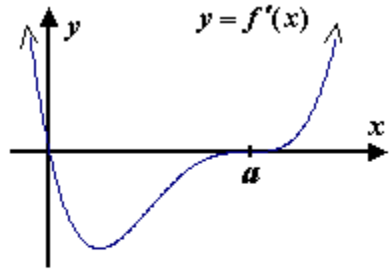
243. *Answer is B.*

Which of the following statements are true about the function f if its derivative f' is defined by $f'(x) = x(x-a)^3$ where $a > 0$

I. The graph of f is increasing at $x = 2a$

II. The function f has a local maximum at $x = 0$

III. The graph of f has an inflection point at $x = a$



Sketch graph of $f'(x)$ and look at zero's

Zeros at $x = 0,\ a$ $\leftarrow$ degree $\mathbf{4}$ (no bounces)

I. The graph of f is increasing at $x = 2a$ ☑ True

$f'(x) > 0 \rightarrow f(x)$ is increasing

II. The function f has a local maximum at $x = 0$ ☑ True

at $x = 0$ $f'(x)$ changes from $\underbrace{\text{positive to negative}}_{\text{maximum}}$

III. The graph of f has an inflection point at $x = a$ ☒ False

at $x = a$ $f'(x)$ changes from $\underbrace{\text{negative to positive}}_{\text{minimum}}$

which ***cannot*** be an inflection point

244. *Answer is D.*

If $f'(x) = x^3(x+2)^2$ then the graph of f has inflection points when $x =$

$f'(x) = x^3(x+2)^2$ $\leftarrow$ product rule

$f''(x) = x^3 2(x+2)^1 + (x+2)^2 3x^2$

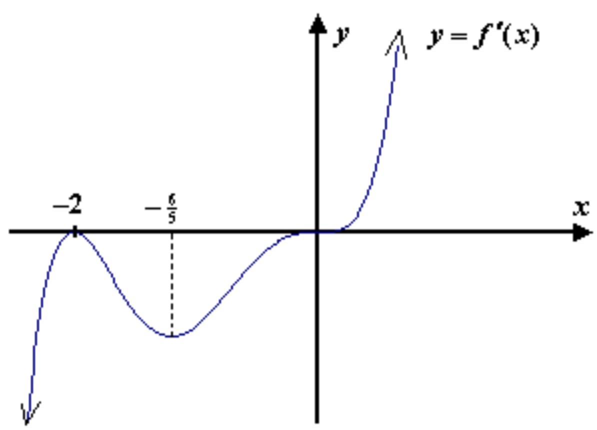
$f''(x) = 2x^3(x+2) + 3x^2(x+2)^2$

$f''(x) = x^2(x+2)[2x + 3(x+2)]$

$f''(x) = x^{2-bounce}(x+2)^{1-cross}(5x+6)^{1-cross}$

Inflection points (concavity changes)

occur at $\boxed{x = 2, -\frac{6}{5}}$



245. *Answer is E.*

If $f'(x) = -5(x-3)^2(x-2)$ which of the following features does the graph of $f(x)$ have ?

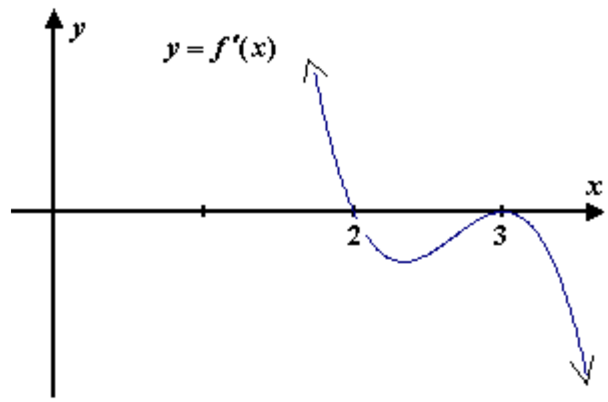
Sketch the graph of $y = f'(x)$ with zero's at $x = 2,\ 3$ and end behaviour ↖↘

$f'(x) = -5(x-3)^{2-bounce}(x-2)^{1cross}$

$\to$ $f(x)$ local maximum at $x = 2$ because $f'(x)$ changes from positive to negative

$\to$ inflection point at $x = 3$ because slope of $f'(x) = 0$

(another inflection point at $x \approx 2.3$)



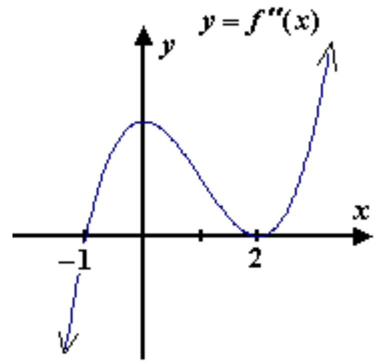
246. *Answer is B.*

A function $f(x)$ exists such that $f''(x) = (x-2)^2(x+1)$
How many points of inflection does $f(x)$ have ?

Sketch $f''(x) = (x-2)^{2-bounce}(x+1)^{1-cross}$

$f''(x-a) = 0$ $\leftarrow$ inflection point if and only if $f''(x)$ changes sign at $x = a$ (no bounce)

$f(x)$ has $\boxed{1}$ point of inflection when $x = -1$



247. *Answer is B.*

$f(x) = x^2 - 3x^3$ has a point of inflection at

$f(x) = x^2 - 3x^3$

$f'(x) = 2x - 9x^2$

$f''(x) = 2 - 18x = 0$

$2 = 18x$

$\frac{1}{9} = x$

Possible Inflection number

Inflection number $\frac{1}{9}$

Interval	$-\infty < x < \frac{1}{9}$	$\frac{1}{9} < x < \infty$
$f''(x)$	$f''(0) = +$	$f''(1) = -$
$f(x)$	Concave ***up***	Concave ***down***

There is **1** inflection point (concavity changes once) at $\boxed{x = \frac{1}{9}}$

248. *Answer is C.*

The graph of $y = 2x^3 + 5x^2 - 6x + 7$ has a point of ***inflection*** at $x =$

$y = 2x^3 + 5x^2 - 6x + 7$

$y' = 6x^2 + 10x - 6$

$y'' = 12x + 10 = 0$

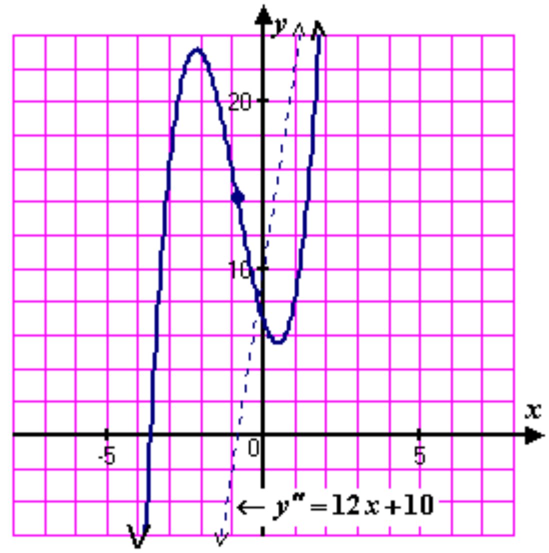
$12x = -10$

$\boxed{x = -\frac{5}{6}}$

Graph is concave ***down*** for $x < -\frac{5}{6}$

and concave ***up*** for $x > -\frac{5}{6}$

Graph of $y'' = 12x + 10$ changes sign at $x = -\frac{5}{6}$



249. *Answer is C.*

The number of ***inflection points*** in the curve $f(x) = x^4 - 4x^2$ is

$f(x) = x^4 - 4x^2$

$f'(x) = 4x^3 - 8x$

$f''(x) = 12x^2 - 8 = 0$

$12x^2 = 8$

$x^2 = \frac{8}{12}$

$x = \pm\sqrt{\frac{2}{3}}$

Possible Inflection numbers

Inflection numbers $-\sqrt{\frac{2}{3}}$ $\sqrt{\frac{2}{3}}$

Interval	$-\infty < x < -\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{2}{3}} < x < \sqrt{\frac{2}{3}}$	$\sqrt{\frac{2}{3}} < x < \infty$
$f''(x)$	$f''(-1) = +$	$f''(0) = -$	$f''(1) = +$
$f(x)$	Concave ***up***	Concave ***down***	Concave ***up***

There are **2** inflection points (concavity changes twice)

250. *Answer is B.*

An equation of the line ***tangent*** to $y = x^3 + 3x^2 + 2$ at it's point of ***inflection*** is

$y = x^3 + 3x^2 + 2$

$y' = 3x^2 + 6x$

$y'' = 6x + 6 = 0$

$6x = -6$

$x = -1$

$y''(0) = \textit{positive}$

$y''(-2) = \textit{negative}$

$\therefore$ $x = -1$ is inflection number

$y = x^3 + 3x^2 + 2$

$y(-1) = (-1)^3 + 3(-1)^2 + 2$

$y(-1) = -1 + 3 + 2 = 4$

Point of inflection $(-1,\ 4)$

$y' = 3x^2 + 6x$

$y'(-1) = 3(-1)^2 + 6(-1)$

$y'(-1) = 3 - 6 = -3$

Slope of tangent $= -3$

$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{-3}{1} = \frac{y-4}{x+1}$

$y - 4 = -3x - 3$

$\boxed{y = -3x + 1}$

251. *Answer is B.*

If the graph of $y = x^3 + ax^2 + bx - 4$ has a point of inflection at $(1, -6)$, what is the value of b

$$y = x^3 + ax^2 + bx - 4$$
$$y' = 3x^2 + 2ax + b$$
$$y'' = 6x + 2a$$
$$y''(1) = 6(1) + 2a = 0 \quad \leftarrow \text{point of inflection}$$
$$2a = -6$$
$$a = -3$$

$y = x^3 - 3x^2 + bx - 4$ point at $(1, -6)$

$$-6 = (1)^3 - 3(1)^2 + b(1) - 4$$
$$-6 = 1 - 3 + b - 4$$
$$\boxed{0 = b}$$

252. *Answer is C.*

Difficulty = **0.40**

At what value of x does the graph of $y = \frac{1}{x^2} - \frac{1}{x^3}$ have a point of inflection ?

$$y = \frac{1}{x^2} - \frac{1}{x^3} = x^{-2} - x^{-3} \qquad x \neq 0$$
$$y' = -2x^{-3} + 3x^{-4}$$
$$y'' = 6x^{-4} - 12x^{-5} = \frac{6}{x^4} - \frac{12}{x^5} = 0$$
$$\frac{6}{x^4} = \frac{12}{x^5}$$
$$12x^4 = 6x^5$$
$$12x^4 - 6x^5 = 0$$
$$6x^4(2 - x) = 0$$
$$\cancel{x = 0} \quad \big| \quad 2 = x$$

Possible Inflection number

Note this is ***not*** a polynomial graph, it is not defined when $x = 0$

$$y'' = \frac{6}{x^4} - \frac{12}{x^5} = \frac{6x - 12}{x^5}$$
$$y''(1) = \frac{6(1) - 12}{1^5} = \textit{negative}$$
$$y''(3) = \frac{6(3) - 12}{3^5} = \textit{positive}$$

Concavity changed at $x = 2$

253. *Answer is C.*

What is the value of k such that the curve $y = x^3 - \frac{k}{x}$ has a point of inflection at $x = 1$

$$y = x^3 - \frac{k}{x} = x^3 - kx^{-1}$$
$$y' = 3x^2 + kx^{-2}$$
$$y'' = 6x - 2kx^{-3} = 6x - \frac{2k}{x^3}$$
$$y''(1) = 6(1) - \frac{2k}{(1)^3} = 0$$
$$6 - 2k = 0$$
$$\boxed{3 = k}$$

254. *Answer is D.*

The curve $y = x^5 + 10x^4 - 5$ has points of inflection at $x =$

$$y = x^5 + 10x^4 - 5$$
$$y' = 5x^4 + 40x^3$$
$$y'' = 20x^3 + 120x^2 = 0$$
$$20x^2(x+6) = 0$$
$$x = 0 \quad\Big|\quad x = -6$$

Possible inflection points

Inflection numbers -6 0

Interval	$-\infty < x < -6$	$-6 < x < 0$	$0 < x < \infty$
$f''(x)$	$f''(-7) = -$	$f''(-1) = +$	$f''(1) = +$
$f(x)$	Concave ***down***	Concave ***up***	Concave ***up***

There is **1** inflection point (concavity changes once)

when $\boxed{x = -6}$

255. *Answer is E.*

The curve $y = 1 - 6x^2 - x^4$ has inflection points at $x =$

$$y' = -12x - 4x^3$$
$$y'' = -12 - 12x^2$$
$$y'' = -12(1 + x^2) = 0$$

No solution, therefore $\boxed{\textit{no}}$ inflection points.

256. *Answer is A.*

The slope of the line tangent to the curve $f(x) = x^3 + 3x^2 - 24x + 4$ at the point of inflection is

$$f(x) = x^3 + 3x^2 - 24x + 4$$
$$f'(x) = 3x^2 + 6x - 24$$
$$f''(x) = 6x + 6 = 0$$
$$6(x+1) = 0$$
$$x = -1$$

Inflection point

$$f'(x) = 3x^2 + 6x - 24$$
$$f'(-1) = 3(-1)^2 + 6(-1) - 24$$
$$f'(-1) = 3 - 6 - 24 = \boxed{-27}$$

257. *Answer is E.*

The curve $y = 3x^4 - 8x^3 + 6x^2 - 1$ has points of inflection at $x =$

$$y' = 12x^3 - 24x^2 + 12x$$
$$y'' = 36x^2 - 48x + 12 = 0$$

$3x^2 - 4x + 1 = 0$ $\leftarrow$ parabola opening up with zero's at $x = \frac{1}{3}, 1$

$(3x-1)(x-1) = 0$ $\therefore$ $y = f''(x)$ must change sign twice.

$$x = \tfrac{1}{3} \quad\Big|\quad x = 1$$

Inflection numbers

258. *Answer is E.*

The equation of the line tangent to the curve $f(x)=2x^3-3x^2$ at the point of inflection is

$f(x)=2x^3-3x^2$

$f'(x)=6x^2-6x$

$f''(x)=12x-6=0$

$12x=6$

$x=\frac{1}{2}$

Inflection number

$f(x)=2x^3-3x^2$

$f(\frac{1}{2})=2(\frac{1}{2})^3-3(\frac{1}{2})^2$

$f(\frac{1}{2})=\frac{1}{4}-\frac{3}{4}=-\frac{1}{2}$

Inflection point $(\frac{1}{2},-\frac{1}{2})$

$f'(x)=6x^2-6x$

$f'(\frac{1}{2})=6(\frac{1}{2})^2-6(\frac{1}{2})=\frac{3}{2}-3=-\frac{3}{2}$

Slope $=\dfrac{-3}{2}=\dfrac{y+\frac{1}{2}}{x-\frac{1}{2}}$

$2y+1=-3x+\frac{3}{2}$

$4y+2=-6x+3$

$\boxed{6x+4y=1}$

259. *Answer is D.*

An equation for the line tangent to the curve $f(x)=-x^3+12x+5$ at the point of inflection is

$f(x)=-x^3+12x+5$

$f'(x)=-3x^2+12$

$f''(x)=-6x=0$

$x=0$

Inflection number

$f(x)=-x^3+12x+5$

$f(0)=5$

$(0,\ 5)$ inflection point

$f'(x)=-3x^2+12$

$f'(0)=12$

Equation for the line tangent at $(0,\ 5)$

Slope $=\dfrac{rise}{run}=\dfrac{12}{1}=\dfrac{y-5}{x}$

$y-5=12x$

$\boxed{y-12x=5}$

260. *Answer is C.*

The curve $y=3x^5-5x^4+3x-2$ has a point of inflection at

$y=3x^5-5x^4+3x-2$

$y'=15x^4-20x^3+3$

$y''=60x^3-60x^2=0$

$60x^2(x-1)=0$

$x=0$ | $\boxed{x=1}$

Inflection number at $x=1$ (no bounce)

$y=3x^5-5x^4+3x-2$

$y(1)=3(1)^5-5(1)^4+3(1)-2$

$y(1)=3-5+3-2=-1$

Inflection point $\boxed{(1,-1)}$

261. *Answer is E.*

If the graph of $f(x)=2x^2+\frac{k}{x}$ has a point of inflection at $x=-1$ then the value of k is

$$f(x)=2x^2+kx^{-1}$$
$$f'(x)=4x-kx^{-2}$$
$$f''(x)=4+2kx^{-3}=4+\frac{2k}{x^3}$$
$$f''(-1)=4+\frac{2k}{(-1)^3}=0 \quad \leftarrow x=-1 \text{ at inflection point}$$
$$4-2k=0$$
$$4=2k$$
$$\boxed{2=k}$$

262. *Answer is A.*

The function $y=x^4+bx^2+8x+1$ has a horizontal tangent and a point of inflection for the same value of x What must be the value of b

$$y=x^4+bx^2+8x+1$$
$$y'=4x^3+2bx+8=0 \quad \rightarrow b=\frac{-8-4x^3}{2x}$$
$$y''=12x^2+2b=0 \quad \rightarrow b=\frac{-12x^2}{2}$$

$$\frac{-8-4x^3}{2x}=\frac{-12x^2}{2}$$
$$-16-8x^3=-24x^3$$
$$-16=-16x^3$$
$$1=x^3$$
$$1=x \ ☑$$
$$b=\frac{-12(1)^2}{2}=\boxed{-6}$$

263. *Answer is C.*

How many points of inflection does the graph of $y=2x^6+9x^5+10x^4-x+2$ have ?

$$y'=12x^5+45x^4+40x^3-1$$
$$y''=60x^4+180x^3+120x^2$$
$$y''=60x^2(x^2+3x+2)$$
$$y''=60x^{2=bounce}(x+1)^{1=cross}(x+2)^{1=cross}$$

y'' (crosses the x-axis twice $\rightarrow$ changes sign) $\rightarrow$ $\boxed{2}$ points of inflection at $x=-1$ and $x=-2$

264. *Answer is D.*

If the graph of $y = x^3 + ax^2 + bx - 8$ has a point of inflection at (2, 0), what is the value of b

$$y' = 3x^2 + 2ax + b$$
$$y'' = 6x + 2a$$
$$y''(2) = 6(2) + 2a = 0$$
$$2a = -12$$
$$\boxed{a = -6} \rightarrow$$

$$y = x^3 + ax^2 + bx - 8 \leftarrow \text{point } (2, 0)$$
$$0 = (2)^3 + (-6)(2)^2 + b(2) - 8$$
$$24 = 2b$$
$$\boxed{12 = b}$$

265. *Answer is A.*

What is the x-coordinate of the point of inflection on the graph of $y = xe^x$

$$y = xe^x$$
$$y' = xe^x + e^x$$
$$y'' = xe^x + e^x + e^x$$
$$y'' = xe^x + 2e^x = e^x(x+2) = 0$$
$$e^x(x+2) = 0$$
$$e^x \neq 0 \quad \Big| \quad \boxed{x = -2}$$

266. *Answer is C.*

What is the x-coordinate of the point of inflection of the graph of $y = x^3 + 3x^2 - 45x + 81$

$$y = x^3 + 3x^2 - 45x + 81$$
$$y' = 3x^2 + 6x - 45$$
$$y'' = 6x + 6 = 0$$
$$6x = -6$$
$$\boxed{x = -1}$$

Inflection number

267. *Answer is D.*

What are the x-coordinates of the points of inflection on the graph of the function $f(x) = 3x^4 - 4x^3 + 6$

$$f'(x) = 12x^3 - 12x^2$$
$$f''(x) = 36x^2 - 24x = 0$$
$$12x(3x - 2) = 0$$
$$\boxed{x = 0} \quad \Big| \quad \boxed{x = \tfrac{2}{3}}$$ ← x-coordinates of the points of inflection

268. *Answer is A.*

Given the function $h(x)=6x^3-8x^2+2$, at what x value(s) is/are the inflection point(s) ?

$$h(x)=6x^3-8x^2+2$$
$$h'(x)=18x^2-16x$$
$$h''(x)=36x-16=0$$
$$36x=16$$
$$\boxed{x=\frac{4}{9}}$$

Inflection number

269. *Answer is C.*

How many inflection points does $3x^4-5x^3-9x+2$ have ?

$$y=3x^4-5x^3-9x+2$$
$$y'=12x^3-15x^2-9$$
$$y''=36x^2-30x=0$$
$$6x(6x-5)=0$$
$$x=0 \quad\Big|\quad x=\tfrac{5}{6}$$

$\Rightarrow$

Interval	$-\infty<x<0$	$0<x<\frac{5}{6}$	$\frac{5}{6}<x<\infty$
y''	$y''(-1)=+$	$y''(\frac{1}{2})=-$	$y''(1)=+$
y	concave up	concave down	concave up

Both $x=0$ and $x=\frac{5}{6}$ have changes in concavity so there are **2** inflection points

270. *Answer is B.*

What is the x-coordinate of the point of inflection on the graph of $y=\dfrac{2}{3}x^3-2x^2+7$

$$y=\frac{2}{3}x^3-2x^2+7$$
$$y'=2x^2-4x$$
$$y''=4x-4=0$$
$$4x=4$$
$$\boxed{x=1}$$

271. *Answer is B.*

What is the x-coordinate of the point of inflection for the graph of $y=x^3+3x^2-1$

$$y=x^3+3x^2-1$$
$$y'=3x^2+6x$$
$$y''=6x+6=0$$
$$6x=-6$$
$$\boxed{x=-1}$$

272. *Answer is C.*

Difficulty = **0.78 U**

A particle moves along the x-axis so that its position at time t is $x(t) = 2t^2 - 7t + 3$ (x in cm and t in seconds). What is the velocity (in cm/sec) at time $t = 2$ seconds ?

$$x(t) = 2t^2 - 7t + 3$$
$$v(t) = x'(t) = 4t - 7$$
$$v(2) = 4(2) - 7 = \boxed{1 \text{ cm/sec}}$$

273. *Answer is C.*

Difficulty = **0.76 U**

A particle moves along the x-axis according to the function $x(t) = t^2 - 4t + 3$, where x (meters) is the position of the particle at time t (seconds). At what time t does the particle have a velocity of $6\ m/s$

$$x(t) = t^2 - 4t + 3$$
$$v(t) = 2t - 4 = 6$$
$$2t = 10$$
$$\boxed{t = 5}$$

274. *Answer is D.*

Difficulty = **0.75 U**

A particle moves along the x-axis so that its position at time t is $x(t) = 3t^3 + 2t^2 + 7$ where x is in meters and t is in seconds. Find the velocity at $t = 2$ seconds.

$$x(t) = 3t^3 + 2t^2 + 7$$
$$v(t) = 9t^2 + 4t$$
$$v(2) = 9(2)^2 + 4(2) = 36 + 8 = \boxed{44 \text{ m/s}}$$

275. *Answer is D.*

Difficulty = **0.71 U**

A particle moves along the x-axis according to the position function $x(t) = 2t^3 - 6t^2 + 9$ where x is in meters and t is in seconds. Find the value(s) of t when the particle is stationary.

$$x(t) = 2t^3 - 6t^2 + 9$$
$$v(t) = 6t^2 - 12t = 0 \quad \leftarrow \text{particle is stationary}$$
$$6t(t - 2) = 0$$
$$t = 0 \mid t = 2$$

276. *Answer is B.*

Difficulty = **0.69 U**

A particle moves along the x-axis so that its position at time t is $x(t) = t^2 - 2t + 5$ where x is in centimeters and t is in seconds. At what time is the particle's velocity $4\ cm/s$

$$x(t) = t^2 - 2t + 5$$
$$v(t) = 2t - 2 = 4 \quad \leftarrow \text{velocity } 4\ cm/s$$
$$2t = 6$$
$$\boxed{t = 3}$$

277. *Answer is B.*

Difficulty = **0.69 U**

An object moves along the x-axis so that its position at time t is $x = t^2 - 3t + 5$ where x is in meters and t is in seconds. At what time(s) is its velocity $5\ m/s$

$$x(t) = t^2 - 3t + 5$$
$$v(t) = 2t - 3 = 5 \quad \leftarrow \text{velocity } 5\ m/s$$
$$2t = 8$$
$$\boxed{t = 4}$$

278. *Answer is B.*

Difficulty = **0.51 H**

A particle moves along the x-axis according to the position function $x(t) = 2t^3 - 6t + 1$ where x is in meters and t is in seconds. For what values of t is the particle moving to the right ?

$$x(t) = 2t^3 - 6t + 1$$
$$v(t) = 6t^2 - 6 = 0$$
$$6t^2 = 6$$
$$t^2 = 1$$
$$t = \pm 1$$

$v(t) = 6t^2 - 6$

Parabola opening ***up*** with

zeros of $t = \pm 1$

moving to the ***right*** means $v(t) > 0$

$$\boxed{x < -1 \text{ or } x > 1}$$

279. *Answer is D.*

The position of an object moving in a straight path is given by $x(t) = kt^2 + 12t$, where x is in meters and t is in seconds. Find the value of k if the velocity of the object is $4\ m/s$ when $t = 2$ seconds.

$$x(t) = kt^2 + 12t$$
$$v(t) = 2kt + 12$$
$$v(2) = 2k(2) + 12 = 4 \quad \leftarrow \text{velocity of the object is } 4\ m/s$$
$$4k = -8$$
$$\boxed{k = -2}$$

280. *Answer is C.*

A particle moves along the x-axis according to the position function $x(t)=t^3-4t^2+3$ (x in meters, t in seconds). Determine the velocity in m/s at $t=-2$

$$x(t)=t^3-4t^2+3$$
$$v(t)=3t^2-8t$$
$$v(-2)=3(-2)^2-8(-2)=12+16=\boxed{28\text{ m/sec}}$$

281. *Answer is C.*

A particle moves along the x-axis according to the position function $x(t)=t^2-t$ (x in cm, t in sec). Determine the time t (in sec) when the velocity is $12\ cm/s$

$$x(t)=t^2-t$$
$$v(t)=2t-1=12 \quad \leftarrow \text{velocity is } 12\ cm/s$$
$$2t-1=12$$
$$2t=13$$
$$\boxed{t=6.5}$$

282. *Answer is C.*

As a particle moves along the x-axis, its distance from the origin is given by $x(t)=3t^2-4t+10$ where x is in meters and t is in seconds. At what time is the velocity $14\ m/s$

$$x(t)=3t^2-4t+10$$
$$v(t)=6t-4=14 \quad \leftarrow \text{velocity } 14\ m/s$$
$$6t=18$$
$$\boxed{t=3}$$

283. *Answer is A.*

An object moves so that its distance in metres, at time t seconds, is given by $f(t)$. What does $f'(2)$ represent ?

Position function $\rightarrow f(t)$ $\leftarrow$ position at any time t
Velocity function $\rightarrow f'(x)$ $\leftarrow$ velocity at any time t
Velocity at $t=2 \rightarrow f'(2)$

284. *Answer is B.*

As a particle moves along the x-axis, its distance from the origin is given by $x(t)=t^2-6t+5$ At what time t (in seconds) is the velocity of the particle zero ?

$$x(t)=t^2-6t+5$$
$$v(t)=2t-6=0 \quad \leftarrow \text{velocity of the particle zero}$$
$$2t=6$$
$$\boxed{t=3}$$

285. *Answer is C.*

A particle moves along a line according to the distance function $s(t) = 2t^3 - 21t^2 + 60t + 13$ During the time interval from $t = 1$ to $t = 12$, how many times does the paticle reverse its direction of movement ?

$$s(t) = 2t^3 - 21t^2 + 60t + 13$$
$$v(t) = 6t^2 - 42t + 60 = 0$$
$$t^2 - 7t + 10 = 0$$
$$(t-2)(t-5) = 0$$
$t = 2$ | $t = 5$ ← **Twice** in interval from $t = 1$ to $t = 12$

286. *Answer is E.*

Difficulty = **0.67**

A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 5$ At what time t is the particle at rest ?

$$x'(t) = 6t^2 - 42t + 72$$
$x'(t) = 6t^2 - 42t + 72 = 0$ ← at rest
$$t^2 - 7t + 12 = 0$$
$$(t-3)(t-4) = 0$$
$\boxed{t = 3}$ | $\boxed{t = 4}$

287. *Answer is B.*

The position of a particle moving along a straight line at any time t is given by $s(t) = t^2 + 4t + 4$ What is the acceleration of the particle when $t = 4$

$$s(t) = t^2 + 4t + 4$$
$$v(t) = s'(t) = 2t + 4$$
$$a(t) = s''(t) = 2$$
$$a(4) = s''(4) = \boxed{2}$$

288. *Answer is C.*

A particle moves along the x-axis so that at any time t its position is given by $x(t) = te^{-2t}$ For what values of t is the particle at rest ?

$$x(t) = te^{-2t}$$
$$v(t) = t(-2e^{-2t}) + e^{-2t} = 0$$
$$e^{-2t}(-2t + 1) = 0$$
$e^{-2t} \neq 0$ | $-2t + 1 = 0$, $\boxed{\frac{1}{2} = t}$ ← particle at rest when $t = \frac{1}{2}$

289. *Answer is C.*

Difficulty $= \mathbf{0.74}$

A particle moves along the x-axis so that at any time $t \ge 0$ its position is given by $x(t) = t^3 - 3t^2 - 9t + 1$ For what values of t is the particle at rest

$$x(t) = t^3 - 3t^2 - 9t + 1$$
$$v(t) = 3t^2 - 6t - 9 = 0 \quad \leftarrow \text{at rest}$$
$$t^2 - 2t - 3 = 0$$
$$(t-3)(t+1) = 0$$
$$\boxed{t = 3} \quad \Big| \quad t = -1$$

290. *Answer is C.*

A particle starts at time $t = 0$ and moves along a number line so that its position, at time $t \ge 0$ is given by $x(t) = (t-2)^3(t-6)$ The particle is $\underbrace{\textit{moving to the right}}_{x(t)>0}$ for

$$x'(t) = (t-2)^3 + (t-6)3(t-2)^2$$
$$x'(t) = (t-2)^2\left[(t-2) + 3(t-6)\right]$$
$$x'(t) = (t-2)^2\left[t - 2 + 3t - 18\right]$$
$$x'(t) = (t-2)^2\left[4t - 20\right]$$
$$x'(t) = 4(t-2)^2(t-5) = 0$$

critical $\rightarrow$ $t = 2$ | $t = 5$

$cn = 2$ $\quad$ $cn = 5$

interval	$0 < t < 2$	$2 < t < 5$	$t > 5$
$x'(t)$	$x'(1) = -16$	$x'(3) = -8$	$x'(6) = 64$
direction	*left*	*left*	*right*

291. *Answer is C.*

The formula $x(t) = \ln t + \frac{t^2}{18} + 1$ gives the position of an object moving along the x-axis during the time interval $1 \le t \le 5$ At the instant when the acceleration of the object is zero, the velocity is

$$x'(t) = v(t) = \frac{1}{t} + \frac{t}{9}$$
$$v'(t) = a(t) = \frac{-1}{t^2} + \frac{1}{9} = 0$$
$$\frac{1}{9} = \frac{1}{t^2}$$
$$t^2 = 9$$
$$1 \le t \le 5 \quad \rightarrow \quad t = 3$$

$$v(t) = \frac{1}{t} + \frac{t}{9}$$
$$v(3) = \frac{1}{3} + \frac{3}{9} = \boxed{\frac{2}{3}}$$

292. *Answer is A.*

Which of the following must be true about a particle that starts at $t=0$ and moves along a number line if its position at time t is given by $s(t)=(t-2)^3(t-6)$

I. The particle is moving to the right for $t>5$

II. The particle is at rest at $t=2$ and $t=6$

III. The particle changes direction at $t=2$

$$s(t)=(t-2)^3(t-6)$$
$$s'(t)=(t-2)^3+(t-6)3(t-2)^2$$
$$=(t-2)^2\left[(t-2)+3(t-6)\right]$$
$$v(t)=(t-2)^2\left[t-2+3t-18\right]$$
$$=(t-2)^2\left[4t-20\right]=4(t-2)^2\left[t-5\right]$$

293. *Answer is D.*

A particle starts at time $t=0$ and moves along a number line so that its position, at time $t\geq 0$, is given by $x(t)=(t-2)(t-6)^3$ The particle is moving to the left for

$$x'(t)=(t-2)3(t-6)^2+(t-6)^3$$
$$x'(t)=(t-6)^2\left[3(t-2)+(t-6)\right]$$
$$x'(t)=(t-6)^2\left[3t-6+t-6\right]$$
$$x'(t)=(t-6)^2\left[4t-12\right]$$
$$x'(t)=4(t-6)^2\left[t-3\right]$$

Sketch graph

294. *Answer is E.*

The position function of a moving particle on the x-axis is given as $s(t)=t^3+t^2-8t$ for $0\leq t\leq 10$ For what values of t is the particle $\underbrace{\text{moving to the right}}_{v(t)>0}$?

$$s(t)=t^3+t^2-8t$$
$$v(t)=3t^2+2t-8$$
$$v(t)=(t+2)(3t-4)$$

$t=-2$	$t=\frac{4}{3}$

If $t>\frac{4}{3}$ then the velocity is ***positive*** and the particle is moving to the ***right*** !

295. *Answer is B.*

A particle is moving along the x-axis. Its position at time $t > 0$ is e^{2-t} What is its acceleration when $t = 2$

$$s(t) = e^{2-t}$$
$$v(t) = e^{2-t}(-1) = -e^{2-t}$$
$$a(t) = e^{2-t}$$
$$a(2) = e^{2-2} = e^0 = \boxed{1}$$

296. *Answer is C.*

297. A particle moves along the x-axis so that its position at time t is given by $x(t) = 4t^3 - 33t^2 + 30t + 12$, where t is measured in seconds and x is measured in meters.
a) Determine the velocity, in m/s, of the particle at time $t = 2$ seconds
b) Determine the time(s), in seconds, when the particle is stationary

a) $x(t) = 4t^3 - 33t^2 + 30t + 12$
$$v(t) = 12t^2 - 66t + 30$$
$$v(2) = 12(2)^2 - 66(2) + 30 = \boxed{-54\ m/\sec}$$

b) $v(t) = 12t^2 - 66t + 30 = 0$ ← particle is stationary
$$2t^2 - 11t + 5 = 0$$
$$(2t-1)(t-5) = 0$$
$$\boxed{t = \tfrac{1}{2}} \quad \boxed{t = 5}$$

298. A particle moves along the x-axis such that its distance from the origin is given by $x(t) = 2t^2 + 60t$ where x is in centimeters and t is in seconds. When the particle's velocity is $72\ cm/\sec$, determine its distance $x(t)$ from the origin.

$$x(t) = 2t^2 + 60t$$
$$v(t) = 4t + 60 = 72$$
$$4t = 12$$
$$t = 3$$

$$x(t) = 2t^2 + 60t$$
$$x(3) = 2(3)^2 + 60(3) = 18 + 180 = 198$$

Distance $x(t)$ from the origin when its velocity is $72\ cm/\sec$ is **198** cm to the ***right*** of the origin.

299. A particle moves along the x-axis so that its position at time t is $x(t) = 4t^3 - 21t^2 + 30t$ where t is measured in seconds, and x is measured in meters.

a) Determine the time(s) when the particle is stopped.

b) Determine when the particle is moving to the left

a) $x(t) = 4t^3 - 21t^2 + 30t$

$v(t) = 12t^2 - 42t + 30 = 0$ ← particle stopped

$2t^2 - 7t + 5 = 0$

$(2t-5)(t-1) = 0$

$\boxed{t = \frac{5}{2}}$ | $\boxed{t = 1}$

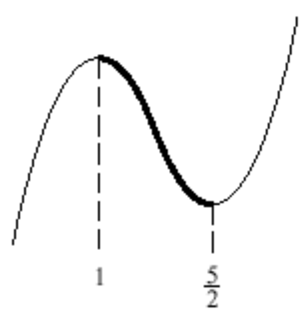
b) $v(t) = 12t^2 - 42t + 30$ ← parabola opening up with zero's of **1** and **2.5**

$12t^2 - 42t + 30 < 0$ ← $v(t)$ negative (moving left) $\boxed{1 < t < \frac{5}{2}}$

Solution:

Consider graph of position function x

$x = 4t^3 - 21t^2 + 30t$

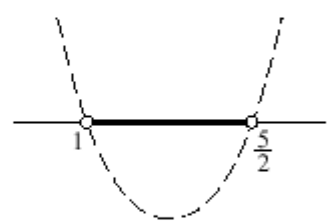


decreasing ⇒ moving left

or

Consider sign of x'

$x' = 6(2t-5)(t-1) < 0$ ← $\frac{1}{2}$ **mark**



$x' < 0 \Rightarrow$ moving left

∴ particle is moving left when

$1 < t < \frac{5}{2}$ ← $\frac{1}{2}$ **mark**

300\. A particle moves along the x-axis so that its position at time t is $x(t) = 2t^3 - 5t^2 - 4t + 3$ (x in cm and t in seconds.)

a) At what time(s) is the particle stationary ?

b) At what time(s) is the particle moving to the left ?

a) $x(t) = 2t^3 - 5t^2 - 4t + 3$

$v(t) = x'(t) = 6t^2 - 10t - 4$

$v(t) = 6t^2 - 10t - 4 = 0$ ← stationary

$3t^2 - 5t - 2 = 0$

$(3t + 1)(t - 2) = 0$

$\boxed{t = -\frac{1}{3}}$ | $\boxed{t = 2}$

$6t^2 - 10t - 4 < 0$ ← moving left

Sketch parabola opening up with zeros $t = -\frac{1}{3},\ 2$ and parabola is negative when

$\boxed{-\frac{1}{3} < t < 2}$

301\. A particle moves along the x-axis so that its position at time t is $x(t) = 2t^3 - 9t^2 + 12t$ (x in cm and t in seconds)

a) Determine the time(s) when the particle is stopped

b) Determine the velocity of the particle at time $t = 3$ seconds

a) $f(x) = x^3 - 3x + 5$

$f'(x) = 3x^2 - 3$

$f'(2) = 3(2)^2 - 3 = 9$ ← slope

$f(2) = 2^3 - 3(2) + 5 = 7$

point of tangency $(2,\ 7)$

Equation of the tangent line

$$\text{Slope} = \frac{rise}{run} = \frac{9}{1} = \frac{y-7}{x-2}$$

$y - 7 = 9x - 18$

$\boxed{y = 9x - 11}$

b) $f'(x) = 3x^2 - 3 = 0$ ← slope $= 0$

$3x^2 = 3$

$x^2 = 1$

$\boxed{x = \pm 1}$

302. A particle moves along the x-axis so that its position at time t is $x(t) = 2t^3 - 9t^2 + 12t$
(x in cm and t in seconds)

a) Determine the time(s) when the particle is stopped

b) Determine the velocity of the particle at time $t = 3$ seconds

a)

$$x(t) = 2t^3 - 9t^2 + 12t$$

$$v(t) = x'(t) = 6t^2 - 18t + 12 \quad \rightarrow$$

$$v(t) = 6t^2 - 18t + 12 = 0 \leftarrow \text{stopped}$$

$$t^2 - 3t + 2 = 0$$

$$(t-1)(t-2) = 0$$

$$\boxed{t = 1} \quad \boxed{t = 2}$$

b) Velocity of the particle at $t = 3$ seconds

$$v(t) = 6t^2 - 18t + 12$$

$$v(3) = 6(3)^2 - 18(3) + 12$$

$$\boxed{v(3) = 12\ cm/\sec}$$

303. A particle moves along the x-axis so that its position at time t is $x(t) = 4t^3 - 21t^2 + 18t + 3$
where t is measured in seconds and x is measured in meters.

a) Determine an equation for the velocity function.

b) Determine the velocity at time $t = 2$

c) Determine the time(s) when the particle is stationary.

a) $x(t) = 4t^3 - 21t^2 + 18t + 3$

$$\boxed{v(t) = 12t^2 - 42t + 18} \leftarrow \text{first derivative}$$

b) $v(2) = 12t^2 - 42t + 18 = 12(2)^2 - 42(2) + 18 = \boxed{18\ m/s}$

c) $v(t) = 12t^2 - 42t + 18 = 0 \leftarrow$ particle is stationary

$$2t^2 - 7t + 3 = 0$$

$$(2t-1)(t-3) = 0$$

$$\boxed{t = \tfrac{1}{2}} \quad \boxed{t = 3}$$

304. A particle moves along the x-axis in such a way that its position at time t is given by

$x(t) = 3t^4 - 16t^3 + 24t^2$ for $-5 \le t \le 5$

a) Determine the velocity and acceleration of the particle at time t

b) At what values of t is the particle at rest ?

c) At what values of t does the particle change direction ?

d) What is the velocity when the acceleration is first zero ?

(a) $v = \frac{dx}{dt} = 12t^3 - 48t^2 + 48t = 12t(t^2 - 4t + 4) = 12t(t-2)^2$

$a = \frac{dv}{dt} = 36t^2 - 96t + 48 = 12(3t^2 - 8t + 4) = 12(3t-2)(t-2)$

(b) The particle is at rest when $v = 0$. This occurs when $t = 0$ and $t = 2$.

(c) The particle changes direction at $t = 0$ only.

(d) $a = 0$ when $t = \frac{2}{3}$ and $t = 2$. The acceleration is first zero at $t = \frac{2}{3}$.

$v\left(\frac{2}{3}\right) = 12\left(\frac{2}{3}\right)\left(-\frac{4}{3}\right)^2 = \frac{128}{9}$

305. A particle moves along the x-axis in such a way that its position at time t for $t \geq 0$ is given by $x(t) = \frac{1}{3}t^3 - 3t^2 + 8t$

$a)$ Show that at time $t = 0$, the particle is moving to the right.

$b)$ Find all values of t for which the particle is moving to the left.

$c)$ What is the position of the particle at time $t = 3$

$d)$ When $t = 3$, what is the total distance the particle has traveled ?

(a) $v = \frac{dx}{dt} = t^2 - 6t + 8$

$v(0) = 8 > 0$ and so the particle is moving to the right at $t = 0$.

(b) The particle is moving to the left when $v(t) = t^2 - 6t + 8 = (t-4)(t-2) < 0$.
Therefore the particle moves to the left for $2 < t < 4$.

(c) At time $t = 3$, $x = \frac{1}{3}(3)^3 - 3(3)^2 + 8(3) = 6$.

(d) The particle changes direction at $t = 2$.

$x(0) = 0$

$x(2) = \frac{1}{3}(2)^3 - 3(2)^2 + 8(2) = \frac{20}{3}$

$x(3) = 6$

Distance $= (x(2) - x(0)) + (x(2) - x(3)) = \frac{20}{3} + \frac{2}{3} = \frac{22}{3}$